

## Unit 2

# LOGARITHMS

### Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

- ◆ Express a number in standard form of scientific notation and vice versa.
- ◆ Define logarithm of a real number to a base  $a$  as a power to which  $a$  must be raised to give the number (i.e.,  $a^x = y \Leftrightarrow \log_a y = x$ ,  $a > 0$ ,  $y > 0$  and  $a \neq 1$ ).
- ◆ Define a common logarithm, characteristic and mantissa of log of a number.
- ◆ Use tables to find the log of a number.
- ◆ Give concept of antilog and use tables to find the antilog of a number.
- ◆ Use of calculator to find the log and antilog of a number.
- ◆ Differentiate between common and natural logarithms.
- ◆ Write,  $\log_{10} y = \log y$  or simply  $\log y$  and  $\log_e(y)$  as  $\ln y$ ,
  - (i)  $\log_{10} y = x \Leftrightarrow y = 10^x$ ,
  - (ii)  $\ln y = x \Leftrightarrow y = e^x$ .
- ◆ Prove the following laws of logarithms.
  - (i)  $\log_a(mn) = \log_a m + \log_a n$
  - (ii)  $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$ ,
  - (iii)  $\log_a m^n = n \log_a m$ ,
  - (iv)  $\log_a m, \log_m n = \log_a n$ .
- ◆ Apply these laws of logarithms to convert lengthy processes of multiplication, division and exponentiation into easier processes of addition and subtraction etc.

## Introduction :

Logarithms were introduced by the great muslim mathematician **Abu Muhammad Musa Al-Khawarizmi**. Letter on in the seventeen century and **John Napier** developed the concept of logarithm further and prepared tables for it. In these table the base " $e$ " was used.  $e$  is an irrational number whose approximate value is 2.71828... . In 1631, Professor **Henry Briggs** developed the tables with base " $10$ ".

By the use of logarithms the enormous labour of calculations is reduced and it is performed with great ease.

## 2.1 Scientific Notation

Scientific notation is special form to write very large or very small numbers conveniently.

### 2.1.1 Express a number in standard form of scientific notation and vice versa

In the world of science and technology; we deal with very large and small numbers and quantities, the distance from the earth to the sun is **150,000,000 km** approximately and weight of hydrogen atom is **0.000,000,000,000,000,000,000,001,7g**. The writing of such type of numbers in **ordinary notation**(Standard notation) is too difficult for everyone and it is time consuming. Scientists have developed a convenient method to write very small and very large numbers that is called **scientific notation**.

The above mentioned number in section 2.1.1 can be simply and written in scientific notation as:  $1.5 \times 10^8$  km and  $1.7 \times 10^{-27}$ g respectively.

The following examples will help to understand the scientific notation.

**Example 01**

Express the following numbers in scientific notation.

(i) 400900

(ii) 0.0000075

**Solution:**

(i) 400900

In given number, decimal is after the unit digit, so move decimal point up to five digits from right to left, and write as  $400900 = 4.009 \times 10^5$ , which is the required scientific notation.

(ii) 0.0000075

There are 7 digits after decimal point in the given decimal number. There is '7' first non-zero digit in it, so, we move decimal point up to 6 digits from left to right and write as  $0.0000075 = 7.5 \times 10^{-6}$ , which is the required scientific notation of the given number.

**Example 02** Write the following in ordinary notation

(i)  $2.76 \times 10^6$

(ii)  $5.24 \times 10^{-4}$

**Solution:**

(i)  $2.76 \times 10^6$

The power of 10 is 6, so we move decimal point from left to right up to six decimal places but there are 2 digits, so, we put 4 zeros from right side then the required ordinary form is  $2.76 \times 10^6 = 2760000$ , is the required ordinary notation.

**Solution:**

(ii)  $5.24 \times 10^{-4}$

There is negative 4<sup>th</sup> power of 10, so we move decimal point from right to left up to 4 decimal places but there is already one digit before decimal point, so, we put three zeros before the digit 5, and then we get the required notation.

Thus,  $5.24 \times 10^{-4} = 0.000524$  is the required notation.

## Exercise 2.1

1. Express each of the following numbers in scientific notation.  
(i) 9700 (ii) 4,980,000 (iii) 96,000,000  
(iv) 4169 (v) 84,000 (vi) 0.00643  
(vii) 0.00643 (viii) 0.0074 (ix) 0.0074 (x) 275,000
2. Express the following numbers in ordinary notation (Standard notation).  
(i)  $7 \times 10^4$  (ii)  $8.072 \times 10^{-10}$  (iii)  $6.018 \times 10^6$   
(iv)  $7.865 \times 10^8$  (v)  $2.05 \times 10^{-4}$  (vi)  $7.25 \times 10^{10}$   
(vii)  $4.502 \times 10^6$  (viii)  $2.865 \times 10^{-8}$  (ix)  $3.056 \times 10^6$

## 2.2 Logarithms

Logarithms is a method of reducing complicated problems of multiplication/ division into simple form.

**2.2.1 Define logarithm of a real number to a base  $a$  as a power to which  $a$  must be raised to give the number**  
**(i.e.  $a^x = y \Leftrightarrow \log_a y = x$ ,  $a > 0$ ,  $y > 0$  and  $a \neq 1$ )**

If  $a^x = y$ , then  $x$  is called the logarithm of  $y$  to the base ' $a$ ' and is written as  $\log_a y = x$ , where,  $a > 0$ ,  $y > 0$  and  $a \neq 1$ .

Thus,  $a^x = y \Leftrightarrow \log_a y = x$ .

It is noted that  $a^x = y$  is an exponential form and  $\log_a y = x$  is a logarithmic form.

The following examples will help to understand the concept of exponential and logarithmic forms.

**Example 01** Write  $2^{-4} = \frac{1}{16}$  in logarithmic form.

**Solution:**  $2^{-4} = \frac{1}{16} \Rightarrow \log_2 \frac{1}{16} = -4$

**Example 02** Write  $\log_3 81 = 4$  in exponential form.

**Solution:**  $\log_3 81 = 4 \Rightarrow 3^4 = 81$

**Example 03** Find the value of  $\log_4 2$ .

**Solution:** Let  $x = \log_4 2$

Exponential form is

$$\therefore 4^x = 2$$

$$\Rightarrow (2^2)^x = 2^1$$

Equating exponents on both the sides, we have

$$2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

**Example 04** Find the value of  $x$ . If  $\log_x 8 = \frac{3}{2}$

**Solution:**  $\log_x 8 = \frac{3}{2}$

Exponential form is

$$\Rightarrow (x)^{\frac{3}{2}} = 8$$

$$\Rightarrow (x)^{\frac{3}{2}} = 2^3$$

Taking power  $\frac{2}{3}$  on both sides

$$\Rightarrow (x^{\frac{3}{2}})^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}$$

$$\Rightarrow x = 2^2$$

$$\Rightarrow x = 4$$

**Example 05** Find the value of  $x$  if  $\log_{64} x = \frac{-2}{3}$

**Solution:**

Exponential form is

$$(64)^{\frac{-2}{3}} = x \Rightarrow (4^3)^{\frac{-2}{3}} = x$$

$$4^{-2} = x \Rightarrow \frac{1}{4^2} = x$$

$$\frac{1}{16} = x \Rightarrow x = \frac{1}{16}$$

### Exercise 2.2

1. Write the following in logarithmic form.

(i)  $7^3 = 343$

(ii)  $3^{-4} = \frac{1}{81}$

(iii)  $10^{-3} = 0.001$

(iv)  $\sqrt[3]{m^2} = 1$

2. Write the following in exponential form.

(i)  $\log_{27} 81 = \frac{4}{3}$

(ii)  $\log_2 \frac{1}{8} = -3$

(iii)  $\log_{10} 1 = 0$

(iv)  $\log_{10}(0.01) = -2$

3. Find the unknown in the following.

(i)  $\log_{32} x = \frac{1}{2}$

(ii)  $\log_a 3 = \frac{1}{2}$

(iii)  $\log_{\sqrt{5}} 25 = y$

(iv)  $\log_4 x = \frac{3}{2}$

(v)  $\log_{10} 100 = y$

(vi)  $\log_a 64 = 3$

(vii)  $\log_a 1 = 0$

(viii)  $\log_{55} 55 = y$

(xi)  $\log_{64} 8 = \frac{x}{2}$

### 2.2.2 Define a common logarithm, characteristic and mantissa of log of a number

#### Common logarithms

Common logarithms have base 10, it is also named as artificial logarithms or Briggs logarithm.

Common log written as  $\log_{10} y$  or simply  $\log y$ .

$$\text{If } \log y = x \Leftrightarrow y = 10^x$$

#### Characteristic and Mantissa of log of a number

Logarithms of a number consist of two parts. One part is integer and the second part is decimal fraction. Integral part is called Characteristic and decimal part is called Mantissa.

It is noted that characteristic of logarithm may be positive or negative, but mantissa is always positive, for this we use logarithmic tables.

In scientific notation, the power of 10 is called characteristic and mantissa is found by using log table which would be discussed later.

**Example 01** Find the characteristic of the following numbers.

0.765, 0.04, 0.004567, 2.134, 23.56 and 3456.

| Nos. | Number   | Scientific Notation     | Characteristic  |
|------|----------|-------------------------|-----------------|
| 1    | 0.765    | $7.65 \times 10^{-1}$   | -1 or $\bar{1}$ |
| 2    | 0.04     | $4.0 \times 10^{-2}$    | -2 or $\bar{2}$ |
| 3    | 0.004567 | $4.5467 \times 10^{-3}$ | -3 or $\bar{3}$ |
| 4    | 2.134    | $2.134 \times 10^0$     | 0               |
| 5    | 23.56    | $2.356 \times 10^1$     | 1               |
| 6    | 3456     | $3.456 \times 10^3$     | 3               |

**We observe that**

- Characteristic of logarithm of a number greater than 1 is always non-negative integer.
- Characteristic of logarithm of a number less than 1 is always negative.

**Mantissa:** The mantissa is found by using logarithmic tables. These tables are constructed to obtain the logarithms up to 7-decimal digits. But at this level, for practical purposes, a **four figure logarithmic table** is useful for accuracy to find the logarithm of a number.

**Do you understand?**

Negative characteristic of logarithm is written as:  $\bar{3}, \bar{2}$  or  $\bar{1}$  instead of -3, -2 or -1 respectively. When mantissa becomes negative, then, we must change it into +ve number, because mantissa is always positive.

### 2.2.3 Use table to find the log of a number.

The following will help us to find the logarithm by using table.

**Example 01** Find the Mantissa of the following logarithmic numbers

- (i)  $\log(43.254)$     (ii)  $\log(0.002347)$ .

**Solution (i):**  $\log(43.254)$

**Step 1:** Ignore decimal and round off the number up to 4 digits. Then we have number is 4325.

- Step 2:** Locate the row corresponding to 43 in log table.
- Step 3:** Proceed horizontally to third digit i.e. 2. The number at the intersection of 43<sup>rd</sup> row and 2<sup>nd</sup> column is 6355.
- Step 4:** Again, proceed horizontally till mean difference column till 4<sup>th</sup> digit i.e.5, we get number 5 at the intersection of 5<sup>th</sup> column and 43<sup>rd</sup> row.
- Step 5:** Add 5 in 6355; we will get 0.6360 as the mantissa of  $\log (43.25)$ .

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Logarithm Table

|    |      |      |      |      |      |      |      |      |      |      | Mean Differences |   |    |    |    |    |    |    |    |
|----|------|------|------|------|------|------|------|------|------|------|------------------|---|----|----|----|----|----|----|----|
|    | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 10 | 0000 | 0043 | 0086 | 0128 | 0171 | 0212 | 0253 | 0294 | 0334 | 0374 | 4                | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4                | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3                | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3                | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3                | 6 | 9  | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3                | 6 | 8  | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3                | 5 | 8  | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2                | 5 | 7  | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2                | 5 | 7  | 9  | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2                | 4 | 7  | 9  | 11 | 13 | 16 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2                | 4 | 6  | 8  | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2                | 4 | 6  | 8  | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2                | 4 | 6  | 8  | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2                | 4 | 6  | 7  | 9  | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2                | 4 | 5  | 7  | 9  | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2                | 3 | 5  | 7  | 9  | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2                | 3 | 5  | 7  | 8  | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2                | 3 | 5  | 6  | 8  | 9  | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2                | 3 | 5  | 6  | 8  | 9  | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1                | 3 | 4  | 6  | 7  | 9  | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1                | 3 | 4  | 6  | 7  | 9  | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1                | 3 | 4  | 6  | 7  | 8  | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1                | 3 | 4  | 5  | 7  | 8  | 9  | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1                | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1                | 3 | 4  | 5  | 6  | 8  | 9  | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1                | 2 | 4  | 5  | 6  | 7  | 9  | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1                | 2 | 4  | 5  | 6  | 7  | 8  | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1                | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1                | 2 | 3  | 5  | 6  | 7  | 8  | 9  | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1                | 2 | 3  | 4  | 6  | 7  | 8  | 9  | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1                | 2 | 3  | 4  | 5  | 6  | 8  | 9  | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1                | 2 | 3  | 4  | 5  | 6  | 7  | 7  | 8  |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1                | 2 | 3  | 4  | 5  | 5  | 6  | 7  | 8  |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1                | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1                | 2 | 3  | 4  | 4  | 5  | 6  | 7  | 8  |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1                | 2 | 3  | 3  | 4  | 5  | 6  | 7  | 8  |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1                | 2 | 3  | 3  | 4  | 5  | 6  | 7  | 8  |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 7  | 7  |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1                | 2 | 2  | 3  | 4  | 5  | 6  | 6  | 7  |

# Logarithm Table

| Logarithm Table |      |      |      |      |      |      |      |      |      |      |                  |   |   |   |   |   |   |   |   |
|-----------------|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|---|---|---|---|---|
|                 |      |      |      |      |      |      |      |      |      |      | Mean Differences |   |   |   |   |   |   |   |   |
| 54              | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1                | 2 | 2 | 3 | 4 | 5 | 6 | 7 |   |
| 55              | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56              | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57              | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1                | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58              | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59              | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60              | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 61              | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1                | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 62              | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 63              | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 64              | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 65              | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 66              | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 67              | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1                | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 68              | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1                | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69              | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1                | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 70              | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 71              | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 72              | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 73              | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1                | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 74              | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 75              | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 76              | 8808 | 8814 | 882  | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 77              | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78              | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79              | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80              | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81              | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82              | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83              | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84              | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85              | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86              | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1                | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87              | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88              | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89              | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 90              | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 91              | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 92              | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93              | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94              | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 95              | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96              | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97              | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98              | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99              | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 0                | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |

**Solution (ii):**

For  $\log(0.002347)$  we ignore decimal and zeros before the digit 2 and see in the log table at the intersection of row 23 and column 4 is 3692. Add mean difference column corresponding to the digit 7 is 13 in 3692, we get 3705. The required Mantissa is 0.3705.

So, Mantissa of  $\log(0.002347)$  is 0.3705.

**Example 02** Find the log of the following numbers:

- (i) 278.27      (ii) 0.07058

**Solution:** Let  $x = 278.27$

Taking log both sides,

$$\therefore \text{Log } x = \log(278.27),$$

**Step 1:** Round off the number up to 4 Digits i.e. 278.3.

**Step 2:**  $278.3 = 2.783 \times 10^2$

So, characteristics is = 2.

**Step 3: For finding** mantissa ignore decimal point, we get 2783.

By using log table, we get mantissa of  $\log(2783) = 0.4445$ .

**Step 4:** Add characteristic and mantissa.

We get,  $\log x = 2.4445$ .

**Solution(ii):** Let  $x = 0.07058$

**Step 1:** No need to round off here. Four digits are 7058.

**Step 2:** Convert the given number into scientific notation

$$\text{i.e., } 7058 \times 10^{-2}.$$

so, characteristic = -2 or  $\bar{2}$ .

**Step 3:** Ignore decimal point and find mantissa of 7058.

By using log table, mantissa of 7058 is 0.8487.

**Step 4:** Add characteristic and mantissa.

We get,  $\text{Log } x = \log(0.07058) = \bar{2}.8487$ .

**Remember that:**

The logarithms of numbers of the same sequence of significant digits have the same mantissa.

For example, the numbers 0.004576, 0.04576, 0.4576, 45.76 etc. have the same mantissa.

### Exercise 2.3

- Find the characteristics and mantissa of the following Logarithm.
 

|            |             |                |
|------------|-------------|----------------|
| (i) 8      | (ii) 5054   | (iii) 9.992    |
| (vi) 765.3 | (v) 0.00329 | (vi) 0.0000300 |
- Find the logarithms of the following numbers.
 

|            |             |               |
|------------|-------------|---------------|
| (i) 9      | (ii) 55.56  | (iii) 29.592  |
| (vi) 405.3 | (v) 0.00469 | (vi) 0.000076 |
- If  $\log 31.09 = 1.4926$ , find the value of the following without using log table.
 

|                    |                    |                       |
|--------------------|--------------------|-----------------------|
| (i) $\log 3.109$   | (ii) $\log 310.9$  | (iii) $\log 0.003109$ |
| (iv) $\log 3109.0$ | (v) $\log 310.942$ | (vi) $\log 310926$    |

#### 2.2.4 Give concept of antilog and use of tables to find the antilog of a number.

If  $\log x = y$ , then  $x$  is called anti log of  $y$ . It is written as  $x = \text{antilog } y$ . If the common logarithm of a number  $x$  is  $y$ , i.e. if  $\log x = y$ , then we find the number  $x$  by using the tables of antilogarithms and with the help of following two rules.

**Rule 1.** If the characteristic is positive  $n$ , then antilog must have  $n+1$  digits in integral part.

**Rule 2.** If the characteristic is negative  $n$ , then antilog must have  $n-1$  zeros immediately following the decimal point.

The procedure of finding antilogarithms is explained by the following examples

| Antilogarithm Table |      |      |      |      |      |      |      |      |      |      |                  |   |   |   |   |   |   |   |
|---------------------|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|---|---|---|---|
|                     |      |      |      |      |      |      |      |      |      |      | Mean Differences |   |   |   |   |   |   |   |
|                     | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 1                | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| .00                 | 1002 | 1005 | 1007 | 1009 | 1012 | 1014 | 1016 | 1019 | 1021 | 0000 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 |
| .01                 | 1023 | 1026 | 1028 | 1030 | 1033 | 1035 | 1038 | 1040 | 1042 | 1045 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 |
| .02                 | 1047 | 1050 | 1052 | 1054 | 1057 | 1059 | 1062 | 1064 | 1067 | 1069 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 |
| .03                 | 1072 | 1074 | 1076 | 1079 | 1081 | 1084 | 1086 | 1089 | 1091 | 1094 | 0                | 0 | 1 | 1 | 1 | 1 | 2 | 2 |
| .04                 | 1096 | 1099 | 1102 | 1104 | 1107 | 1109 | 1112 | 1114 | 1117 | 1119 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .05                 | 1122 | 1125 | 1127 | 1130 | 1132 | 1135 | 1138 | 1140 | 1143 | 1146 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .06                 | 1148 | 1151 | 1153 | 1156 | 1159 | 1161 | 1164 | 1167 | 1169 | 1172 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 |
| .07                 | 1175 | 1178 | 1180 | 1183 | 1186 | 1189 | 1191 | 1194 | 1197 | 1199 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 |

Antilogarithm Table

|     |      |      |      |      |      |      |      |      |      |      | Mean Differences |   |   |   |   |   |   |   |   |  |
|-----|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|---|---|---|---|---|--|
| .08 | 1202 | 1205 | 1208 | 1211 | 1213 | 1216 | 1219 | 1222 | 1225 | 1227 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |  |
| .09 | 1230 | 1233 | 1236 | 1239 | 1242 | 1245 | 1247 | 1250 | 1253 | 1256 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |  |
| .10 | 1259 | 1262 | 1265 | 1268 | 1271 | 1274 | 1276 | 1279 | 1282 | 1285 | 0                | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 3 |  |
| .11 | 1288 | 1291 | 1294 | 1297 | 1300 | 1303 | 1306 | 1309 | 1312 | 1315 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .12 | 1318 | 1321 | 1324 | 1327 | 1330 | 1334 | 1337 | 1340 | 1343 | 1346 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .13 | 1349 | 1352 | 1355 | 1358 | 1361 | 1365 | 1368 | 1371 | 1374 | 1377 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .14 | 1380 | 1384 | 1387 | 1390 | 1393 | 1396 | 1400 | 1403 | 1406 | 1409 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .15 | 1413 | 1416 | 1419 | 1422 | 1426 | 1429 | 1432 | 1435 | 1439 | 1442 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .16 | 1445 | 1449 | 1452 | 1455 | 1459 | 1462 | 1466 | 1469 | 1472 | 1476 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .17 | 1479 | 1483 | 1486 | 1489 | 1493 | 1496 | 1500 | 1503 | 1507 | 1510 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .18 | 1514 | 1517 | 1521 | 1524 | 1528 | 1531 | 1535 | 1538 | 1542 | 1545 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .19 | 1549 | 1552 | 1556 | 1560 | 1563 | 1567 | 1570 | 1574 | 1578 | 1581 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .20 | 1585 | 1589 | 1592 | 1596 | 1600 | 1603 | 1607 | 1611 | 1614 | 1618 | 0                | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 3 |  |
| .21 | 1622 | 1626 | 1629 | 1633 | 1637 | 1641 | 1644 | 1648 | 1652 | 1656 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .22 | 1660 | 1663 | 1667 | 1671 | 1675 | 1679 | 1683 | 1687 | 1690 | 1694 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .23 | 1698 | 1702 | 1706 | 1710 | 1714 | 1718 | 1722 | 1726 | 1730 | 1734 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .24 | 1738 | 1742 | 1746 | 1750 | 1754 | 1758 | 1762 | 1766 | 1770 | 1774 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .25 | 1778 | 1782 | 1786 | 1791 | 1795 | 1799 | 1803 | 1807 | 1811 | 1816 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .26 | 1820 | 1821 | 1828 | 1832 | 1837 | 1841 | 1845 | 1849 | 1854 | 1858 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .27 | 1862 | 1866 | 1871 | 1875 | 1879 | 1884 | 1888 | 1892 | 1897 | 1901 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .28 | 1905 | 1910 | 1914 | 1919 | 1923 | 1928 | 1932 | 1936 | 1941 | 1945 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .29 | 1950 | 1954 | 1959 | 1963 | 1968 | 1972 | 1977 | 1982 | 1986 | 1991 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .30 | 1995 | 2000 | 2004 | 2009 | 2014 | 2018 | 2023 | 2028 | 2032 | 2037 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .31 | 2042 | 2046 | 2051 | 2056 | 2061 | 2065 | 2070 | 2075 | 2080 | 2084 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .32 | 2089 | 2094 | 2099 | 2104 | 2109 | 2113 | 2118 | 2123 | 2128 | 2133 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .33 | 2138 | 2143 | 2148 | 2153 | 2158 | 2163 | 2168 | 2173 | 2178 | 2183 | 0                | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .34 | 2188 | 2193 | 2198 | 2203 | 2208 | 2213 | 2218 | 2223 | 2228 | 2234 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .35 | 2239 | 2244 | 2249 | 2254 | 2259 | 2265 | 2270 | 2275 | 2280 | 2286 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .36 | 2291 | 2296 | 2301 | 2307 | 2312 | 2317 | 2323 | 2328 | 2333 | 2339 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .37 | 2344 | 2350 | 2355 | 2360 | 2366 | 2371 | 2377 | 2382 | 2388 | 2393 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .38 | 2399 | 2404 | 2410 | 2415 | 2421 | 2427 | 2432 | 2438 | 2443 | 2449 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .39 | 2455 | 2460 | 2466 | 2472 | 2477 | 2483 | 2489 | 2495 | 2500 | 2506 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .40 | 2512 | 2518 | 2523 | 2529 | 2535 | 2541 | 2547 | 2553 | 2559 | 2564 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .41 | 2570 | 2576 | 2582 | 2588 | 2594 | 2600 | 2606 | 2612 | 2618 | 2624 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .42 | 2630 | 2636 | 2642 | 2649 | 2655 | 2661 | 2667 | 2673 | 2679 | 2685 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .43 | 2692 | 2698 | 2704 | 2710 | 2716 | 2723 | 2729 | 2735 | 2742 | 2748 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |
| .44 | 2754 | 2761 | 2767 | 2773 | 2780 | 2786 | 2793 | 2799 | 2805 | 2812 | 1                | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |  |



| Antilogarithm Table |      |      |      |      |      |      |      |      |      |      |                  |   |   |   |   |   |    |    |    |  |
|---------------------|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|---|---|----|----|----|--|
|                     |      |      |      |      |      |      |      |      |      |      | Mean Differences |   |   |   |   |   |    |    |    |  |
| .45                 | 2818 | 2825 | 2831 | 2838 | 2844 | 2851 | 2858 | 2864 | 2871 | 2877 | 1                | 1 | 2 | 3 | 3 | 4 | 5  | 5  | 6  |  |
| .46                 | 2884 | 2891 | 2897 | 2904 | 2911 | 2917 | 2924 | 2931 | 2938 | 2944 | 1                | 1 | 2 | 3 | 3 | 4 | 5  | 5  | 6  |  |
| .47                 | 2951 | 2958 | 2965 | 2972 | 2970 | 2985 | 2992 | 2999 | 3006 | 3013 | 1                | 1 | 2 | 3 | 3 | 4 | 5  | 5  | 6  |  |
| .48                 | 3020 | 3027 | 3034 | 3041 | 3048 | 3055 | 3062 | 3069 | 3076 | 3083 | 1                | 1 | 2 | 3 | 4 | 4 | 5  | 6  | 6  |  |
| .49                 | 3090 | 3097 | 3105 | 3112 | 3119 | 3126 | 3133 | 3141 | 3148 | 3155 | 1                | 1 | 2 | 3 | 4 | 4 | 5  | 6  | 6  |  |
| .50                 | 3162 | 3170 | 3177 | 3184 | 3192 | 3199 | 3206 | 3214 | 3221 | 3228 | 1                | 1 | 2 | 2 | 4 | 4 | 5  | 6  | 7  |  |
| .51                 | 3236 | 3243 | 3251 | 3258 | 3266 | 3273 | 3281 | 3289 | 3296 | 3304 | 1                | 2 | 2 | 3 | 4 | 5 | 5  | 6  | 7  |  |
| .52                 | 3311 | 3319 | 3327 | 3334 | 3342 | 3350 | 3357 | 3365 | 3373 | 3381 | 1                | 2 | 2 | 3 | 4 | 5 | 5  | 6  | 7  |  |
| .53                 | 3388 | 3396 | 3404 | 3412 | 3420 | 3428 | 3436 | 3443 | 3451 | 3459 | 1                | 2 | 2 | 3 | 4 | 5 | 6  | 6  | 7  |  |
| .54                 | 3467 | 3475 | 3483 | 3491 | 3499 | 3508 | 3516 | 3524 | 3532 | 3540 | 1                | 2 | 2 | 3 | 4 | 5 | 6  | 6  | 7  |  |
| .55                 | 3548 | 3556 | 3565 | 3573 | 3581 | 3589 | 3597 | 3606 | 3614 | 3622 | 1                | 2 | 2 | 3 | 4 | 5 | 6  | 7  | 7  |  |
| .56                 | 3631 | 3639 | 3648 | 3656 | 3664 | 3673 | 3681 | 3690 | 3698 | 3707 | 1                | 2 | 3 | 3 | 4 | 5 | 6  | 7  | 8  |  |
| .57                 | 3715 | 3724 | 3733 | 3741 | 3750 | 3758 | 3767 | 3776 | 3784 | 3793 | 1                | 2 | 3 | 3 | 4 | 5 | 6  | 7  | 8  |  |
| .58                 | 3802 | 3811 | 3819 | 3828 | 3837 | 3846 | 3855 | 3864 | 3873 | 3882 | 1                | 2 | 3 | 4 | 4 | 5 | 6  | 7  | 8  |  |
| .59                 | 3890 | 3899 | 3908 | 3917 | 3926 | 3936 | 3945 | 3954 | 3963 | 3972 | 1                | 2 | 3 | 4 | 5 | 5 | 6  | 7  | 8  |  |
| .60                 | 3981 | 3990 | 3999 | 4009 | 4018 | 4027 | 4036 | 4046 | 4055 | 4064 | 1                | 2 | 3 | 4 | 5 | 6 | 6  | 7  | 8  |  |
| .61                 | 4074 | 4083 | 4093 | 4102 | 4111 | 4121 | 4130 | 4140 | 4150 | 4159 | 1                | 2 | 3 | 4 | 5 | 6 | 7  | 8  | 9  |  |
| .62                 | 4169 | 4178 | 4188 | 4198 | 4207 | 4217 | 4227 | 4236 | 4246 | 4256 | 1                | 2 | 3 | 4 | 5 | 6 | 7  | 8  | 9  |  |
| .63                 | 4266 | 4276 | 4285 | 4295 | 4305 | 4315 | 4325 | 4335 | 4345 | 4355 | 1                | 2 | 3 | 4 | 5 | 6 | 7  | 8  | 9  |  |
| .64                 | 4365 | 4374 | 4385 | 4395 | 4406 | 4416 | 4426 | 4436 | 4446 | 4457 | 1                | 2 | 3 | 4 | 5 | 6 | 7  | 8  | 9  |  |
| .65                 | 4467 | 4477 | 4487 | 4498 | 4508 | 4519 | 4529 | 4539 | 4550 | 4560 | 1                | 2 | 3 | 4 | 5 | 6 | 7  | 8  | 9  |  |
| .66                 | 4571 | 4581 | 4592 | 4603 | 4613 | 4624 | 4634 | 4645 | 4656 | 4667 | 1                | 2 | 3 | 4 | 5 | 6 | 7  | 9  | 10 |  |
| .67                 | 4677 | 4688 | 4699 | 4710 | 4721 | 4732 | 4742 | 4753 | 4764 | 4775 | 1                | 2 | 3 | 4 | 5 | 7 | 8  | 9  | 10 |  |
| .68                 | 4786 | 4797 | 4808 | 4819 | 4831 | 4842 | 4853 | 4864 | 4875 | 4887 | 1                | 2 | 3 | 4 | 6 | 7 | 8  | 9  | 10 |  |
| .69                 | 4898 | 4909 | 4920 | 4932 | 4943 | 4955 | 4966 | 4977 | 4989 | 5000 | 1                | 2 | 3 | 5 | 6 | 7 | 8  | 9  | 10 |  |
| .70                 | 5012 | 5023 | 5035 | 5047 | 5058 | 4070 | 5082 | 5093 | 5105 | 5117 | 1                | 2 | 4 | 5 | 6 | 7 | 8  | 9  | 11 |  |
| .71                 | 5129 | 5140 | 5152 | 5164 | 5176 | 5188 | 5200 | 5212 | 5224 | 5236 | 1                | 2 | 4 | 5 | 6 | 7 | 8  | 10 | 11 |  |
| .72                 | 5248 | 5260 | 5272 | 5284 | 5297 | 5309 | 5321 | 5333 | 5346 | 5358 | 1                | 2 | 4 | 5 | 6 | 7 | 9  | 10 | 11 |  |
| .73                 | 5370 | 5383 | 5395 | 5408 | 5420 | 5433 | 5445 | 5458 | 5470 | 5483 | 1                | 3 | 4 | 5 | 6 | 8 | 9  | 10 | 11 |  |
| .74                 | 5495 | 5508 | 5521 | 5534 | 5546 | 5559 | 5572 | 5585 | 5598 | 5610 | 1                | 3 | 4 | 5 | 6 | 8 | 9  | 10 | 12 |  |
| .75                 | 5623 | 5636 | 5649 | 5662 | 5675 | 5689 | 5702 | 5715 | 5728 | 5741 | 1                | 3 | 4 | 5 | 7 | 8 | 9  | 10 | 12 |  |
| .76                 | 5754 | 5768 | 5781 | 5794 | 5808 | 5821 | 5834 | 5848 | 5861 | 5875 | 1                | 3 | 4 | 5 | 7 | 8 | 9  | 11 | 12 |  |
| .77                 | 5888 | 5902 | 5916 | 5929 | 5943 | 5957 | 5970 | 5984 | 5998 | 6012 | 1                | 3 | 4 | 5 | 7 | 8 | 10 | 11 | 12 |  |
| .78                 | 6026 | 6039 | 6053 | 6067 | 6081 | 6095 | 6109 | 6124 | 6138 | 6152 | 1                | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 13 |  |
| .79                 | 6166 | 6180 | 6194 | 6209 | 6223 | 6237 | 6252 | 6266 | 6281 | 6295 | 1                | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |  |
| .80                 | 6310 | 6324 | 6339 | 6353 | 6368 | 6383 | 6397 | 6415 | 6427 | 6442 | 1                | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |  |
| .81                 | 6457 | 6471 | 6486 | 6501 | 6516 | 6531 | 6546 | 6561 | 6577 | 6592 | 2                | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |  |



### Antilogarithm Table

|     |      |      |      |      |      |      |      |      |      |      | Mean Differences |   |   |   |    |    |    |    |    |  |
|-----|------|------|------|------|------|------|------|------|------|------|------------------|---|---|---|----|----|----|----|----|--|
| .82 | 6607 | 6622 | 6637 | 6653 | 6668 | 6683 | 6699 | 6714 | 6730 | 6745 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 12 | 14 |  |
| .83 | 6761 | 6776 | 6792 | 6808 | 6823 | 6839 | 6855 | 6871 | 6887 | 6902 | 2                | 3 | 5 | 6 | 8  | 9  | 11 | 13 | 14 |  |
| .84 | 6918 | 6934 | 6950 | 6966 | 6982 | 6998 | 7015 | 7031 | 7047 | 7063 | 2                | 3 | 5 | 6 | 8  | 10 | 11 | 13 | 15 |  |
| .85 | 7079 | 7096 | 7112 | 7129 | 7145 | 7161 | 7178 | 7196 | 7211 | 7228 | 2                | 3 | 5 | 7 | 8  | 10 | 12 | 13 | 15 |  |
| .86 | 7244 | 7261 | 7278 | 7295 | 7311 | 7328 | 7345 | 7362 | 7379 | 7396 | 2                | 3 | 5 | 7 | 8  | 10 | 12 | 13 | 15 |  |
| .87 | 7413 | 7430 | 7447 | 7464 | 7482 | 7499 | 7516 | 7534 | 7551 | 7568 | 2                | 3 | 5 | 7 | 9  | 10 | 12 | 14 | 16 |  |
| .88 | 7586 | 7603 | 7621 | 7638 | 7656 | 7674 | 7691 | 7709 | 7727 | 7745 | 2                | 4 | 5 | 7 | 9  | 11 | 12 | 14 | 16 |  |
| .89 | 7762 | 7780 | 7798 | 7816 | 7834 | 7852 | 7870 | 7889 | 7907 | 7925 | 2                | 4 | 5 | 7 | 9  | 11 | 13 | 14 | 16 |  |
| .90 | 7943 | 7962 | 7980 | 7998 | 8017 | 8035 | 8054 | 8072 | 8091 | 8110 | 2                | 4 | 6 | 7 | 9  | 11 | 13 | 15 | 17 |  |
| .91 | 8128 | 8147 | 8166 | 8185 | 8204 | 8222 | 8241 | 8260 | 8279 | 8299 | 2                | 4 | 6 | 8 | 9  | 11 | 13 | 15 | 17 |  |
| .92 | 8318 | 8337 | 8356 | 8375 | 8395 | 8414 | 8433 | 8453 | 8472 | 8492 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |  |
| .93 | 8511 | 8531 | 8551 | 8570 | 8590 | 8610 | 8630 | 8650 | 8670 | 8690 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |  |
| .94 | 8710 | 8730 | 8750 | 8770 | 8790 | 8810 | 8831 | 8851 | 8872 | 8892 | 2                | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |  |
| .95 | 8913 | 8933 | 8954 | 8974 | 8995 | 9016 | 9036 | 9057 | 9078 | 9099 | 2                | 4 | 6 | 8 | 10 | 12 | 15 | 17 | 19 |  |
| .96 | 9120 | 9141 | 9162 | 9183 | 9204 | 9226 | 9247 | 9268 | 9290 | 9311 | 2                | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |  |
| .97 | 9333 | 9354 | 9376 | 9397 | 9419 | #### | 9462 | 9484 | 9506 | 9528 | 2                | 4 | 7 | 9 | 11 | 13 | 15 | 17 | 20 |  |
| .98 | 9550 | 9572 | 9594 | 9616 | 9638 | 9661 | 9683 | 9705 | 9727 | 9750 | 2                | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |  |
| .99 | 9772 | 9795 | 9817 | 9840 | 9863 | 9886 | 9908 | 9931 | 9954 | 9977 | 2                | 5 | 7 | 9 | 11 | 14 | 16 | 18 | 20 |  |

**Example 01** Find the number whose logarithm is,

(i) 1.3247      (ii)  $\bar{2}.1324$

**Solution (i):** Let  $x = \text{antilog}(1.3247)$ , Here  $\log x = 1.3247$

**Step 1:** Now characteristic = 1 and mantissa = 0.3247

**Step 2:** Now locate the row corresponding to .32 in the antilog table

**Step 3:** Proceed horizontally to third digit that is 4. The number at the intersection of row 32<sup>nd</sup> and column 4<sup>th</sup> is 2109.

**Step 4:** Again, proceed horizontally go to mean difference 7<sup>th</sup> column where the value is 3.

**Step 5:** Add 3 in 2109, we get 2112 as the required digits.

**Step 6:** Since characteristic is 1, so put decimal after two places from left to right, thus require antilog is 21.12.

**Solution (ii):** Let  $x = \text{antilog}(\bar{2}.1324)$ , Here  $\log \bar{x} = \bar{2}.1324$

Here, Characteristic =  $\bar{2}$  and mantissa = 0.1324

Now see .13 in anti-log table corresponding to column 2, we found 1355 and mean difference in 4<sup>th</sup> column is 1, so it is  $1355+1=1356$ .

Characteristics is  $\bar{2}$ ,  
thus required number is 0.01356.

## 2.2.5 Use of calculator to find the log and antilog of a number

**Example 1.** By using calculator, determine the value of  $\log(41230)$ .

**Solution:** Let  $x = \log(41230)$ . Our first step is to press the 'Log' key.

Now enter (41230), (We want to determine its value.)

Finally, close the parenthesis and press the "=" key.

Now, we can see the value of the  $\log(41230)$  on the screen which is, 4.615213335.

Thus,  $\log(41230) = 4.615213335$



**Example 02:** By using calculator, determine the value of anti-log (4.615213335).

**Solution:** We have to use the antilog function key.

- (i) Press 2<sup>nd</sup> function key or shift key.
- (ii) Press the 'Log' key
- (iii) Enter 4.615213335 followed by the right parenthesis symbol
- (iv) Press 'ENTER' key

The answer of antilog 4.615213335 is 41230.00002. This number is rounded off to 41230.

Thus,  $\text{antilog}(4.615213335) = 41230.00002$

### 2.3 Differentiate between common and natural logarithm.

The common logarithm has base 10, and is represented as  $\log(x)$  instead of  $\log_{10}(x)$ , while natural logarithm has base  $e$  ( $e$  is an irrational number whose value is 2.718281...) and is represented as  $\ln x$  instead of  $\log_e(x)$ .

#### Exercise 2.4

1. By using table, find the numbers whose common logarithms are.
 

|             |                     |               |
|-------------|---------------------|---------------|
| (i) 3.56721 | (ii) $\bar{1}.7427$ | (iii) 0.35749 |
|-------------|---------------------|---------------|
2. Find the Logarithm of the following numbers by using calculator.
 

|            |             |               |
|------------|-------------|---------------|
| (i) 900    | (ii) 45.54  | (iii) 36582   |
| (vi) 826.3 | (v) 0.00851 | (vi) 0.000097 |
3. Find the value of  $x$  from the following, using calculator.
 

|                               |                                |                               |
|-------------------------------|--------------------------------|-------------------------------|
| (i) $\log x = 1.7505$         | (ii) $\log x = 0.6609$         | (iii) $\log x = 2.2132$       |
| (iv) $\log x = 1.9009$        | (v) $\log x = 0.0009$          | (vi) $\log x = 3.8505$        |
| (vii) $\log x = \bar{1}.6132$ | (viii) $\log x = \bar{2}.7777$ | (ix) $\log x = \bar{3}.3465$  |
| (x) $\log x = \bar{4}.8455$   | (xi) $\log x = \bar{6}.7835$   | (xii) $\log x = \bar{9}.6875$ |
| (xiii) $\log x = 3.4800$      | (xiv) $\log x = \bar{7}.0038$  | (xv) $\log x = \bar{9}.687$   |

### 2.4 Laws of Logarithms.

#### 2.4.1 Prove the following laws of logarithms.

- (i)  $\log_a(mn) = \log_a m + \log_a n$
- (ii)  $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$$(iii) \log_a m^n = n \log_a m$$

$$(iv) \log_a n = \frac{\log_b n}{\log_b a}$$

(i) For real numbers  $m, n, a$  and  $a > 0, a \neq 1, \log_a(mn) = \log_a m + \log_a n$

**Proof:** Let  $\log_a m = x$  and  $\log_a n = y$ . Then

$$m = a^x \text{ and } n = a^y$$

$$\text{Now } mn = a^x \cdot a^y$$

$$mn = a^{x+y} \quad (\text{Rule of indices})$$

By changing exponential form into logarithmic form

$$= \log_a(mn) = x + y$$

$$\text{Hence, } \log_a(mn) = \log_a m + \log_a n$$

**The logarithm of the product of two numbers is the sum of their logarithms.**

(ii) For real numbers  $m, n, a$  and  $a > 0, a \neq 1,$

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

**Proof:** Let  $\log_a m = x$  and  $\log_a n = y$ . Then

$$m = a^x \text{ and } n = a^y$$

$$\text{Now } \frac{m}{n} = \frac{a^x}{a^y}$$

$$\frac{m}{n} = a^{x-y}$$

By changing exponential form into logarithmic form

$$\Rightarrow \log_a \frac{m}{n} = x - y$$

$$\text{Hence, } \log_a \frac{m}{n} = \log_a m - \log_a n$$

**The logarithm of the quotient of two numbers is the difference of their logarithm.**

(iii) For real numbers  $m, n, a$  and  $a > 0, a \neq 1,$

$$\log_a m^n = n \log_a m$$

**Proof:** Let  $\log_a m = x$  Then  $m = a^x$

$$\text{Now } m^n = (a^x)^n$$

$$m^n = a^{nx}$$

By changing exponential form into logarithmic form

$$\Rightarrow \log_a m^n = nx$$

$$\text{Hence } \log_a m^n = n \log_a m$$

**The logarithm of a number raised to a power n is the product of the exponent n and the logarithm of the number.**

**(iv) Change of base property**

For real numbers  $a, b, n$  and  $a > 0, a \neq 1$ ,

$$\log_a n = \frac{\log_b n}{\log_b a}$$

**Proof:** Let  $\log_a n = x$

$$\text{So, } n = a^x$$

Taking logarithms of both sides to the base  $b$ , we have

$$\log_b n = \log_b a^x$$

$$\log_b n = x \log_b a \quad \therefore \log_b m^n = n \log_b m$$

$$x = \frac{\log_b n}{\log_b a}$$

$$\text{Hence } \log_a n = \frac{\log_b n}{\log_b a}$$

**Example 01** Express  $\log_a (2bc)$  as a sum of logarithms.

**Solution:** Using the Law of logarithm,

$$\log_a (2bc) = \log_a 2 + \log_a b + \log_a c,$$

Hence expressed as sum of the logarithms.

**Example 02** Express  $\log(52.5 \times 63 \times 4.567)$  as a sum of the logarithms.

**Solution:** Using the Law of logarithm,

$$\log(52.5 \times 63 \times 4.567) = \log 52.5 + \log 63 + \log 4.567$$

Hence expressed as some of the logarithms.

**Note that**

- (i)  $\log_a(mn) \neq \log_a m \times \log_a n$
- (ii)  $\log_a m + \log_a n \neq \log_a (m + n)$

**Example 03** Express  $\log\left(\frac{213.1}{34.22}\right)$  as a difference of logarithms

**Solution:** Apply difference law of log, on  $\log\left(\frac{213.1}{34.22}\right)$ , we have,

$$\log\left(\frac{213.1}{34.22}\right) = \log 213.1 - \log 34.22.$$

Hence expressed as a difference of logarithms.

**Example 04** Express  $\log_a 2^x$  as a product.

**Solution:** We know that  $\log_a m^n = n \log_a m$

$$\therefore \log_a 2^x = x \log_a 2.$$

$\therefore$

**Exercise 2.5**

1. Express the following logarithm in terms of  $\log_a x$ ,  $\log_a y$  and  $\log_a z$ .

(i)  $\log_a \frac{x^3 y}{z^2}$

(ii)  $\log_a \sqrt{xy^2 z}$

(iii)  $\log_a \left( \sqrt[3]{x^{-1} y^3} \div \sqrt{y^3 x} \right)$

(iv)  $\log_a \left\{ \left( \frac{yz^{-2}}{y^{-4} z^3} \right)^{-3} \div \left( \frac{y^{-1} z}{y^2 z^{-3}} \right)^5 \right\}$

(v)  $\log_a \frac{\sqrt[6]{xy^{-1} z^{-2}}}{(x^{-1} y^{-2} z^{-3})^{\frac{1}{6}}}$

(vi)  $\log_a \frac{x \sqrt{y^3}}{\sqrt[3]{z^2 x^5}}$

2. Reduce each of the following into a single term.

(i)  $\log_a 20 - \log_a 15 + \frac{1}{2} \log_a \frac{9}{2}$

(ii)  $\frac{1}{3} \log_a (x-1)^3 + \frac{10}{9} \log_a (x+1) - \frac{1}{9} \log_a (x+1)$

(iii)  $\log x - 2 \log x + 3 \log(x+1) - \log(x^2-1)$ .

3. If  $\log 2 = 0.3010$ ,  $\log 3 = 0.4771$  and  $\log 5 = 0.6990$ , then find the values of the following without using table.

(i)  $\log 15$  (ii)  $\log 64$  (iii)  $\log \sqrt{5 \times 2}$  (iv)  $\log 48$

(v)  $\log \sqrt{18}$  (vi)  $\log 30$  (vii)  $\log \frac{8}{3}$  (viii)  $\log \frac{5}{\sqrt{3}}$

4. Prove the following:

(i)  $\log_b m \times \log_m a = \log_b a$  (ii)  $\log_a b \times \log_c a = \log_c b$

(iii)  $\log_b a \cdot \log_c b \cdot \frac{1}{\log_c a} = 1$  (iv)  $\log_a b = \frac{1}{\log_b a}$

5. Verify the following:

(i)  $\log_5 7 \times \log_7 25 = 2$  (ii)  $\log_3 2 \times \log_2 81 = 4$

(iii)  $\log_5 343 \times \log_7 25 = 6$  (iv)  $\log_6 16 \times \log_2 216 = 12$

## 2.5 Application of Laws of Logarithm

**2.5.1 Apply laws of logarithm to convert lengthy processes of multiplication, division and exponential into easier process of addition and subtraction etc.**

The following examples will help to understand the application of laws of logarithm.

**Example 01** Find the value of  $(8.573)(28.74)$  by using logarithm.

**Solution:**

Let  $x = (8.573)(28.74)$

Taking log on both the sides, we have,

$$\begin{aligned}\therefore \log x &= \log(8.573 \cdot 28.74) \\ \Rightarrow \log x &= \log(8.573) + \log(28.74) \\ \Rightarrow \log x &= 0.9332 + 1.4585 \\ \Rightarrow \log x &= 2.3917 \\ \therefore x &= \text{antilog}(2.3917) \\ \text{Thus, } x &= 246.4\end{aligned}$$

**Example 02** Find the value of  $\frac{213.1}{34.22}$  by using logarithm.

**Solution:** Let  $x = \frac{213.1}{34.22}$

Taking log on both the sides, we have,

$$\begin{aligned}\log x &= \log\left(\frac{213.1}{34.22}\right) \\ \Rightarrow \log x &= \log\left(\frac{213.1}{34.22}\right) \\ \Rightarrow \log x &= \log 213.1 - \log 34.22, \quad \left( \log\left(\frac{a}{b}\right) = \log a - \log b \right)\end{aligned}$$

By referring log table, we have,

$$\log x = 2.3286 - 1.5343 = 0.7943$$

$$\Rightarrow \log x = 0.7943,$$

by antilog, we have,

$x = \text{antilog}(0.7943)$ , by referring antilog table we have,

$$x = 6.227, \quad (\because \text{characteristic} = 0 \text{ and mantissa} = 0.7943).$$

Thus, required value of  $\frac{213.1}{34.22}$  is found 6.227.

**Example 03** Calculate  $\sqrt{\frac{3.41 \times 37.92}{2.34}}$  by using logarithmic rules.

**Solution:** Let  $x = \sqrt{\frac{3.41 \times 37.92}{2.34}}$

Take log on both sides,

$$\begin{aligned}\log x &= \log \left( \frac{3.41 \times 37.92}{2.34} \right)^{\frac{1}{2}} \\&= \frac{1}{2} \log \left( \frac{3.41 \times 37.92}{2.34} \right) \\&= \frac{1}{2} (\log 3.41 + \log 37.92 - \log 2.34) \\&= \frac{1}{2} (0.5325 + 1.5788 - 0.3692) \\&= \frac{1}{2} (1.7424) \\&= 0.8712 \\x &= \text{antilog } (0.8712) \\&= 7433 \\&= 7.433\end{aligned}$$

**Example 04** Find the number of digits in  $4^5$

**Solution:** Let  $n = 4^5$

Taking log on both the sides, we have,

$$\begin{aligned}\therefore \log n &= \log 4^5, & (\because \log a^n &= n \log a) \\ \Rightarrow \log n &= 5 \log 4, \\ \Rightarrow \log n &= 5 \times 0.6021, & (\text{since } \log 4 &= 0.6021) \\ \Rightarrow \log n &= 3.0105,\end{aligned}$$

Since number of digits = characteristic + 1,

so, number of digits in  $4^5 = 3 + 1 = 4$ .

Exercise 2.6

1. Find the values of the following by using logarithms.

(i)  $57.86 \times 4.385$

(ii)  $25.753 \times 0.5341 \times 490.8$

(iii)  $\frac{25.753}{0.5341}$

(iv)  $\frac{(790.6 \times 30.32)}{25.753}$

(v)  $\frac{(2391 \times 30.72)}{23.34}$

(vi)  $\frac{99.87}{(8.369) \times (0.785)}$

(vii)  $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

(viii)  $\frac{(26.62)^{\frac{1}{2}} \times (87.19)^3}{\sqrt{69.53}}$

(ix)  $\frac{(4308)^3 \times \sqrt{80.06}}{(0.3387)^3}$

(x)  $\frac{(43.68)^5 \times \sqrt{(0.057)^3}}{(3.048)^4}$

2. Find the number of digits in the following.

(i)  $4^{12}$

(ii)  $7^{25}$

(iii)  $3^{30}$

(iv)  $3^5$

(v)  $9^4$

## Review Exercise 2

1. Read the following sentences carefully and encircle "T" in case of True and "F" in case of False statement.

- (i) 0.025 can be written in scientific notation as  $2.5 \times 10^3$  T/F
- (ii) Logarithm was invented by Al- Beruni. T/F
- (iii) Integral part in the logarithm of a number is called its characteristic. T/F
- (iv) Mantissa in the logarithm of a number is always negative. T/F
- (v)  $\log_a x = y \Leftrightarrow a^y = x$ . T/F

2. Fill in the blanks.

- (i) Logarithms having base 10 is called \_\_\_\_\_.
- (ii)  $\log 1 =$  \_\_\_\_\_.
- (iii) Fractional part of logarithm is called \_\_\_\_\_.
- (iv)  $\log 512 =$  \_\_\_\_\_.
- (v)  $\log_a m \times \log_m n =$  \_\_\_\_\_.
- (vi) The exponential form of  $x = \log_a y$  is \_\_\_\_\_.
- (vii) The logarithmic form of  $a^{10} = y$  is \_\_\_\_\_.
- (viii)  $\log_b a \times \log_a b =$  \_\_\_\_\_.
- (ix)  $\log_a \left( \frac{m}{n} \right) =$  \_\_\_\_\_.
- (x)  $\log(10 \times 10) =$  \_\_\_\_\_.

3. Tick (✓) the correct answers.

- (i) If  $\log_{10} x = 4$ , then  $x =$  \_\_\_\_\_ .  
 (a) 500      (b) 100      (c) 1000      (d) 10000
- (ii) The characteristic of  $\log 54.58$  is \_\_\_\_\_ .  
 (a) 0      (b) 1      (c) 2      (d) 4
- (iii) The base of common logarithm is \_\_\_\_\_ .  
 (a) 5      (b) 10      (c)  $e$       (d) 100
- (iv)  $\log xyz =$  \_\_\_\_\_ .  
 (a)  $\log x \log y \log z$       (b)  $\log x + \log y + \log z$   
 (c)  $\log (xy)^z$       (d)  $\log x - \log y - \log z$
- (v) Scientific notation of 0.00789 is \_\_\_\_\_ .  
 (a)  $7.89 \times 10^{-3}$       (b)  $7.89 \times 10^3$   
 (c)  $0.789 \times 10^{-2}$       (d)  $78.9 \times 10^{-4}$
- (vi) If  $\log x = 2$  then  $x =$  \_\_\_\_\_ .  
 (a) 200      (b) 1000      (c) 100      (d)  $\frac{2}{10}$
- (vii) If  $\log_2 8 = x$  then  $x =$  \_\_\_\_\_ .  
 (a) 64      (b)  $3^2$       (c) 3      (d)  $2^8$
- (viii) Base in the Natural logarithm is \_\_\_\_\_ .  
 (a) 10      (b)  $e$       (c)  $\pi$       (d) 5
- (ix)  $3^5 = 243$ , can be written in logarithmic form as \_\_\_\_\_ .  
 (a)  $\log_3 5 = 243$       (b)  $\log_3 243 = 5$   
 (c)  $\log_5 243 = 12$       (d)  $\log_5 3 = 243$

### Summary

- ◆ If  $a^x = y$ , then  $x$  is called the logarithm of  $y$  to the base ' $a$ ' and is written as  $\log_a y = x$ , where  $a > 0, y > 0$  and  $a \neq 1$
- ◆ Common logarithms have base **10**, it is also named as *Brigg's* logarithm and usually written as  $\log x$  instead of  $\log_{10} x$ , natural logarithms have base  $e$ , (an irrational number) whose value is 2.7182818.... and written as  $\ln x$  instead of  $\log_e x$ .
- ◆  $\log x = y \Leftrightarrow 10^y = x$
- ◆  $\ln x = y \Leftrightarrow e^y = x$ .
- ◆ The integral part of logarithm of any number is called characteristic and fractional part is called mantissa.
- ◆ Characteristic of logarithm of a number  $> 1$  is always positive.
- ◆ Characteristic of logarithm of a number  $< 1$  is always negative.
- ◆ Negative characteristic of logarithm can be written as  $\bar{3}, \bar{2}$  or  $\bar{1}$  instead of  $-3, -2$  or  $-1$ .
- ◆ The logarithms of a number having the same sequence of digits have same mantissa.
- ◆ The number corresponding to a given log is called anti-logarithm.
- ◆ Laws of logarithms
  - (i)  $\log_a (mn) = \log_a m + \log_a n$
  - (ii)  $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
  - (iii)  $\log_a m^n = n \log_a m$
  - (iv)  $\log_a n = \frac{\log_b n}{\log_b a}$

this law can also be written as  $\log_b a \cdot \log_a n = \log_b n$ .