

Unit

10

PARALLELOGRAM AND TRIANGLES

Student Learning Outcomes (SLOs)

After completing this unit, students will be able to:

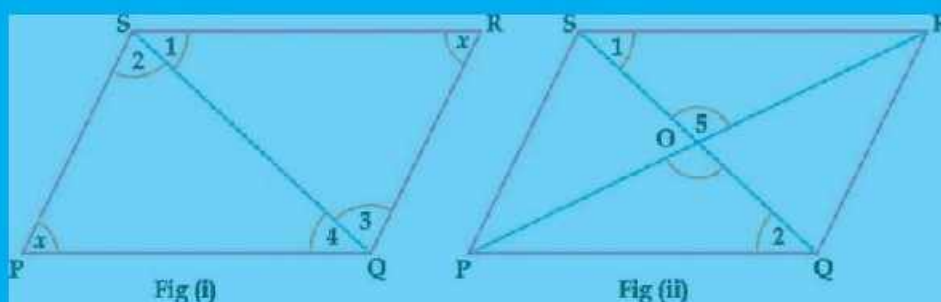
- ◆ Understand the following theorems along with their corollaries and apply them to solve allied problems.
 - a) In a parallelogram:
 - ◆ The opposite sides are congruent,
 - ◆ The opposite angles are congruent,
 - ◆ The diagonals bisect each other.
 - b) If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.
 - c) The line segments joining the midpoints of two sides of a triangle, is parallel to the third side and it is equal to one half of its length.
 - d) The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.
 - e) If three or more parallel lines make congruent intercepts on the transversal, they also intercept congruent segments on any other line that cuts them.

Introduction

In the previous classes students learned and construct many kinds of polygons like triangles, parallelogram, square, rectangle, rhombus, trapezium etc. Also observe the congruency related to their sides and angles. In this unit, we will discuss and understand the theorems related to parallelograms and Triangles.

Parallelograms and Triangles

Theorem 10.1.1



Given:

▯ PQRS

To Prove:

1. $\overline{PQ} \cong \overline{RS}$; $\overline{PS} \cong \overline{QR}$
2. $\angle P \cong \angle R$; $\angle S \cong \angle Q$
3. Diagonals $\overline{PR}$ and $\overline{SQ}$ bisect each other at point O. [fig (ii)]

Construction:

In figure (i) join points S and Q.

Proof:

Statements	Reasons
In figure (i)	
(1)	
$\overline{SR} \parallel \overline{PQ}$, $\overline{SQ}$ is transversal,	Alternate angles of $\parallel$ lines
$m\angle 1 = m\angle 2$	
Similarly, $m\angle 3 = m\angle 4$	$\overline{SR} \parallel \overline{PQ}$, $\overline{SQ}$
$\therefore m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	Addition property of postulate
or $\angle S \cong \angle Q$	Angle addition postulate
Similarly, $\angle P \cong \angle R$	By above the same process
i.e opposite angles are congruent	
(2)	
$\triangle SPQ \leftrightarrow \triangle QRS$	
$\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$	Proved in (1) above
$\overline{SQ} \cong \overline{SQ}$	Common
$\therefore \triangle SPQ \cong \triangle QRS$	A.S.A $\cong$ A.S.A
$\therefore \overline{PQ} \cong \overline{RS}$ and $\overline{PS} \cong \overline{QR}$	By the congruence of $\triangle$ s
i.e opposite sides are congruent	
In figure (ii)	
(3)	
$\triangle SOQ \leftrightarrow \triangle ROS$	
$\angle 1 \cong \angle 2$	Proved in (1) above
$\angle POQ \cong \angle SOR$	Vertically opposite angles
$\overline{PQ} \cong \overline{SR}$	Proved in (2) above
$\therefore \triangle POQ \cong \triangle ROS$	A.A.S $\cong$ A.A.S
$\therefore \overline{PO} \cong \overline{OR}$ and $\overline{OQ} \cong \overline{OS}$	By the congruence of $\triangle$ s
$\therefore \overline{PR}$ and $\overline{RS}$ diagonals bisect each other	

Q.E.D

1. The line joining the mid-points of two opposite sides of parallelogram is parallel to the other sides.
2. Interior angles on any side of a parallelogram are supplementary.
3. Prove that the bisectors of two angles on same side of a parallelogram cut each other at right angle.
4. If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
5. In parallelogram corresponding angles on any sides are equal.

If two opposite sides of a quadrilateral are congruent and parallel, it is a parallelogram.

Given:

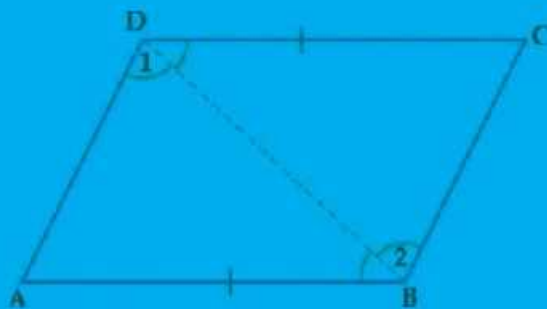
In a quadrilateral ABCD,
 $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

To prove:

Quadrilateral ABCD is a $\parallel^m$

Construction: Join B and D.

Proof:



Statements	Reasons
$\overline{AB} \parallel \overline{CD}$, $\overline{BD}$ is transversal	Alternate angles of $\parallel$ lines
$\therefore \angle ABD \cong \angle CBD$	
In $\triangle ADB \leftrightarrow \triangle CBD$	
i. $\overline{AB} \cong \overline{CD}$	i. Given
ii. $\angle ABD \cong \angle CBD$	ii. Proved above
iii. $\overline{BD} \cong \overline{BD}$	iii. Common
$\therefore \triangle ADB \cong \triangle CBD$	S.A.S $\cong$ S.A.S
$\therefore \angle 1 \cong \angle 2$	By the congruence of triangles.
But these are alternate triangles	By definition of alternate angles.
$\overline{AD} \parallel \overline{BC}$	Alternate angles are congruent
$\overline{AB} \parallel \overline{CD}$	Given
ABCD is a $\parallel^m$	Opposite sides are parallel

Q.E.D

1. Prove that a quadrilateral is a parallelogram, if its opposite angles are congruent
2. Prove that a quadrilateral is a parallelogram, if its diagonals bisect each other.
3. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
4. If a quadrilateral is a parallelogram, it has diagonals which form two congruent triangles.
5. If the angles formed with every side of a quadrilateral are supplementary, it is a parallelogram.

The line segment joining the mid-points of two sides of a triangle, is parallel to the third side and it is equal to one half of its length.

Given:

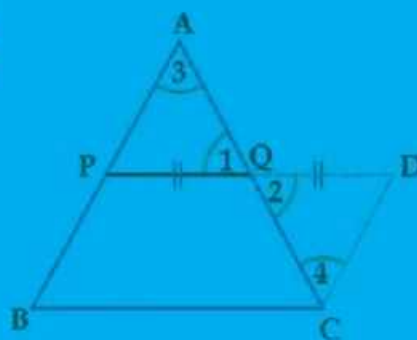
P and Q are midpoints of $\overline{AB}$ and $\overline{AC}$ in $\triangle ABC$, $\overline{PQ}$ is the line segment joining them.

To prove:

$$\overline{PQ} \parallel \overline{BC} \text{ and } m\overline{PQ} = \frac{1}{2} m\overline{BC}$$

Construction:

Produce $\overline{PQ}$ to D such that $\overline{QD} \cong \overline{PQ}$.
Join D and C.



Proof:

Statements		Reasons
In	$\triangle APQ \leftrightarrow \triangle CDQ$	
i.	$\overline{PQ} \cong \overline{QD}$	i. Construction
ii.	$\angle 1 \cong \angle 2$	ii. Vertical angles
iii.	$\overline{AQ} \cong \overline{QC}$	iii. Given

$\therefore \Delta APQ \cong \Delta CDQ$
 $\therefore \overline{AP} \cong \overline{CD}$ and $\angle 3 \cong \angle 4$
 But $\overline{PB} \cong \overline{AP}$
 $\therefore \overline{PB} \cong \overline{CD}$
 $\angle 3$ and $\angle 4$ are alternate $\angle$ s
 $\therefore \overline{AB} \parallel \overline{CD}$ i.e. $\overline{PB} \parallel \overline{CD}$
 $\therefore$ PBCD is a $\parallel^m$
 $\therefore \overline{PD} \parallel \overline{BC}$ and $\overline{PD} \cong \overline{BC}$
 $\therefore \overline{PQ} \cong \overline{BC}$
 and $m\overline{PQ} = \frac{1}{2} m\overline{BC}$

S.A.S postulate

By the congruence of Δ s

Given

Each is congruent to $\overline{AP}$

By definition of alternate $\angle$ s

Alternate $\angle$ s are congruent

A pair of opposite sides $\parallel$ and $\cong$

Opposite sides of a $\parallel^m$ are parallel and congruent.

$\overline{PD}$ and $\overline{PQ}$ are same line and

$$m\overline{PQ} = \frac{1}{2} m\overline{PD}$$

Q.E.D

1. If the line segments joining the mid-points of two sides of a triangle, is parallel to the third side and its length is 4 cm. What is the length of third side.
2. Prove that the line-segment joining the mid-points of the opposite sides of a quadrilateral bisect each other.
3. The line segments, joining the mid-points of the sides of a quadrilateral, taken in order, form a parallelogram.
4. Prove that the line segment passing through the mid-point of one side and parallel to another side of a triangle also bisect the third side.
5. Prove that four triangles obtained by joining the mid-points of the three sides of a triangle are all congruent to each other.

The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.

Given:

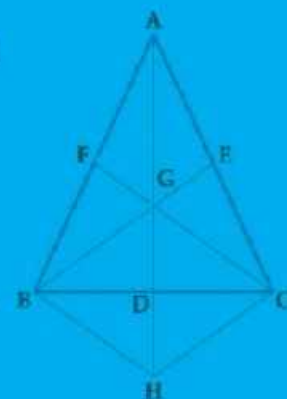
$\triangle ABC$, in which medians $\overline{BE}$ and $\overline{CF}$ meet in G .

To prove:

- $\overline{AG}$ produces bisects $\overline{BC}$ in D , and
- G is the point of trisection of each median.

Construction:

Draw $\overline{CH}$ $\overline{EB}$ meeting $\overline{AD}$ produced in H .
Join points B and H .

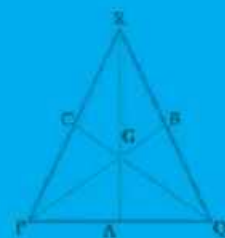


Proof:

Statements	Reasons
In $\triangle ACH$, $\overline{AE} \cong \overline{EC}$	Given
and $\overline{EG} \cong \overline{CH}$	Construction
$\therefore \overline{AG} \cong \overline{GH}$	By converse of theorem 10.1.3
Further in $\triangle ABH$	
$\overline{AG} \cong \overline{GH}$	Proved above
$\overline{AF} \cong \overline{FB}$	Given
$\overline{FG} \cong \overline{BH}$	By theorem 10.1.3
Hence $BGCH$ is a $\square$	Opposite sides are parallel
Diagonals $\overline{BC}$ and $\overline{GH}$ bisect each other	By theorem 10.1.1
i.e. $\overline{GH} \cong \overline{DH}$, $\overline{BD} \cong \overline{DC}$	
$\overline{AD}$ median of $\triangle ABC$	$\overline{BD} \cong \overline{DC}$ (Proved above)
Also $m\overline{AG} = m\overline{GH} = 2m\overline{GD}$	$\overline{GD} \cong \overline{DH} \Rightarrow m\overline{GH} = 2m\overline{GD}$
G is a point of trisection of $\overline{AD}$	As $\overline{AG}$ is twice of $\overline{GD}$
Similarly we can prove that G is a point of trisection of $\overline{BE}$ and $\overline{CF}$ as well.	By the above process

Q.E.D

1. If three medians of a triangle are congruent, prove that the triangle is an equilateral.
2. The medians $\overline{PB}$, $\overline{QC}$ and $\overline{RA}$ of $\triangle PQR$ meet in point G , show that G is the centroid of $\triangle PQR$.
3. In the given figure, the length $\overline{GD}$ is of 2cm then find the length of $\overline{AG}$.



If three or more parallel lines makes congruent intercepts on the transversal, they also intercept congruent segments on any other line that cuts them.

Given:

AB, CD and EF cut transversal GH at points P, Q and R respectively such that:

$$\overline{PQ} \cong \overline{QR}$$

XY is another transversal cutting AB, CD, EF in points M, N, O, C respectively.



To prove: $\overline{NM} \cong \overline{NO}$

Construction:

Draw $\overline{MJ}$ and $\overline{NK}$ each parallel to GH meeting $\overline{CD}$ and $\overline{EF}$ respectively in points J and K .

Proof:

Statements	Reasons
$\overline{PM} \parallel \overline{QJ}$	$\overline{AB} \parallel \overline{CD}$ (Given)
and $\overline{PQ} \parallel \overline{MJ}$	Construction
$\therefore PMJQ$ is a $\parallel^m$	Opposite sides parallel
$\therefore \overline{PQ} \cong \overline{MJ}$	Opposite sides of a $\parallel^m$
Similarly, $QRKN$ is a $\parallel^m$	$\overline{QR} \parallel \overline{NK}$ and $\overline{QN} \parallel \overline{RK}$

$$\therefore \overline{QR} \cong \overline{NK}$$

$$\text{But } \overline{PQ} \cong \overline{QR}$$

$$\therefore \overline{MJ} \cong \overline{NK}$$

$$\text{Now } \overline{MJ} \parallel \overline{NK}$$

$$\therefore \angle 1 \cong \angle 2$$

$$\text{In } \triangle MNJ \leftrightarrow \triangle NOK$$

$$\text{i. } \angle 1 \cong \angle 2$$

$$\text{ii. } \angle 3 \cong \angle 4$$

$$\text{iii. } \overline{MJ} \cong \overline{NK}$$

$$\therefore \triangle MNJ \cong \triangle NOK$$

$$\therefore \overline{MN} \cong \overline{NO}$$

By above reason

Given

Transitive property of equation

Each parallel to $\overline{GH}$

Corresponding $\angle$ s of $\parallel$ lines

i. Proved above

ii. Corresponding $\angle$ s of $\parallel$ lines

iii. Proved above

A.A.S $\cong$ A.A.S

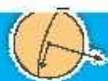
By the congruence of $\triangle$ s.

Q.E.D

1. The triangle formed by joining the mid-points of the sides of a triangle is equivalent to the original triangle.
2. The line segment joining the mid-points of the non-parallel sides of a trapezium is parallel to the parallel sides and is equal to half their sum.
3. Every line segment drawn from the vertex to the base of a triangle is bisected by the line joining the mid-point of the other two sides.

1. Fill in the blanks:

- (i) In a parallelogram, opposite sides are _____.
- (ii) In a parallelogram, opposite angles are _____.
- (iii) In a triangle, medians are _____.
- (iv) In a parallelogram, the diagonals _____ each other.
- (v) In a parallelogram, corresponding angles are _____.
- (vi) The medians of a triangle are _____.
- (vii) Sum of the measures of interior angles of a quadrilateral is equal to _____.



2. Tick (✓) the correct answer.

- (i) Diagonals of a square are _____ to each other.
a) Perpendicular b) Non congruent
c) Congruent d) Both 'a' and 'c'
- (ii) Sum of the measures of interior angles of a quadrilateral is
a) 2 right angles b) 4 right angles
c) 3 right angles d) none of these
- (iii) Measure of a line segment joining the mid points of $\overline{AB}$ and $\overline{AC}$ of $\triangle ABC$ is 3.5cm, then $m\overline{BC}$ =
a) 4.5cm b) 5.5cm
c) 6cm d) 7cm
- (iv) Two medians $\overline{AD}$ and $\overline{BE}$ of $\triangle ABC$ intersect each other at G. If $m\overline{GD}$ = 1.7cm, then $m\overline{AG}$ =
a) 2.7cm b) 8.85cm
c) 3.4cm d) 5.1cm
- (v) If sum of the measures of $\angle A$ and $\angle C$ of a parallelogram ABCD is 130° , then $m\angle B$ =
a) 25° b) 50° c) 65° d) none of these
- (vi) If opposite angles of a quadrilateral are equal in measures and none of them is a right angle, then the quadrilateral is a
a) Square b) Parallelogram
c) Trapezium d) Rectangle
- (vii) Centroid is the common point of intersection of
a) Medians of a triangle
b) Diagonals of a parallelogram
c) Angle bisectors of a triangle
d) Perpendicular bisectors of a triangle
- (viii) A point on median of a triangle is
a) equidistant from its vertices
b) equidistant from the mid points of its sides
c) equidistant from its altitudes
d) none of these

- ◆ Opposite sides of parallelogram are congruent.
- ◆ Opposite angles of parallelogram are congruent.
- ◆ Supplementary angles property holds for consecutive angles.
- ◆ Diagonals of a parallelogram bisect each other and each diagonal separates it into two congruent triangles
- ◆ If the angles of a parallelogram is right angle, then all the angles are right angles.
- ◆ Diagonals of a parallelogram divide it into four congruent triangles.
- ◆ Sum of the angles of a parallelogram is 360° .
- ◆ Sum of the interior angles of a triangle is 180° .
- ◆ Sum of the exterior angles of a triangle is 360° .
- ◆ If three or more parallel lines make congruent segments on a transversal they also intercept congruent segments on any other lines that cuts them.
- ◆ The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.
- ◆ The line segment joining the mid-points of two sides of a triangle, is parallel to the third side and is equal to one half of its length.