

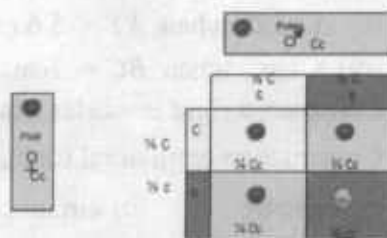
UNIT 11

BASIC STATISTICS

In this unit the students will be able to:

- Construct grouped frequency table, Histogram(unequal intervals) and frequency polygon.
- Calculate Arithmetic mean, Median, Mode of grouped data.
- Calculate weighted mean.
- Related real life word problems
- Probability of single events.
- Probability of complementary events.
- Relative frequency as an estimate of probability.
- Expected frequency.
- Related real life word problems.

The famous Biologist Gregor Mendel demonstrated that pea plant characteristics were transmitted as discrete units from parent to offspring i.e. in a genetic cross, the probability of a pea plant of purple flower with a pea plant of yellow flowers, the dominant trait is expressed in 3 out of 4 of the offspring and recessive trait is expressed in one out of 4 as shown in the adjoining figure.



Statistic plays vital role in our day to day life as in our lives we have to come across frequent decisions based on some relevant past experiences or the related statistics already available. For instance we may use statistics in efficient budgeting of our family, in finding the best prices for sale or purchase of products of our interest, to predict risks in our business, to estimate best time to perform various routine activities. Online shopping is an easily understandable example of this era where the customers see reviews of others about a product and make decision to purchase or not to purchase. If more than 50% reviews are in the favour it may be predicted that the product will be out of stock soon. Similarly, to estimate the chances of failure or success of a project, probability has great application. Quantifying and estimating investment and risk assessment in a project are very important for financial planning, investment comparison and decision making.

THINK MORE

Hadiya is on the holy trip of umrah. while her stay she observed the best time to perform TWWAF E KAABAH PAK when the crowd is least is soon after Fajar prayer when of the people go back to rest. What do you guess for another, such time.



11.1 Frequency Distribution

In statistics it is very important to represent a data in a manageable way so that it becomes easy to understand, analyze and to draw useful results. This useful presentation of data can be made by various methods. Grouped frequency distribution is one of the most important methods to represent the quantitative data. Data can be classified as:

Raw Data / Ungrouped Data

Data which has not been arranged in a systematic order is called raw data or ungrouped data.

Grouped Data

A tabular arrangement of data in which various items / values are arranged or distributed into some classes and the number of items falling in each class is stated is called grouped data or a frequency distribution.

Let us try to understand the above two categories of data with the help of an example. The points scored by 50 participants of Maths Olympiad out of 100 are recorded below.

10 18 38 15 30 27 41 39 16 50 50 45 34 37 34 09 12 45 19 23 29 31 28 17 08
19 25 39 42 48 11 30 21 34 37 50 46 08 15 15 16 21 28 32 37 41 19 29 48 50

This presentation of data is called ungrouped data. It is difficult to draw any useful conclusions from the above data. To analyze the above results easily, it is better to distribute the students in some groups according to their marks as follows.

Marks obtained	No. of students
1 – 10	4
11 – 20	12
21 – 30	11
31 – 40	11
41 – 50	12
Total = 50	

From the table it is easy to get the following results.

There are 4 participants whose scores are from 1 to 10.

There are 12 participants whose scores are from 11 to 20.

There are 11 participants whose scores are from 21 to 30.

There are 11 participants whose scores are from 31 to 40.

There are 12 participants whose scores are from 41 to 50.

34 participants scored more than 20 points.

Data represented in this form is called grouped data.

To convert an ungrouped data into grouped form, the students should be familiar with the following terms.

(i) Number of Groups

A number of groups depends upon the size of data. There is no hard and fast rule for a number of groups, but it is usually between 5 and 15.

(ii) Class Limits

A group or class is defined by two numbers. The smaller number is called lower class limit and larger one is called upper class limit. In above example first group is (1 – 10), 1 is its lower class limit and 10 the upper class limit. Both the lower and upper limits are included in the respective groups.

(iii) Class Boundaries

In above example the values / marks between 10 and 11 are not included in any group. If marks of a student are 10.5, then above mentioned classes are not suitable. To include such values we should write the classes as

0.5 – 10.5, 10.5 – 20.5, 20.5 – 30.5, 30.5 – 40.5, 40.5 – 50.5.

The participants having scores between 0.5 and 10.5 including 0.5 are considered in the first group. The participants having scores 10.5 will be in second group. The largest value of 0.5 – 10.5 and smallest value of 10.5 – 20.5 are same.

0.5 and 10.5 are lower and upper boundaries of first group. The class boundaries can be formed from class limits as follows. First of all find difference between the lower class limit of second class and upper class limit of first class. Then subtract half of this difference from lower limits of all classes and add it to the upper limits of all classes.

Adjustment factor = $\frac{1}{2}$ (lower limit of a class – upper limit of the preceeding class) = 0.5

To understand the difference between class limits and class boundaries, look at the following table.

scores (class intervals)	Lower limits	Upper limits	Lower boundaries	Upper boundaries	Class boundaries
1 – 10	1	10	$1 - 0.5 = 0.5$	$10 + 0.5 = 10.5$	0.5 – 10.5
11 – 20	11	20	$11 - 0.5 = 10.5$	$20 + 0.5 = 20.5$	10.5 – 20.5
21 – 30	21	30	$21 - 0.5 = 20.5$	$30 + 0.5 = 30.5$	20.5 – 30.5
31 – 40	31	40	$31 - 0.5 = 30.5$	$40 + 0.5 = 40.5$	30.5 – 40.5
41 – 50	41	50	$41 - 0.5 = 40.5$	$50 + 0.5 = 50.5$	40.5 – 50.5

(iv) Size of Class Interval

Size of a class interval is the difference of lower limits or upper limits of two consecutive classes. It is usually denoted by 'h'.

e.g. in 1 – 10, 11 – 20, 21 – 30, the size of class intervals is 10.

(v) Class Marks / Mid Values

It is the average of lower and upper limits or boundaries of a class. Class marks are denoted by x .

Class mark of 1 – 10 is $\frac{1+10}{2} = 5.5$ and that of 11 – 20 is $\frac{11+20}{2} = 15.5$

Note:

1. The difference of class marks of two consecutive classes is equal to the size of class interval h .
2. Class boundaries may be derived from the class marks x and size of class interval h by using the formula $(x \pm \frac{h}{2})$.

(vi) Frequency

Number of repetitions of an individual value in an ungrouped data is called its frequency. In a grouped data, number of values present in a group is called frequency of group. In Table 1 frequency of group 1 – 10 is 4, because there are four students whose obtained marks are between 1 and 10 both inclusive.

(vii) Cumulative Frequency

The sum of frequency of a class and the frequencies of all the preceding classes is called cumulative frequency.

10.1.1 Construction of Frequency Distribution Table

We can construct a frequency distribution table in two ways.

- (i) Tally bar method (ii) Direct observation method

Tally Bar Method

Step I: Decide how many groups should be made in the table.

Step II: Find approximate size of class interval by the formula $h \approx \frac{x_l - x_s}{n}$,

Where h = size of class interval

x_l = largest value in ungrouped data

x_s = smallest value in ungrouped data

n = approximate number of classes

Step III: Draw a table and write the class limits in first column according to the size of class interval.

Step IV: Choose the values from ungrouped data one by one and draw small tally marks in second column against the respective group. If 5 or more values fall in a same group, then draw 5th tally mark diagonally. It will make the counting of tally marks easier.

Step V: Count tally marks of each group and write their number against the respective class in third column (column of frequencies).

Step VI: Find sum of all frequencies and check that it must be equal to the number of values present in ungrouped data i.e

number of total items in ungrouped data (n) = sum of frequencies in grouped data ($\sum f$)

Example 1:

The following are masses in pounds of 40 boys at a gym.

147, 138, 164, 150, 132, 144, 125, 149, 157, 146, 158, 140, 136, 148, 152, 144, 168, 126, 138, 176, 163, 126, 154, 165, 146, 173, 142, 147, 135, 153, 140, 135, 161, 145, 135, 142, 150, 156, 145, 128.

Make a frequency distribution table using an appropriate size of class interval by tally bar method.

Solution:

Step I: Let us distribute the data in 5 classes.

Step II: $x_l = 176, x_s = 125, n = 40$.

$$h = \frac{x_l - x_s}{n} = \frac{176 - 125}{5} = 10.2 \approx 11$$

Step III:

Masses in pounds	Tally marks	Frequencies
125 – 135		8
136 – 146		13
147 – 157		11
158 – 168		6
169 – 179		2
Total		40

History a Mystery

Tally sticks are the ancient tools used for recording numerical data. These elongated pieces of bone, wood or ivory were marked with counts or lines. Marco Polo have documented the use of tally sticks in regions like Yunnan (China).

Direct Observation Method

In this method the values of ungrouped data are written in second column of table against the respective group instead of tally marks. The remaining process of construction of frequency table by direct method is same as that of tally bar method. When ungrouped data are not arranged in any order, then it is better to use tally bar method.

In example 1, the frequency table by direct method can be constructed as follows.

Masses in pounds	Observation	Frequencies
125 – 135	132, 125, 126, 126, 135, 135, 135, 128	8
136 – 146	138, 144, 146, 140, 136, 144, 138, 146, 142, 140, 145, 142, 145	13
147 – 157	147, 150, 149, 157, 148, 152, 154, 147, 153, 150, 156	11
158 – 168	164, 158, 168, 163, 165, 161	6
169 – 179	176, 173	2
Total		40

Discrete Frequency Distribution

If data is not in fractions and some values occur most often to constitute the whole data, then data is called a discrete data there is no need of class boundaries and size of class interval is 1.

Example 2:

A cubical die is rolled 30 times and numbers of dots on front face recorded each time are as follows.

2, 6, 2, 3, 2, 4, 5, 4, 3, 1, 2, 6, 5, 3, 4, 6, 2, 2, 4, 6, 5, 1, 3, 4, 6, 5, 1, 1, 6, 3

Make a discrete frequency distribution. Also make a column of cumulative frequency

Solution:

No. of dots	Tally marks	Frequencies	Cumulative frequency
1		4	4
2		6	4+6=10
3		5	4+6+5=15
4		5	4+6+5+5=20
5		4	4+6+5+5+4=24
6		6	4+6+5+5+4+6=30
Total	—	30	

11.1.2 Construction of Histogram

Different types of charts and graphs are used to represent a data effectively. It becomes very easy to get a quick idea about the data and to analyze by means of graphs and charts. In histogram, data are represented by adjacent rectangles having bases along horizontal axis (marked off by class boundaries) and areas proportional to the class frequencies. If the class interval sizes is the same, the heights of rectangles are proportional to the class frequencies as you have done earlier in grade 8. If the class interval size is unequal, then the heights of rectangles are to be adjusted.

Construction of Histogram with Unequal Size of Class Intervals

If sizes of class intervals are unequal, then to make areas of rectangles of a histogram proportional to their frequencies, the heights of rectangles have to be adjusted. In a histogram area of rectangle is proportional to class frequency.

$$\text{Area} \propto \text{frequency} \quad \text{or} \quad \text{height} \times \text{width} \propto \text{frequency}$$

$$\Rightarrow \text{height} \propto \frac{\text{frequency}}{\text{width}}$$

Thus height of rectangle may be found by dividing the frequency by width of class interval. The ratio $\frac{\text{frequency}}{\text{width}}$ is called **frequency density**.

To make a histogram with unequal sizes of class intervals, we plot class boundaries on x – axis (horizontal) and frequency densities on y – axis (vertical).

Example 3: The heights in cm of 44 students of a school is given below.

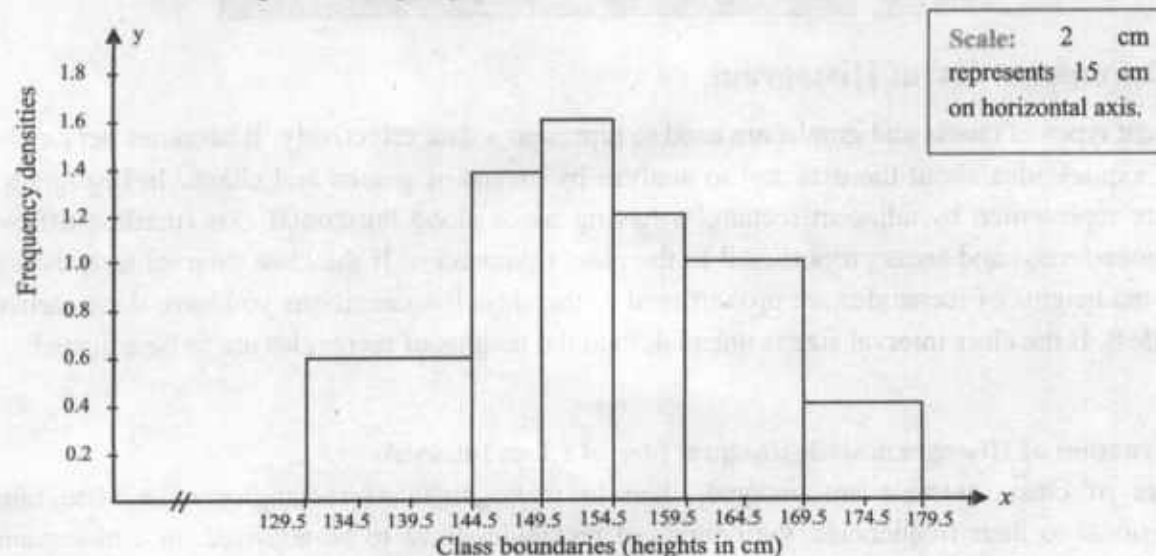
Height	130–144	145–149	150–154	155–159	160–169	170–179
No. of students	9	7	8	6	10	4

Represent the data by a histogram.

Solution:

Height (cm)	f	Size of class interval	F.D	Class boundaries
130 – 144	9	15	$9 \div 15 = 0.6$	129.5 – 144.5
145 – 149	7	5	$7 \div 5 = 1.4$	144.5 – 149.5
150 – 154	8	5	$8 \div 5 = 1.6$	149.5 – 154.5
155 – 159	6	5	$6 \div 5 = 1.2$	154.5 – 159.5
160 – 169	10	10	$10 \div 10 = 1$	159.5 – 169.5
170 – 179	4	10	$4 \div 10 = 0.4$	169.5 – 179.5
Total	44	—	—	—

Title: A histogram showing heights (cm) of 44 students in a secondary school.



11.1.3 Construction of a Frequency Polygon

Polygon is a many sided closed figure. To draw a frequency polygon mid points (class marks) are marked on the horizontal axis and frequencies are on the vertical axis. Points are plotted with class frequencies and their corresponding class marks. These points are joined by straight line segments. To complete a closed polygon we add extra classes at both ends with zero frequencies. In this way two extra points on x-axis are obtained on both ends. The points are joined by their nearer plotted points. Finally a closed frequency polygon is obtained. It is easy to draw a frequency polygon on a histogram. To do so we find mid points of the top edges of rectangles and join them by segments. Then find two extra points on both ends on x-axis to complete the closed polygon.

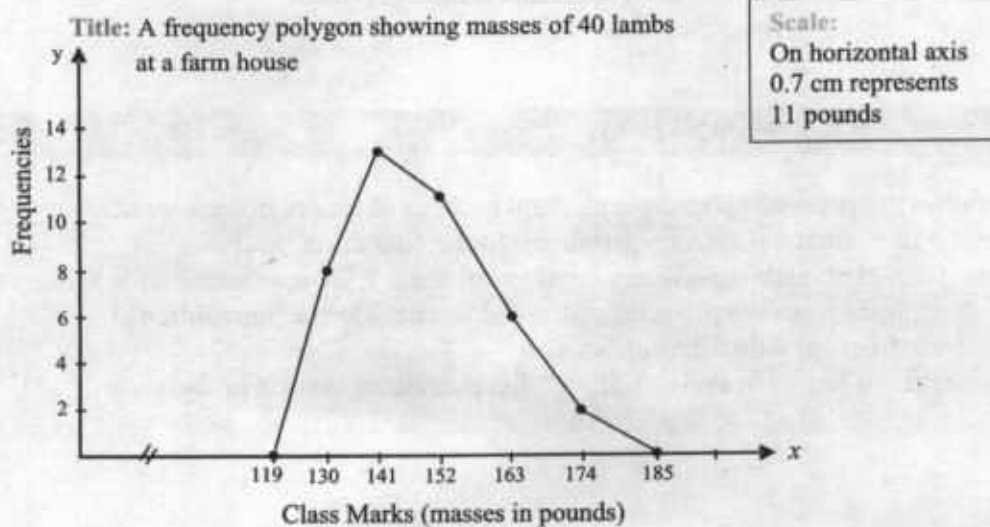
Example 4:

At a farm house there are 40 lambs, their masses are recorded as given below. Represent the following frequency distribution by a frequency polygon.

Masses in pounds	No. of lambs
125 – 135	8
136 – 146	13
147 – 157	11
158 – 168	6
169 – 179	2
Total	40

Solution:

Masses	Frequency	Class Marks
114 – 124	0	119
125 – 135	8	130
136 – 146	13	141
147 – 157	11	152
158 – 168	6	163
169 – 179	2	174
180 – 190	0	185
Total	40	



Example 5:

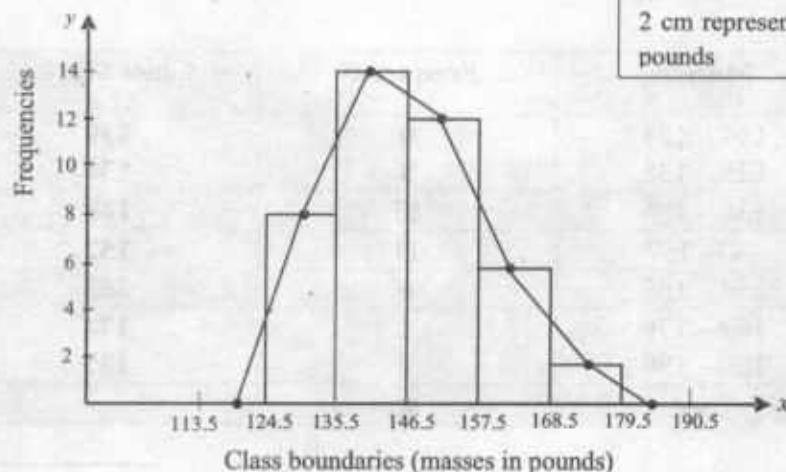
Use the frequency distribution in example 4 and draw a frequency polygon on a histogram.

Solution:

Masses	Frequency	Class boundaries	Class Marks
125 – 135	8	124.5 – 135.5	130
136 – 146	14	135.5 – 146.5	141
147 – 157	12	146.5 – 157.5	152
158 – 168	6	157.5 – 168.5	163
169 – 179	2	168.5 – 179.5	174

Title: A frequency polygon on a histogram showing masses of 40 lambs in pounds

Scale:
On horizontal axis
2 cm represents 11
pounds

**Check Point**

Read the following passage carefully and count number of letters in each word.

"Statistics is a mathematical science pertaining to the collection, analysis, interpretation or explanation and presentation of data. It is applicable to a wide variety of academic disciplines from the physical and social sciences to the humanities".

Make a discrete frequency distribution for

- (i) the number of letters in each word, (ii) the number of vowels in each word.

EXERCISE 11.1

1. The given table shows number of regular readers of a library who have completely read the books.

Class Interval (books)	Frequency (No. of readers)
1 – 10	5
11 – 20	4
21 – 30	8
31 – 40	9
41 – 50	2
51 – 60	2

- (i) What is the total number of readers in the data?
 - (ii) Which group contains highest number of readers?
 - (iii) Which group contains least number of readers?
 - (iv) What is the lower limit of the last class?
 - (v) What is the lower boundary of the last class?
 - (vi) What is the size of the class interval?
 - (vii) Find class marks (mid points) of all groups.
2. Number of gratitude cards made by 80 student of class 9 is given below.
 79, 60, 74, 59, 55, 98, 61, 67, 89, 71, 71, 46, 63, 66, 69, 42, 75, 62, 71, 77, 78, 65, 87, 57, 78,
 91, 82, 73, 65, 94, 48, 87, 62, 81, 63, 66, 65, 49, 45, 51, 69, 56, 84, 93, 63, 60, 68, 51, 73, 54,
 50, 88, 76, 93, 48, 70, 40, 76, 95, 57, 63, 94, 82, 54, 89, 64, 77, 94, 72, 69, 51, 56, 67, 88, 81, 70,
 81, 54, 66, 87
- (i) Prepare a frequency table to represent the above data using the classes 40 – 49 etc.
 - (ii) Add cumulative frequency column in the table .
 - (iii) How many students made less than 70 cards?
 - (iv) What percent of students made less than 50 cards?
 - (v) Which group has the greatest frequency?
 - (vi) What is the size of the class interval?
3. The number of medals won by 45 players in a certain sports festival is given as :
 0, 2, 1, 0, 1, 2, 3, 5, 6, 3, 2, 1, 3, 4, 2, 6, 1, 5, 2, 4, 3, 0, 1, 2, 3, 0, 0, 2, 3, 4, 1, 5, 6, 2, 4, 5, 1, 3,
 4, 6, 2, 3, 1, 2, 5 .
 Prepare a discrete frequency distribution. Also make a column of cumulative frequencies.
4. The ages of workers in a factory were recorded as below.

Ages in Years	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
No. of workers	5	16	12	10	8	4

Draw a frequency polygon to represent the data.

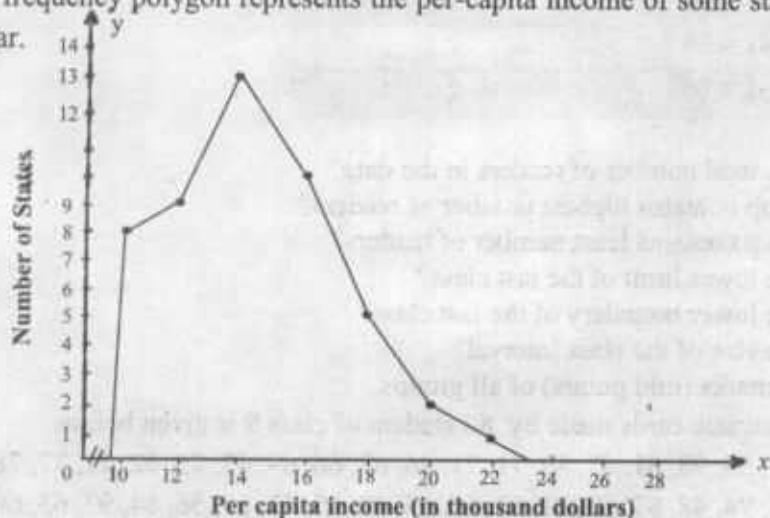
5. Represent the following data by a histogram.

Class Interval	0 – 5	5 – 15	15 – 30	30 – 40	40 – 45
Frequency	5	12	16	10	2

6. In a saving group, there are 400 members, and the number of savings certificates held by them is shown in the following table. Construct a histogram of the distribution of saving certificates.

Class Interval	1 – 50	51–100	101–150	151–200	201–300	301–400	401–500
No. of members	10	15	30	40	120	100	85

7. The following frequency polygon represents the per-capita income of some states, recorded in a particular year.



- What is the total number of states?
 - What is the number of states which have the least per-capita income?
 - What is the number of states having highest per-capita income?
 - What percent of states have per-capita income \$14000 and \$16000?
 - How many states have per-capita income \$18000 and above?
 - Represent the frequency polygon by a frequency table.
8. The masses measured to the nearest kg of 50 boys are given in the frequency distribution as :

Mass (kg)	60 – 64	65 – 69	70 – 79	80 – 89	90 – 94	95 – 99
f	2	6	12	14	10	6

- Construct a histogram.
 - Construct a frequency polygon.
9. At a sale, number of items placed according to the price range is given below:

Price(Rs)	600 – 999	1000 – 1999	2000–2499	2500 – 2999	3000– 3499
f	50	70	75	65	70

Construct a histogram.



11.2 Measures of Central Tendency

In this unit we are looking for a better representation of data. The purpose of summarizing a data is to draw meaningful results, to analyze the data and to compare two or more data having same characteristics. When the data have been arranged into a frequency distribution the information contained in it is easily understood. The data becomes clearer when it is represented by graphs. Still we need a simpler way to represent a data in more efficient way to analyze and understand better. A single value that represents a set of data is called an average. Since the averages tend to lie in the center of a data, they are called measures of central tendency. They are also called measures of location because they tend to be located at the center of the data. Here we will discuss 3 very common measures of central tendency.

- (i) Arithmetic Mean
- (ii) Median
- (iii) Mode

11.2.1 (a) Arithmetic Mean

There are two sections A and B of class 9 in a public school consisting of 10 and 12 students respectively. The marks secured by each student in a test out of 100 are given below :

A : 35, 40, 90, 45, 50, 80, 70, 65, 55, 50

B : 25, 50, 30, 35, 60, 45, 45, 85, 90, 65, 45, 35

It is required to compare the performance of two sections and we have to decide that which one of them is better than the other. It is difficult and lengthy to compare the individual performance of students. Let us see the total marks secured by the students in each section. The total of both sections are same. It is not good to say that performance of both sections is same, because they have different number of students. We have to compare sum of scores and numbers of students in each section simultaneously. To do this we will find the ratio of sum of scores and number of students in each section separately and then we will compare the two results obtained.

In section A, sum of scores: number of students = $610:10 = 61$

In section B, sum of scores: number of students = $610:12 = 50.83$

As $61 > 50.83$ so average performance of section A is better than that of section B.

"The ratio of sum of values of a data and number of values is called an arithmetic mean."

Arithmetic Mean for Ungroup Data

The values of data are usually denoted by x . In case of more than one data, their values are denoted by x, y or z etc. The sum of the values of the data ' x ' is denoted by " $\sum x$ " and number of values is denoted by n . The arithmetic mean of data x is symbolized a $\bar{x}$ and by above

definition:

$$\bar{x} = \frac{\sum x}{n} = \frac{\text{sum of values}}{\text{numbers of values}}$$

Arithmetic Mean for Group Data

If values of data occur more than once, the sum of values can be found by using frequencies e.g.

If $x = 2, 3, 1, 2, 2, 3, 2, 3, 4, 2, 5, 2, 3, 4, 5, 2, 1, 3, 5, 1$

Then, $\sum x = 2+3+1+2+2+3+2+3+4+2+5+2+3+4+5+2+1+3+5+1 = 55$

In this data frequency of 1 is 3 i.e. 1 occurs three times. Similarly frequencies of 2, 3, 4, 5, are 7, 5, 2, 3 respectively.

$$\sum x = 1 \times 3 + 2 \times 7 + 3 \times 5 + 4 \times 2 + 5 \times 3 = 3 + 14 + 15 + 8 + 15 = 55$$

Also $n = \text{number of values} = \text{sum of frequencies} = 3 + 7 + 5 + 2 + 3 = 20$

$$\text{So, } \bar{x} = \frac{\sum x}{n} = \frac{55}{20} = 2.75$$

The above data can be summarized in frequency distribution as follows:

x	1	2	3	4	5	Total
f	3	7	5	2	3	$\sum f = 20$
$f \cdot x$	$1 \times 3 = 3$	$2 \times 7 = 14$	$3 \times 5 = 15$	$4 \times 2 = 8$	$5 \times 3 = 15$	$\sum f \cdot x = 55$

$$\text{The arithmetic mean is } \bar{x} = \frac{\text{sum of values}}{\text{numbers of values}} = \frac{\sum f \cdot x}{\sum f} = \frac{55}{20} = 2.75$$

The arithmetic mean for a grouped data can also be found by using class marks ' x ' and class frequencies ' f ' with the help of formula $\bar{x} = \frac{\sum f \cdot x}{\sum f}$, if size of class interval is greater than 1

Example 6: Find the arithmetic mean of the following data

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of students	2	5	7	3	8

Solution:

Marks	f	Class marks	$f \cdot x$
0 – 10	2	5	10
10 – 20	5	15	75
20 – 30	7	25	175
30 – 40	3	35	105
40 – 50	8	45	360
Total	$\sum f = 25$	–	$\sum f \cdot x = 725$

$$\bar{x} = \frac{\sum f \cdot x}{\sum f} = \frac{725}{25} = 29$$

11.2.1 (b) Median

When data is arranged in some order, it can be represented by a single value that lies exactly at the centre of data. "The central or middle most value of an **arranged data** is called its median represented by $\tilde{x}$ read as x tilde .

Median for Ungroup Data

- If the number of values of a data is odd, then the middle value of the arranged data is the median.
- If the number of values of a data is even, then there are two middle values, the arithmetic mean of these two middle values is called median of the data. These definitions of median can be expressed for an ungrouped data as following formulas.

(i) If n (number of values) is odd, then median = $\frac{n+1}{2}$ th value of the data

(ii) If n is even, then Median = Arithmetic mean of the two middle values

$$= \frac{1}{2} \left\{ \frac{n}{2} \text{th value} + \frac{n+2}{2} \text{th value} \right\}$$

Example 7:

The selling prices of the seven articles sold by a shopkeeper in rupees were 1075, 1050, 2050, 1030, 2045, 1090 and 1530. Find the median selling price of these articles.

Solution:

Arranging these values in the ascending order as 1030, 1050, 1075, 1090, 1530, 2045, 2050.

Here $n = 7$ (odd), so

$$\text{Median} = \frac{n+1}{2} \text{th value} = \frac{7+1}{2} \text{th value} = \frac{8}{2} \text{th value} = 4^{\text{th}} \text{ value} = 1090$$

So, median selling price of seven articles is Rs.1090

Example 8:

Find the median of the following set of values.

40, 35, 45, 60, 50, 35, 40, 45, 35, 50

Solution: Arrange the numbers in ascending order as 35, 35, 35, 40, 40, 45, 45, 50, 50, 60

Here $n = 10$ (even), so

Median = $\tilde{x}$ = half of sum of two middle most values

$$\begin{aligned} &= \frac{1}{2} \left\{ \frac{n}{2} \text{th value} + \frac{n+2}{2} \text{th value} \right\} \\ &= \frac{1}{2} \left\{ \frac{10}{2} \text{th value} + \frac{10+2}{2} \text{th value} \right\} \\ &= \frac{1}{2} (5^{\text{th}} \text{ value} + 6^{\text{th}} \text{ value}) \\ &= \frac{1}{2} (40 + 45) = \frac{85}{2} = 42.5 \end{aligned}$$

Median for Grouped Data

In a grouped data or frequency distribution the median is taken as $\frac{n}{2}$ th value. To find median for a grouped data we first find the group which contains the median. To locate the median group we form cumulative frequency table. The formula used to find median of a group data is

$$\text{Median} = \tilde{x} = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

Where l = lower class boundary of median group

h = size of class interval

f = frequency of median group

$n = \Sigma f$ = Number of items

c = Cumulative frequency of class preceding the median class.

Example 9: Marks obtained by 100 students in an examination are given below,

Marks	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No of students	5	25	40	20	10

Find the median marks.

Solution: Median = $\frac{n}{2}$ th value = $\frac{100}{2}$ th value = 50th value.

Marks	f	Class Boundaries	$c.f$
10 – 19	5	9.5 – 19.5	5
20 – 29	25	19.5 – 29.5	$c = 30$
30 – 39	$f = 40$	29.5 – 39.5	70
40 – 49	20	39.5 – 49.5	90
50 – 59	10	49.5 – 59.5	100

→ median group

From the above table, it is clear that 50th value lies in the group 29.5 – 39.5. So 29.5 – 39.5 is the median group.

Median = $l + \frac{h}{f} \left(\frac{n}{2} - c \right)$ where $l = 29.5$, $h = 10$, $n = 100$, $f = 40$, $c = 30$

$$\begin{aligned}\text{Median} &= 29.5 + \frac{10}{40} \left(\frac{100}{2} - 30 \right) \\&= 29.5 + \frac{1}{4} (50 - 30) \\&= 29.5 + \frac{20}{4} \\&= 29.5 + 5 = 34.5 \text{ marks}\end{aligned}$$

Danger zone !!!
Most of the students take c from the median group which gives the negative result. remember c is taken from the group which comes before median group

11.2.1 (c) Mode

The number of pairs of shoes sold in a day according to their sizes at a shoe store are given below.

Sizes	6	7	8	9	10	11
No. of pairs sold	14	25	45	50	30	16

According to the data, the most commonly used shoes are of size 9. In other words we can say that the average size of shoes sold in a day at a shoe store is 9. The average found in this way is called mode. "The most frequent (repeated) value of a data is called mode of the data." If two or more values occur the same numbers of times but more frequently than any of the other values, then there is more than one mode. In this respect, mode differs from mean and median because they have unique values. If each value of data occurs same numbers of times, then there is no mode.

Example 10: Find mode of following data sets.

- (i) 30, 25, 40, 30, 45, 25, 30, 40, 30
- (ii) 1, 2, 3, 2, 1, 3, 1, 2
- (iii) 125, 130, 115, 125, 135, 120, 135, 130, 120

Solution:

- (i) As 30 is the most frequent value so it is mode of the data. The data is called unimodal.
- (ii) As the values 1 & 2 occur equal number of times, so there are 2 modes. This data is called bimodal data.
- (iii) As the values 120, 125 and 130 occur two times each and frequencies of remaining values are lesser. So 120, 125 and 130 are three modes of the data. This data is called trimodal data.

Mode for Group Data

To find mode for a frequency distribution or a group data, we first find the modal class. A class or group having largest number of items (frequency) is called modal group and it contains mode of the data. The mode of a grouped data can be found by the formula.

$$\text{Mode} = l + \frac{(f_m - f_1)}{2f_m - f_1 - f_2} \times h$$

Where l = lower class boundary of the modal class.

f_m = frequency of the modal class (the highest frequency)

f_1 = frequency of the class preceding the modal class

f_2 = frequency of the class following the modal class

h = size of class interval

Note: If first group of the distribution have the largest frequency, then $f_1 = 0$ and if modal group is the last group of frequency distribution, then $f_2 = 0$. But these are very rare cases.

Example 11:

Fifty workers of a factory are distributed in 7 groups according to their wages in rupees per hour.

Wages in Rupees	25-29	30-34	35-39	40-44	45-49	50-54	55-59
No of workers	2	4	8	20	6	6	4

Find mode of the distribution.

Solution:

Wages	f	Class boundaries
25 - 29	2	24.5 - 29.5
30 - 34	4	29.5 - 34.5
35 - 39	8	34.5 - 39.5
40 - 44	20	39.5 - 44.5 → modal class
45 - 49	6	44.5 - 49.5
50 - 54	6	49.5 - 54.5
55 - 59	4	54.5 - 59.5

As 20 is the largest frequency, so (39.5 - 44.5) is the modal group.

$$l = 39.5, f_m = 20, f_1 = 8, f_2 = 6, h = 5$$

$$\text{Mode} = l + \frac{(f_m - f_1)}{2f_m - f_1 - f_2} \times h$$

$$\text{Mode} = 39.5 + \frac{(20 - 8)}{2 \times 20 - 8 - 6} \times 5$$

$$= 39.5 + \frac{12}{40 - 14} \times 5$$

$$= 39.5 + \frac{12}{26} \times 5 = 39.5 + \frac{60}{26}$$

$$= 39.5 + 2.31 = 41.81 \text{ Rupees}$$

Math Play Ground

1. Make such hopscotch in the play ground.
2. Ask a player to start hoping, doing calculations in each box.
3. Decide the winner if he completes without falling.

Winner

Mode of

8, 8, 8?

Median of

8, 8, 8?

mean of

8, 8, 8?

Start

Example 12:

Find median and mode for the following discrete frequency distribution.

x	1	2	3	4	5
f	3	4	5	2	2

Solution:

$$\text{Here } n = \sum f = 16$$

$$\text{Median} = \text{class containing } \frac{n}{2} \text{th observation} = \frac{16}{2} \text{th observation} = 8 \text{th observation}$$

x	f	c.f.
1	3	3
2	4	7
3	5	12
4	2	14
5	2	16

→ 8th observation

So Median = 3 (8th observation lies in the group where c.f. is 12.)

Hence Mode = most frequent observation = 3

11.2.1 (d) Empirical Relation between Mean, Median and Mode

If any two of mean, median and mode of a data are given, then we can find an approximate value of the third measure of central tendency by using an empirical relation.

$$\text{mode} = 3 \text{ median} - 2 \text{ mean}$$

$$\text{Or } \text{median} = \frac{1}{3} (\text{mode} + 2 \text{ mean})$$

$$\text{Or } \text{mean} = \frac{1}{2} (3 \text{ median} - \text{mode})$$

11.2.2 Weighted Mean

If it is required to find the average of some values which are not equally important, then we assign them certain numerical values to express their relative importance. These numerical values are called weights. To find arithmetic mean of data whose values are not equally important, we use the weights instead of frequencies.

If x denotes the items of a data and w denotes their respective weights, then arithmetic mean is

$$\bar{x}_w = \frac{\sum wx}{\sum w}$$

Example 14: The following are the monthly expenditure of a house according to their weights.

Items	Expenditure (Rs)	Weight relative importance
Food	8,000	20
House rent	5,000	10
Clothing	1,000	6
Utility bills	900	4
Education	600	2
Miscellanies	100	1

Find the weighted mean.

Solution:

Items	x = Expenditure (Rs)	w = Weight	wx
Food	8,000	20	160000
House rent	5,000	10	50000
Clothing	1,000	6	6000
Utility bills	900	4	3600
Education	600	2	1200
Miscellanies	100	1	100
Total	—	43	220900

$$\text{Weighted mean, } \bar{x}_w = \frac{\sum wx}{\sum w} = \frac{220900}{43} = \text{Rs. } 5137.21$$

EXERCISE 11.2

1. Find arithmetic mean for the following data by definition

(i) $x = 2, 4, 6, 8, 10, 12$

(ii) $y = 3, 4, -1, 7, -8, -5, 0$

(iii) $z = 0, 4, 8, 12, 16, 20, 24, 28$

(iv) $u = 3.1, 4.2, 5.3, 6.4, 7.5, 8.6, 9.7, 10.8$

(v) $v = 5, 5, 5, 5, 5, 5, 5, 5$

2. Following are the scores made by two batsmen A and B in a series of innings:

A	12	15	6	73	7	19	199	36	84	29
B	47	12	76	48	4	51	37	48	13	0

Find arithmetic mean of scores of both players and state who is better as run getter?

3. Marks of four students of class 9 in four subjects out of 100 are given below. The weights of subjects according to their importance are also given.

Subjects	Weights	Marks			
		Hanzala	Musa	Zaki	Saleh
Mathematics	4	70	80	90	95
Physics	3	95	60	55	70
Chemistry	2	80	65	50	60
Biology	1	55	45	65	60

Find weighted mean of marks of each student.

4. Following frequency distribution represents number of liters of fuel consumed against the distance in km. Find arithmetic mean of this data set.

Class intervals	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequencies	5	7	10	6	2

5. Find arithmetic mean from the following data of number of a detergent packs sold against its mass in kg.

Classes(kg)	1 – 9	10 – 18	19 – 27	28 – 36	37 – 45
No. of packs	10	15	18	15	12

6. Find median for the following data.

(i) 1, 4, 2, 5, 3, 7, 6

(ii) $\pm 2, \pm 4, \pm 6, \pm 8$

(iii) 0, 1, -2, -3, 4, 5, 6, 3

(iv) 4, 3, 1, -3, 2, -3, 3, 4, 1

(v) 4, 4, 4, 4, 4, 4

7. In a practical of geometry and shapes students are required to construct a circle of any radius between 0cm and 2cm. the data given below is recorded.

Radius (xcm)	1.8	1.6	1.4	1.2	1.0	0.8	0.6	0.4	0.2
No. of circles (f)	6	9	15	24	33	20	13	7	5

Find median radius of the circle.

8. At an old age house, there are people of different age groups as given below.

Classes intervals	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
f	3	8	11	5	4

Find an average median age of people residing in that old age house.

9. Find mode for the following data.

(i) 1, 2, 3, 4, 5, 6

(ii) 2, 4, 2, 3, 2, 5, 3, 2, 5, 4, 2

(iii) 120, 130, 140, 225, 125, 225, 120

(iv) 2, 4, 3, 5, 5, 3, 4, 2

10. Find mean, median and mode for the following discrete frequency distribution.

x	0	1	2	3	4	5
f	3	5	9	4	2	1

11. Find modal value for the following frequency distributions.

(a)

Classes	1 – 4	5 – 8	9 – 12	13 – 16	17 – 20
f	9	8	10	6	7

(b)

Classes	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	7	6	3	2	1

11.3 Probability

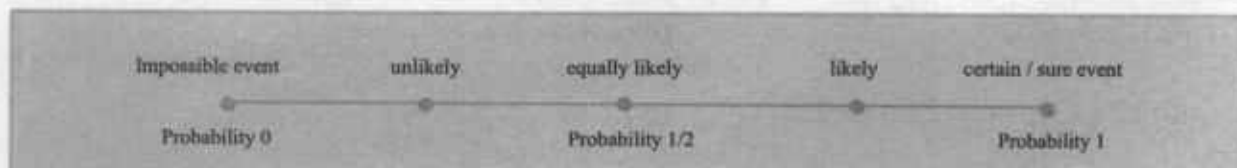
We often come across such situations where we have to make guess about occurrence or non occurrence of something that it would happen or not. For instance we often make such statements :

- 40% students of class 9 are predicted to be absent tomorrow because of severe heat wave.
- You will probably win Maths Olympiad this year .
- I hope you will get at least 90% marks in your board exam .

In these statements one can notice the uncertainty or doubt of occurrence. These are few of the guesses made in our daily life for the possibility of an event that the event is likely to happen or not. Probability is synonymous to possibility. Simply speaking the measure of chance of an event is called probability and its vlues are from 0 to 1. For example

- Probability of a new prophet after Hazrat Muhammad (SAW) is 0.
- Probability that Haleema will win the toss among Haleema, Hajra & Haani is $1/3$.
- Probability of sun setting in west is 1.

In these statements we have seen that probability is between 0 and 1 both inclusive. This range can also be presented pictorially for better understanding as



In previous classes you have learnt about various terms necessary to understand the problems of probability i.e.

Experiment, trial, outcome, sample space, event, impossible event, sure event, equally likely, likely, unlikely, mutually exclusive events, exhaustive events, independent and dependent events etc. Here we will discuss single event and events not happening in a bit detail.

11.3.1 Single Event Probability

An event which has single sample point is called a single event or simple event or singular event. Eg in tossing a coin the sample space is $\{H, T\}$, where $E_1 = \{H\}$ and $E_2 = \{T\}$, as only one out of 2 outcomes is possible. Here both E_1 and E_2 are single events.

Example 15: Write sample space for single roll of a die and enlist all the events. Discuss whether any compound event is present in this case.

Solution: Here we notice if a single die is rolled once then any one of the outcome may occur from 1, 2, 3, 4, 5 and 6. Therefore $S = \{1, 2, 3, 4, 5, 6\}$

Now all possible events are $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$

from events listed above, its evident that all events are single. So no compound event in this case.

The probability of such events is obtained by the formula:

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{total no. of outcomes}}$$

Example 16: Write sample space for single spin of a spinner of 3 primary colours and enlist all the events. Discuss probabilities of all events.

Solution: Here $S = \{\text{blue, red, green}\}$, the pointer of the spinner can stop at only one of the colour so $E_1 = \{\text{blue}\}$, $E_2 = \{\text{red}\}$, $E_3 = \{\text{green}\}$

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Example 17: Ali picked a book at random from a library shelf which had 3 history books of grade 6, 4 geography books grade 7, 9 science books of grade 8, 3 science books of grade 7, 6 geography books of grade 8 and 5 science books of grade 6. Find the probability of getting:

- a science book
- a geography book
- a book of grade 7
- a book of grade 9
- a book of mathematics

Solution: Here total number of books is 30.

- total science books are 17. So $P(E_1) = \frac{17}{30}$
- total geography books are 10. So $P(E_2) = \frac{10}{30} = \frac{1}{3}$
- total books of grade 7 are 7. So $P(E_3) = \frac{7}{30}$
- there is no book of grade 9. So $P(E_4) = \frac{0}{30} = 0$
- there is no book of mathematics. So $P(E_5) = \frac{0}{30} = 0$

11.3.2 Probability of an Event not occurring

An event which describes the non occurrence of another event is called non occurring event.

If 4 boys Affan, Jaffar, Hassan and Maaz are playing ONO, then probability of winning of each boy is $\frac{1}{4}$. If the event of winning of jaffar is represented by J, then $P(J) = \frac{1}{4}$. Now the event that Jaffar won't win is represented by J' and $P(J') = 1 - P(J) = 1 - \frac{1}{4} = \frac{3}{4}$

The two events in which one shows non occurrence of other are called complementary events.

REMEMBER: Sum of Probabilities of 2 complementary events is unity.

Example 18: There are 12 teachers in maths department among them 2 are higher secondary teachers, 4 are secondary teachers and 6 are senior elementary teachers. If good performance award is to be given to only one teacher then find probability

- That a secondary teacher will get the award
- That a secondary teacher will not get the award

Solution: Here total number of teachers is 12. If this is assumed that all teachers are equally hardworking, dedicated and showing excellent performance throughout their career then the events will be equally likely.

- Probability that a secondary teacher will get the award is $P(E) = \frac{4}{12} = \frac{1}{3}$
- Probability that a secondary teacher will not get the award is $P(E') = 1 - P(E) = 1 - \frac{1}{3} = \frac{2}{3}$

11.3.3 Relative Frequency as an Estimate of Probability

When theoretical probability is not certain, we use relative frequency to estimate probabilities, as in most of the cases it is hard to predict the probability of an event happening finely.

Relative frequency is defined as the ratio between no. of times an event is occurring in an experimental trial and total number of trials in the experiment. i.e.

$$\text{Relative Frequency} = \frac{\text{frequency of an event occurring}}{\text{total no. of trials in an experiment}}$$

- The relative frequency is often calculated after performing experiment so it is often called experimental probability.
- More the no. of experiments done, closer the relative frequency becomes to the theoretical probability.
- Empirical probability is another name of relative frequency.

Example 19: A spinner of 5 equally likely colours red, blue, green, yellow and white is spun 50 times and it landed 10 times on red, 12 times on blue, 9 times on green, 9 times on yellow and 10 times on white colour. Find the relative frequency for the event that it lands on green colour.

Solution: Total No. of trials = $n = 50$

No. of favourable outcomes for landing on green colour = $f = 9$

$$\text{Relative Frequency} = \frac{\text{frequency of an event occurring}}{\text{total no. of trials in an experiment}} = \frac{9}{50}$$

Example 20: A fair coin is tossed 150 times and head comes up 70 times. What is the relative frequency of getting head up.

Solution: Total No. of trials = $n = 150$ No. of favourable outcomes for getting head up = $f = 70$

$$\text{Relative Frequency} = \frac{\text{frequency of an event occurring}}{\text{total no. of trials in an experiment}} = \frac{70}{150} = 0.466\ldots$$

11.3.4 Expected Frequency

In probability we often come across the situation where the probability of an event becomes slightly different from its theoretical probability. The expected frequency is defined as how often (we think) an event should happen on the basis of theoretical probability or on the basis of previously happened experiments (relative frequency).

For instance, If Ali rolled a fair die it is expected that 4 will happen $1/6$ times. If he rolls it 120 times, then he expects 4s to happen $120 \times 1/6 = 20$ times. Hence expected frequency is given by the formula: Expected Frequency = No. of trials $\times$ probability of the event = $n \times p$

Example 21: 5 students out of a section of 30 students are expected to be absent everyday in summer camp. What No. of students is expected to be absent every day from 12 such sections of same strength.

Solution: Total no. of students in 12 sections = $n = 12 \times 30 = 360$

$$\text{Probability of absent students} = p = \frac{5}{30} = \frac{1}{6}$$

$$\text{Expected frequency} = \text{No. of trials} \times \text{probability of the event} = n \times p = 360 \times \frac{1}{6} = 60$$

Example 22: If Probability of a person to be Hafiz e Quran in a village is $\frac{3}{10}$. What do you expect the no. of Hafiz e Quran in 2500 such villages?

Solution: Total No. of villages = $n = 2500$ and Probability = $p = \frac{3}{10}$

Expected No. of Hafiz e Quran = No. of villages $\times$ probability of the event

$$= n \times p = 2500 \times \frac{3}{10} = 750$$

EXERCISE 11.3

- A letter is chosen randomly from the word 'ALLAH'. Now find the probability of getting
(i) a vowel (ii) an H (iii) an L (iv) a consonant
- A garment factory shipped an order which contained 7000 jackets, 2000 sweaters, and 3000 trousers in which 7 trousers, 5 jackets and 9 sweaters are faulty. If the quality inspector unpacked one item at random, find the probability that it is
(i) a trouser (ii) not a jacket (iii) a faulty item
- A number wheel is divided into 8 equal sectors labelled as pansy, lily, orchid, tulip, jasmine, rose, marigold and sunflower. Maazz spins the wheel once. Find the probability that pointer:
(i) Stops at rose (ii) stops at a 4 letter flower name
(iii) Doesnot stop at marigold (iv) stops at a 3 letter flower name
(vi) stops at a flower name
- A decagonal die labelled 4, 4, 4, 4, 5, 5, 6, 7, 8, 8 is rolled once. Find the probability of getting:
(i) a 4 (ii) an even number (iii) a multiple of 4
(iv) not a 7 (v) an odd number (vi) a prime number
(vii) LCM of 4 and 8 (viii) HCF of 4 and 8 (ix) factor of 12
- A one digit whole number is chosen at random. Find the probability that it is:
(i) less than 5 (ii) greater than 10
(iii) not the largest 1 digit number (iv) additive identity of whole numbers
(v) HCF of 3 and 5 (vi) multiplicative identity of real numbers
(vii) not a prime number (viii) factor of 9
- A coin is tossed 10 times. Complete the given table and answer the questions below the table.

Event	frequency	Relative frequency
head	6	
tail	4	

- What is the expected frequency of getting head, if it was tossed 520 times?
- What is the expected frequency of getting tail, if it was tossed 305 times?

7. Complete the table and give answers, if a die is rolled 120 times

Number	Frequency	Relative frequency
1	15	...
2	25	...
3	15	...
4	20	...
5	30	...
6	15	...

- (i) if it was rolled 6000 times, how many times occurrence of a 6 is expected?
(ii) if it was rolled 3000 times, how many times occurrence of a 1 is expected?
(iii) if it was rolled 400 times, how many times occurrence of a 2 is expected?
8. There are 6 girls sections of class 9 in a public school, namely Teal, orchid, mauve, hazel, zaffre and denim. Only one girl is to be chosen randomly from all sections for national science olympiad.
- (a) Find the probability of selecting a girl from hazel section.
(b) Find the probability of not selecting a girl from hazel section.
(c) Verify that answers of both parts a and b add up to unity.
(d) If 6 girls are to be selected, find expected frequency of selecting from orchid section.
(e) If 60 girls are to be selected, what number of girls is expected to be chosen from Teal.

KEY POINTS

- Statistics is science of collecting and analyzing numerical data.
- Facts and statistics collected for analysis is called data.
- A table consisting of values of a data along with their frequencies is called a frequency table.
- In a group data, the table consisting of class intervals and their respective frequencies is called frequency table or a frequency distribution.
- The representation of data by a chart having adjacent rectangular bars is called histogram.
- The representation of data by a closed polygon in which class marks are plotted along x-axis and frequencies along y-axis is called a frequency polygon.
- A single value which can represent the whole data is called measure of central tendency, measure of location or an average.
- Measures of central tendency discussed in this grade are arithmetic mean, median and mode.
- Arithmetic mean is a ratio between the sum of items and number of items of the data.
- The middle most value of an arranged data is called median average or simply median.
- The most frequent value of the data is called modal average or mode.
- To find arithmetic mean of data whose values are not equally important, we use the weights instead of frequencies. This is called weighted arithmetic mean.

- The measure of chance of an event is called probability.
- The values of probability are from 0 to 1.
- The two events in which one shows non occurrence of other are called complementary events.
- Relative frequency is defined as the ratio between no. of times an event is occurring in an experimental trial and total number of trials in the experiment.
- The expected frequency of an event is the frequency we expect to see based on probabilities.

MISCELLANEOUS EXERCISE II

1. Encircle the correct option in the following.

- Which of the following is a class mark of the interval (10 – 15)?
(a) 10 (b) 12.5 (c) 15 (d) 16
- What is size of class interval (4 – 7)?
(a) 4 (b) 5 (c) 6 (d) 7
- Which of the following is chart of adjacent rectangles?
(a) bar graph (b) Frequency polygon (c) histogram (d) ogive
- Which of the following is measure of central tendency?
(a) variance (b) standard deviation (c) range (d) arithmetic mean
- Which of the following is a formula of arithmetic mean for grouped data?
(a) $\frac{\sum x}{n}$ (b) $l + \frac{h}{f}(\frac{n}{2} - c)$ (c) $\frac{\sum fx}{\sum f}$ (d) $l + \frac{(f_m - f_1) \times h}{2f_m - f_1 - f_2}$
- If arithmetic mean of 25 values is 10, then what is sum of values?
(a) 250 (b) 125 (c) 25 (d) 2.5
- What is median of the data 4, 3, 0, 2, 1?
(a) 0 (b) 2 (c) 3 (d) 4
- Which measure of central tendency can have more than one value?
(a) weighted mean (b) median (c) mode (d) arithmetic mean
- The probability of getting M in 'MUHAMMAD' is
(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{3}{5}$ (d) none of these
- Probability of picking an ace from a well shuffled pack of 52 playing cards is
(a) $\frac{1}{52}$ (b) $\frac{1}{13}$ (c) $\frac{4}{13}$ (d) none of these
- A normal fair die is rolled 6000 times. The expected number of 5 is
(a) 100 (b) 5000 (c) 1000 (d) none of these
- A fair coin is tossed 500 times. Expected number of tails is
(a) 100 (b) 250 (c) 1000 (d) none of these
- in a group of 5 people, 4 like peach juice. The expected no. of people in a population of 1200 who like peach juice is
(a) 240 (b) 600 (c) 960 (d) none of these

- xiv. How many times Haani should toss a fair coin if he expects to get 100 tails
 (a) 100 (b) 200 (c) 1000 (d) none of these
- xv. An event which can never happen is called
 (a) sure event (b) certain event (c) possible event (d) impossible event
- xvi. Two such events whose probabilities are $\frac{1}{2}$ each are called
 (a) likely (b) equally likely (c) unlikely (d) none of these
- xvii. The probability of a certain event is
 (a) 0 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1
- xviii. The probability of an event can take the value
 (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) all of these
- xix. The probability of an event cannot take the value
 (a) 0 (b) 1 (c) 2 (d) all of these
- xx. The probability of an impossible event is
 (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) all of these

2. For the following data, find mean, mode and median.

x	5	10	15	20	25	30	35	40
f	2	4	6	8	10	7	5	3

3. Find the median for the following frequency distribution.

Marks	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79
Frequency	3	8	11	5	4

4. The arithmetic mean of 10 values is 35.5. If nine values are 20, 23, 37, 48, 29, 33, 45, 40, 45, find the tenth value

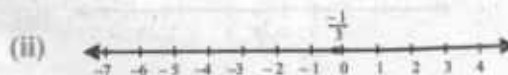
5. Find mean, median and mode for the following frequency distribution.

Classes	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
f	1	5	7	5	1

6. Affan is learning to play dart. He plays it 20 times each day and probability of hitting bull's eye is $\frac{1}{10}$. Find:
 (a) expected number of hits in 25 days. (b) expected number of misses in 25 days.
7. Shifaa usually gets late for her school 2 days out of 6 working days of a week. What is the expected no. of days of her late arrival in 4 weeks.
8. As'ha tries her best to score 100% in each maths assessment, but the probability of getting this is 0.8. Find the expected number of assessments in which she will score 100% out of total 20 assessments.
9. In a lab shelf there are 50 reports of covid 19. Among these 13 are females and 2 of the females are covid positive. If a report is picked at random, find:
 a) The probability of getting a female report
 b) The probability of getting a male report
 c) The probability of getting a female covid negative report.

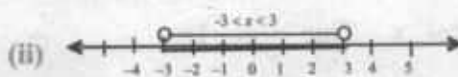
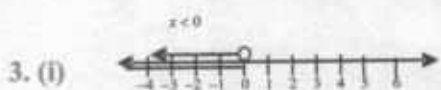
ANSWERS

Exercise 1.1



(iv) to (viii) Do yourself.

2. (i) multiplicative identity (ii) multiplicative inverse (iii) commutative property w.r.t. '+'
 (iv) additive inverse (v) associative property w.r.t. '+'
 (vi) distributive property of multiplication over subtraction



(iv) to (vi) Do yourself

4. (i) reflexive property (ii) transitive property (iii) symmetric property
 (iv) multiplicative property (v) transitive property (vi) cancellation property
 (vii) transitive property (viii) cancellation property

Exercise 1.2

1. (i) $6^{\frac{2}{3}}$ (ii) $\sqrt[3]{2^5}$, 2 (iii) $\sqrt[4]{x^4}$, x (iv) product rule is not applicable

- (v) $\sqrt[4]{x^2}$, $x^{\frac{1}{2}}$ (vi) $\sqrt[3]{10^3}$, 10 (vii) $\sqrt[3]{10^2}$, 10 (viii) product rule is not applicable

2. (i) $\sqrt[3]{216^2}$, 36 (ii) $\sqrt{29}$ (iii) $\sqrt{\frac{1}{32}}$, $\frac{1}{2}$ (iv) $\sqrt[3]{(216)^{-2}}$, $\frac{1}{36}$ (v) $\sqrt[3]{1000}$, 10 (vi) $\sqrt{39}$

3. (i) $5^{\frac{2}{3}}$ (ii) $10^2 = 100$ (iii) $-6^2 = -36$ (iv) $6^2 = 36$ (v) $-5^{\frac{2}{3}}$ (vi) $-10^2 = 100$

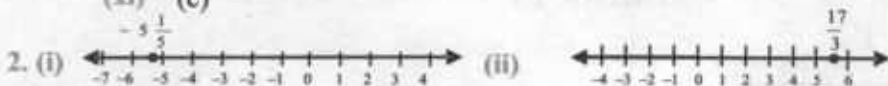
4. (i) 2^3 (ii) $7^{\frac{4}{3}} - 7$ (iii) $2^{\frac{13}{42}}$ (iv) $\frac{1}{\sqrt{3}}$ (v) 1 (vi) $\frac{3b^2}{a}$ (vii) 1

Exercise 1.3

1. 16,289 6. Rs. 77.7
 2. No 7. 1
 3. 43.5 8. Capital = Rs. 12,00,000, debt = Rs. 8,00,000
 4. 42 km 9. F = -128.56, K = 183.8
 5. B 10. Farah = 76,144.83, Maryam = 152,289.66, Tehreem = 228,434.49

Miscellaneous Exercise 1

1. (i) (b) (ii) (d) (iii) (d) (iv) (c) (v) (a)
 (vi) (d) (vii) (c) (viii) (c) (ix) (b) (x) (a)
 (xi) (c)



3. (i) $\sqrt[3]{16}$ (ii) $\sqrt[3]{(-3)^3}, -3$ (iii) $2^8, 128$ (iv) $(-2)^9, -256$ (v) $x^{-6}, \frac{1}{x^6}$

4. (i) $(-2)^3, -8$ (ii) $2^3, 3^3$

5. a. False b. False c. True c. True

Exercise 2.1

1. (i) 5.3407×10^{-4} (ii) 5.340×10^7 (iii) 1.20×10^{-11} (iv) 2.5326×10^0
 2. (i) 0.00009067 (ii) 5.64 (iii) 0.00000653 (iv) 3141500000
 3. (i) 1.4318234×10^4 (ii) 2300000 (iii) 1.2915783×10^{-2} (iv) 9988000
 4. 1389 hrs app. & 58 days app. 5. 8.3535 minutes

Exercise 2.2

1. (i) no (ii) no (iii) yes (iv) no
 2. (i) $6^3 = 216$ (ii) $\log_7 2401 = 4$ (iii) $5^5 = x$ (iv) $\log_b \frac{1}{27} = \frac{-3}{4}$ (v) $\log_{125} 25 = \frac{x}{3}$
 (vi) $10^{12} = 10^y$ (vii) $\log_{256} \frac{1}{64} = \frac{x}{4}$ (viii) $x^J + 1 = 3^2$ (ix) $(2x - 3) = 5^1$ (x) $\log_2 (2x + 1) = 3$
 3. (i) $x = 3$ (ii) $x = 2$ (iii) $x = 4$ (iv) $x = 5$ (v) $x = 16$ (vi) $x = \pm 3$
 4. (iii) 3125 (iv) 81 (v) 2 (vi) 12 (vii) -3
 (viii) 2 (ix) 4 (x) 3.5

Exercise 2.3

1. 3.7253 2. 3.6609 3. 0.9828 4. 2.0449 5. 1.7192
 6. -1.9183 or 2.0817 7. -2.0315 or 3.9685 8. 12.00 9. -4.3979 or 5.6021
 10. 0.6020 11. 3.6020 12. 1.7324 13. 0.0039
 14. $1.0039 = -0.9961$ 15. impossible

Exercise 2.4

1. 270.6 2. 38.82 3. 1.748 4. 3661 5. 10000 6. 1.009
 7. 0.1749 8. 0.08474 9. 0.003326 10. 1.000 11. 0.0010 12. 997700
 13. 309.0 14. 3.090 15. 218.8

Exercise 2.5

1. (i) $2 \log 3 + \log t$ (ii) $\log 59 - \log s$ (iii) $\log 5 + \log p + 2 \log q - \log x - 3 \log y$
 (iv) $\frac{1}{2} (\log 53.3 - \log 46.4)$ (v) $2 \log 5 + 5 \log t + \frac{1}{3} \log a - \frac{1}{3} \log 4.4 - \frac{1}{3} \log t - 3 \log b$
 (vi) $\frac{2}{5} \log 7 + \frac{3}{5} \log t + \frac{1}{5} \log p - \frac{6}{5} \log d - \frac{2}{5} \log b$
 (vii) $\frac{t}{3} \log 5.512 + t \log p + \frac{t}{2} \log m - \frac{t}{4} \log 5.91 - 2t \log a - t \log b$
2. (i) $\log \frac{x^3}{y^5}$ (ii) $\log \frac{\sqrt{t} \times \sqrt[3]{r}}{\sqrt[5]{s}}$ (iii) $\log \sqrt[7]{\frac{57.7 \times (36.6)^4}{(9.24)^3 \times (23.3)^2}}$
 (iv) $\log \frac{6^5 t^{\frac{1}{3}} \times \sqrt[3]{a^2}}{\sqrt{32.2 \times (9.42)^7}}$ (v) $\log^4 \sqrt{\frac{(37.74)^5 \times 28.83}{53.71}}$
3. (i) 3.9069 (ii) $\frac{1}{3}$ (iii) 3.7997 (iv) 7.7942 (v) 6
4. (i) 0.0791 (ii) 1.0458 (iii) 0.1688 (iv) -1.6198 or $\bar{2}.380$ (v) 0.1046

Exercise 2.6

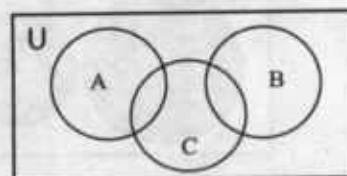
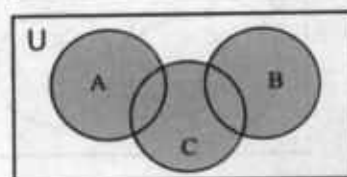
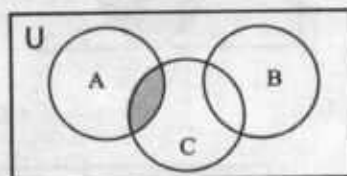
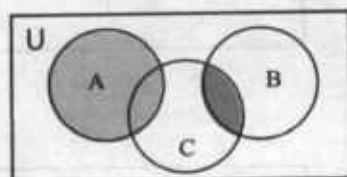
1. (i) 15 (ii) 201 (iii) 4 (iv) 26 (v) 82 (vi) 21
2. (i) 140.6 (ii) 8.914 (iii) 6.209 (iv) 0.5664 (v) 0.6647 (vi) 0.5631
 (vii) 1.090 (viii) 0.03301
3. app. 5 times

Miscellaneous Exercise 2

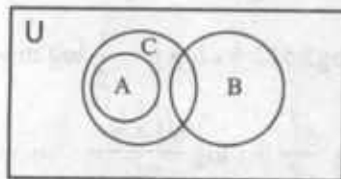
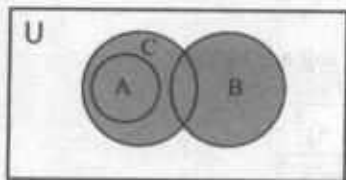
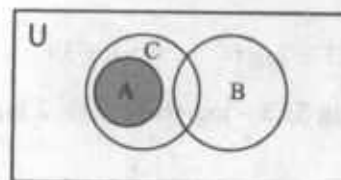
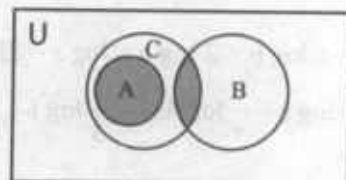
1. (i) (d) (ii) (c) (iii) (d) (iv) (a) (v) (a) (vi) (b)
 (vii) (d) (viii) (c) (ix) (b) (x) (d) (xi) (b) (xii) (b)
 (xiii) (b) (xiv) (b) (xv) (c) (xvi) (a) (xvii) (c) (xviii) (c)
2. (i) 5.336×10^1 (ii) 1.02×10^{-14} (iii) 5.234×10^{-1}
3. (i) 0.07232 (ii) 21386.43 (iii) 5.6 4. (i) 0 (ii) 2 (iii) 1
5. (i) 2 (ii) 16 (iii) ± 2 (iv) 8
6. (i) 1.4771 (ii) 1.3802 (iii) 2.5563 7. 31626957.07

Exercise 3.1

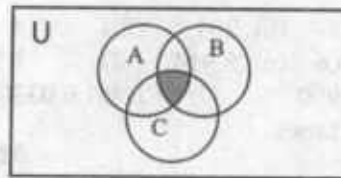
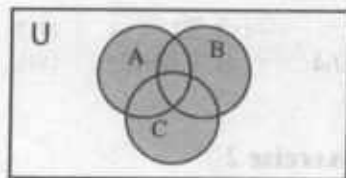
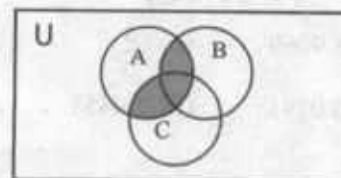
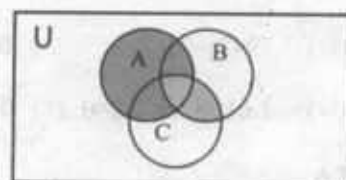
1. (i)



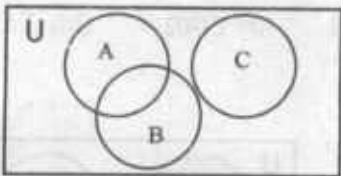
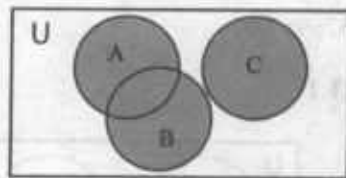
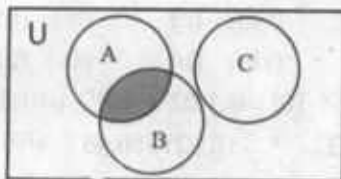
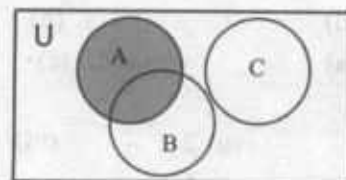
(ii)



(iii)



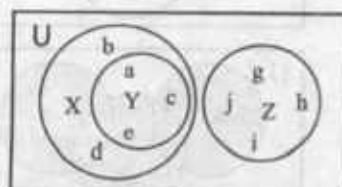
(iv)



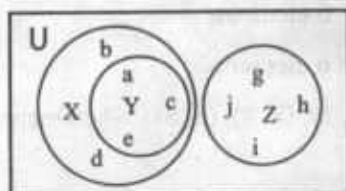
2.

(i) & (ii)

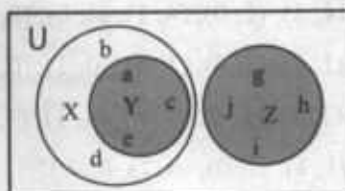
(iii) & (iv)



(v)



(vi)

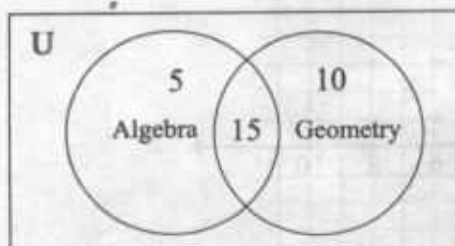


Exercise 3.2

1. 11

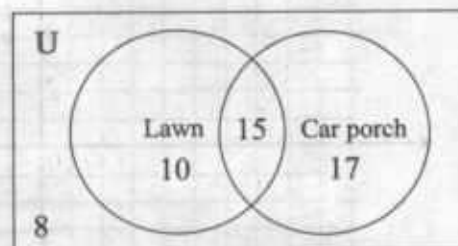
2. $n(B - A) = 19$, $n(B) = 21$

3.



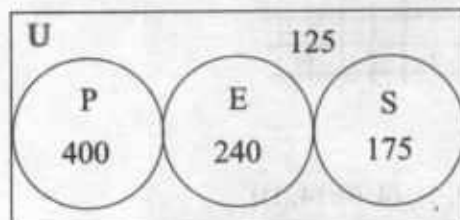
25 students like geometry.

4.



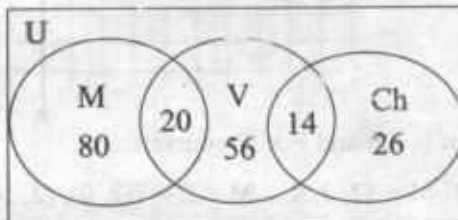
Houses without lawn & porch: 8

5.

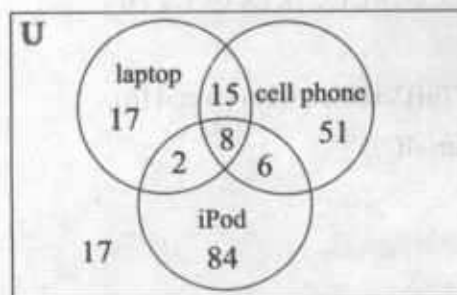


Out of school children: 125

6.

only mango: 80, only vanilla: 56
only chocolate: 26

7.



8. 480

9. 32

10. 18

11. 96

12. Arabic only: 40, French only: 100, Both: 152

13. (i) 210 (ii) 20 (iii) 380

14. 38, 22

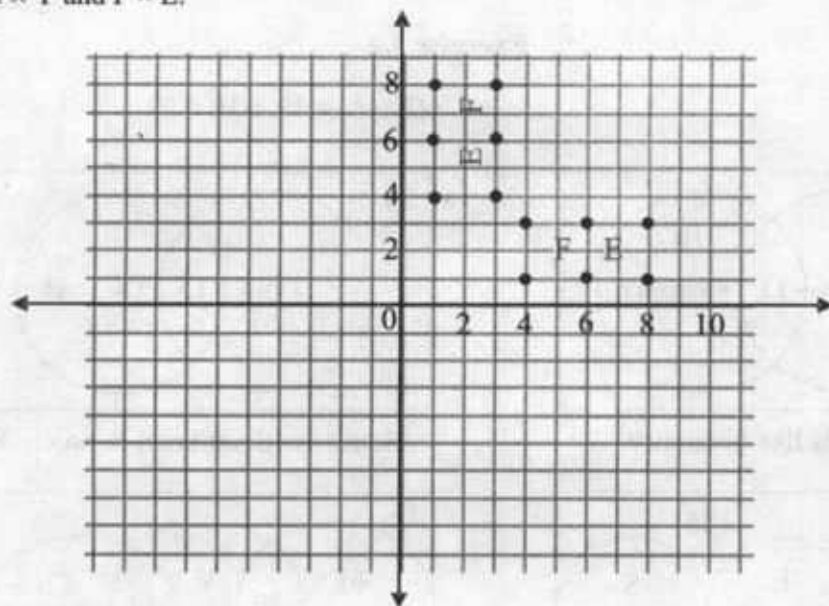
Exercise 3.3

1. (i) $a = 7, b = -1$ (ii) $a = -5, b = -1$ (iii) $a = 6, b = -5$ (iv) $x = -14, y = 8$

(v) $x = -6, y = -16$ (vi) $x = 1, y = \frac{1}{3}$ (vii) $x = \frac{5}{6}, y = \frac{11}{6}$

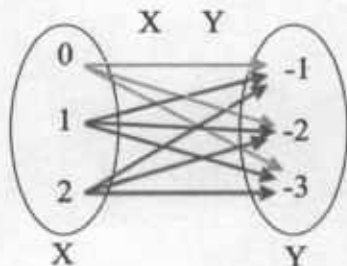
Answers

2. (i) $A \times B = \{(1, 1), (1, 0), (4, 1), (4, 0), (8, 1), (8, 0)\}$ 6 elements
 (ii) $B \times A = \{(1, 1), (1, 4), (1, 8), (0, 1), (0, 4), (0, 8)\}$ 6 elements
 (iii) $A \times A = \{(1, 1), (1, 4), (1, 8), (4, 1), (4, 4), (4, 8), (8, 1), (8, 4), (8, 8)\}$ 9 elements
 (iv) $B \times B = \{(1, 1), (1, 0), (0, 1), (0, 0)\}$ 4 elements
3. Graph of $E \times F$ and $F \times E$:



Draw graphs of $E \times E$ and $F \times F$ yourself.

4. $L = \{0, 1\}$, $M = \{2, 3, 4\}$, $M \times L = \{(2, 0), (2, 1), (3, 0), (3, 1), (4, 0), (4, 1)\}$
5. (i) $\{(1, 2), (1, 4), (1, 6), (1, 7), (3, 2), (3, 4), (3, 6), (3, 7), (5, 2), (5, 4), (5, 6), (5, 7)\}$
 (ii) $\{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4), (1, 6), (1, 7), (3, 6), (3, 7), (5, 6), (5, 7)\}$
 (iii) Verified from (i) and (ii).
6. (i) $\{(a, c), (e, c), (i, c)\}$ (ii) $\{(a, c), (e, c), (i, c)\}$ (iii) Verified from (i) and (ii).
7. (i) $LHS = RHS = \{(1, 1), (2, 1)\}$ (ii) Verify yourself.
- 8.



Draw arrow diagram for $X \times Y$ yourself.

Exercise 3.4

1. (i) $2^6 = 64$ (ii) 2^{49} (iii) 2^9
2. (i) $8, R_1 = \{\}$, $R_2 = \{(\sqrt{2}, \sqrt[3]{5})\}$, $R_3 = \{(\sqrt{3}, \sqrt[3]{5})\}$, $R_4 = \{(\sqrt{5}, \sqrt[3]{5})\}$,
 $R_5 = \{(\sqrt{2}, \sqrt[3]{5}), (\sqrt{3}, \sqrt[3]{5})\}$, $R_6 = \{(\sqrt{2}, \sqrt[3]{5}), (\sqrt{5}, \sqrt[3]{5})\}$, $R_7 = \{(\sqrt{3}, \sqrt[3]{5}), (\sqrt{5}, \sqrt[3]{5})\}$,

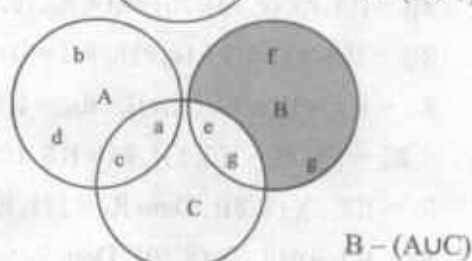
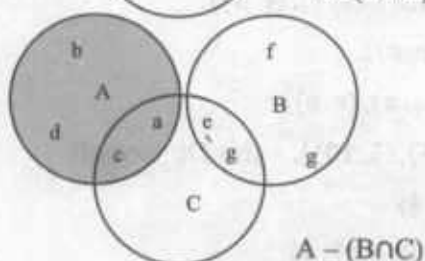
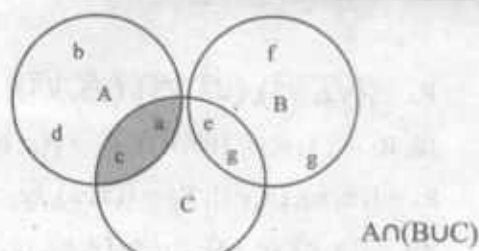
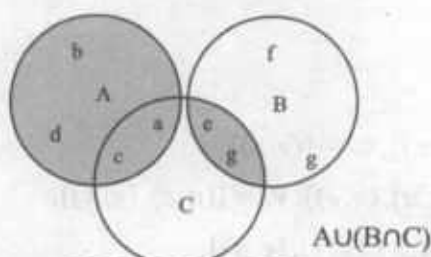
$$R_8 = \{(\sqrt{2}, \sqrt[3]{5}), (\sqrt{3}, \sqrt[3]{5}), (\sqrt{5}, \sqrt[3]{5})\}.$$

- (ii) $16, R_1 = \{ \}, R_2 = \{(\pi, \pi)\}, R_3 = \{(\pi, e)\}, R_4 = \{(e, \pi)\}, R_5 = \{(e, e)\},$
 $R_6 = \{(\pi, \pi), (\pi, e)\}, R_7 = \{(\pi, \pi), (e, \pi)\}, R_8 = \{(\pi, \pi), (e, e)\}, R_9 = \{(\pi, e), (e, \pi)\},$
 $R_{10} = \{(\pi, e), (e, e)\}, R_{11} = \{(e, \pi), (e, e)\}, R_{12} = \{(\pi, \pi), (\pi, e), (e, \pi)\},$
 $R_{13} = \{(\pi, \pi), (\pi, e), (e, e)\}, R_{14} = \{(\pi, \pi), (e, e), (e, \pi)\},$
 $R_{15} = \{(\pi, e), (e, e), (e, \pi)\}, R_{16} = \{(\pi, \pi), (\pi, e), (e, \pi), (e, e)\}$
- (iii) $4, R_1 = \{ \}, R_2 = \{(5, 1)\}, R_3 = \{(5, 10)\}, R_4 = \{(5, 1), (5, 10)\}.$ (iv) Do yourself.
3. $R_1 = \{(7, 7), (7, 8)\}, \text{Dom } R_1 = \{7\}, \text{Rang } R_1 = \{7, 8\}$
 $R_2 = \{(7, 9), (8, 7), (8, 9)\}, \text{Dom } R_2 = \{7, 8\}, \text{Rang } R_2 = \{7, 9\}$
4. (i) $\{(5, 5), (7, 7), (9, 9)\}$
(ii) $\{(5, 7), (5, 9), (5, 11), (6, 7), (6, 9), (6, 11), (7, 9), (7, 11), (8, 9), (8, 11), (9, 11)\}$
(iii) $\{(6, 5), (7, 5), (8, 5), (8, 7), (9, 5), (9, 7)\}$ (iv) $\{(6, 7), (8, 9)\}$
(v) $\{(6, 5), (8, 7)\}$ (vi) $\{(5, 7), (7, 9), (9, 11)\}$
5. (i) $R = \{(2, 4), (2, 6), (2, 8), (2, 12), (4, 4), (4, 8), (4, 12), (6, 12)\}$
(ii) $\text{Dom } R = \{2, 4, 6\}, \text{Rang } R = \{4, 6, 8, 12\}$
(iii) $R^{-1} = \{(4, 2), (6, 2), (8, 2), (12, 2), (4, 4), (8, 4), (12, 4), (12, 6)\}$
(iv) Do yourself.
6. (i) $R = \{(x, y) / x - y = 2\}$ (ii) $\text{Dom } R = \{2, 4, 6, 8, 10\}, \text{Rang } R = \{0, 2, 4, 6, 8\}$
(iii) $R^{-1} = \{(0, 2), (2, 4), (4, 6), (6, 8), (8, 10)\} = \{(y, x) / y - x = 2\}$ (iv) Do yourself.
7. $R = \{(0, 1), (1, 3), (3, 7)\}, R^{-1} = \{(1, 0), (3, 1), (7, 3)\}$
8. (i) $R_1 = \{(1, 3^0)\}, \text{Dom } R_1 = \{1\}, \text{Rang } R_1 = \{3^0\}$
(ii) $R_2 = \{(2, 3^0), (4, 3^0), (4, 3^1), (8, 3^0), (8, 3^1)\}, \text{Dom } R_2 = \{2, 4, 8\}, \text{Rang } R_2 = \{3^0, 3^1\}$
(iii) $R_3 = \{(1, 3^0), (1, 3^1), (1, 3^2)\}, \text{Dom } R_3 = \{1\}, \text{Rang } R_3 = \{3^0, 3^1, 3^2\}$
(iv) $R_4 = \{(1, 3^0), (1, 3^1), (1, 3^2)\}, \text{Dom } R_4 = \{1\}, \text{Rang } R_4 = \{3^0, 3^1, 3^2\}$
(v) $R_5 = \{(1, 3^1), (1, 3^2), (2, 3^2), (4, 3^2)\}, \text{Dom } R_5 = \{1, 2, 4\}, \text{Rang } R_5 = \{3^1, 3^2\}$

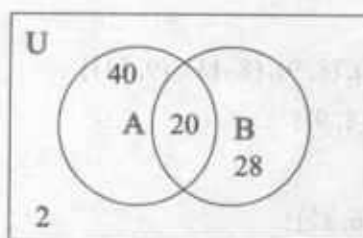
Miscellaneous Exercise 3

1. (i) b (ii) c (iii) c (iv) a (v) b (vi) d (vii) b (viii) c
(ix) a (x) d (xi) d
2. (i) 2^{16} (ii) $R_1 = \{(a, b)\}, R_2 = \{(a, e), (f, j)\}, R_3 = \{(a, j), (f, n), (f, e)\}$
(iii) $\{(a, b), (f, b), (h, e), (s, n), (s, j)\}$ (iv) $\{(a, b), (f, e), (h, j), (s, n)\}$
(v) $\{(a, b), (f, j), (h, e), (s, n)\}$

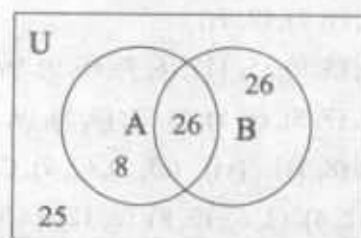
3.



7. (i)



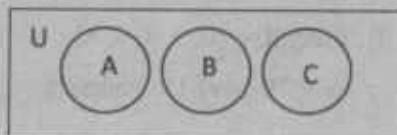
(ii)



8.

- (i) $A = \{\text{Haani, Zubair, Haider}\}$, $B = \{\text{Abdullah, Umer, Bilal, Ali}\}$,
 $C = \{\text{Hassan, Jaffer, Usman}\}$

(ii)



- (iii) $A \cap (B \cap C)^c = \{\text{Haani, Zubair, Haider}\}$

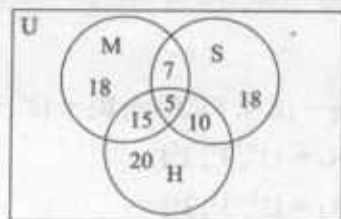
$(A \cup B) \cap C^c = \{\text{Haani, Zubair, Haider, Abdullah, Umer, Bilal, Ali}\}$

$A - (B \cap C) = \{\text{Haani, Zubair, Haider}\}$

$A - (A \cup B) = \{\}$

9.

(i)



(ii) 93

(iii) 7

Exercise 4.1

1. $2xy^2(xy - 3x + y)$

5. $(3x + 13y - 2)(3x - 13y - 2)$

8. $(1 + x + y)(1 - x - y)$

11. (a) $2y - 9$ (b) $5x - 3$

13. $(m^2 + 1 + m)(m^2 + 1 - m)$

16. $4(x^2 + 8y^2 + 4xy)(x^2 + 8y^2 - 4xy)$

19. $(2z - 5)(5z - 2)$

22. $(x - 1)(x + 2)$

25. $(4 + 5x)(2 - x)$

28. $(u + 2)(u - 2)(u + 3)(u - 3)$

2. $3(n - 1)(x - y)$

6. $-2(x + 1)(x - 1)$

9. $x^2 + y^2 + 2xy - 4c^2$

12. $(x^2 + 2m^2 + 2xm)(x^2 + 2m^2 - 2xm)$

14. $-3x^2(x - 1)(x + 8)$

17. $(x - 3)(x - 4)$

20. $(3y - 1)(4 - y)$

23. $(x + 3)(3x + 2)$

26. $(2 + x)(3 - 5x)$

29. $(y - 4)(y + 4)(y^2 + 4)$

3. $18(x^2 + 3y^2)^2$

7. $(x - 3a + 4b)(x - 3a - 4b)$

10. $(x + 2y)^2 - (z)^2$

15. $(x^4 - x^2 + 1)(x^2 + x + 1)(x^2 - x + 1)$

18. $(x - 1)(x - 8)$

21. $(x - 6)(x - 15)$

24. $(2x + y)(x - 3y)$

27. $2(a + 1)(a - 3)$

Exercise 4.2

- $(x-5)(x^2+5x+25)$
- $(2x+1)(4x^2-2x+1)$
- $3(pq-3x)(p^2q^2+3pqx+9x^2)$
- $(3+8x)(9-24x+64x^2)$
- $(t+2)(t-2)(t^2-2t+4)(t^2+2t+4)$
- $(x^2+y^2)(x^4-x^2y^2+y^4)$
- $(y-x)(x^2+y^2-6x-6y+xy+12)$
- $(4x+4y-z)(16x^2+16y^2+32xy+4xz+4yz+z^2)$
- $(3p-4q)^3$
- $(2p+q)^3$
- $(5x-y)^3$
- $(p-3q)^3$
- $(2x^2-3x+11)(x+1)(2x-5)$
- $(y^2+2y+3)(y^2+2y+5)$
- $(k^2+7k-5)(k^2+7k-15)$
- $(y^2-4y+1)(y^2-4y+2)$
- $(k^2+7k-5)(k^2+7k-15)$
- $(x^2-3x-12)(x^2-3x-16)$
- $(x^2-8x-6)(x^2+2x-6)$
- $(x^2-9x+12)(x^2-6x+12)$
- $(10+10x-x^2)(10+2x-x^2)$
- $(a^2+9)(a^4-9a^2+81)$, sum of two cubes because of positive sign between the terms.
- $(t+2)(t+2)(t+2)$, each factor is a binomial.

Exercise 4.3

1(a).

1	p	q	-	pq
5	p ²	q	r	5p ² qr
7	p ²	q	r ²	7p ² qr ²

- b. $12pq(p+q)^2$ c. $(x+y)^2, (x-y)^2$ etc. d. $m-n, (m-n)(m^2+n^2)$ 2. $x+y$ 3. $a^2-2ab+b^2$
 4. $ab(a-b)$ 5. $x-7$ 6. $3x+1$ 7. $cx+d$ 8. m^2-n^2 9. $a(x+a)$

Exercise 4.4

- 1.(i) x^2+2x (ii) x^2+2x (iii) x^2+2x (iv) $5(x^2+2x)$ (v) No, HCF is x^2+2x^2

2.

(i)	8	x^6	y	z	$8x^6yz$
(ii)	24	x^3	y^4	z	$24x^3y^4z$
(iii)	18	x^6	y^2	z^3	$18x^6y^2z^3$
(iv)	28	x^3	y^5	z^5	$28x^3y^5z^5$

3. $a+2$ x^2+4x+4 5. $x-1$ 6. p^2+1 7. $x(3x+2)$
 8. $x+3$ 9. $2ab(9a^2-1)$ 10. $p^2q^2(p+q)(p^3-q^3)$ 11. $xy(2x+1)(2x-1)$
 12. $(x-3)(x-1)(x+2)$ 13. $(m^2+1)(m^6-1)$ 14. x^4-5x^2+4 15. $(x+5)(x-4)(x-6)$

Exercise 4.5

- 1.(i) $x-2, -4(x^2-4)(x+3)$ (ii) HCF: a^2+a+1 , LCM: $(a^3-1)(a^2-a+1)(a-1)$
 (iii) $x(x+3), x(x+3)(x-1)(2x-1)$
 2. $x^4-17x^3+81x^2-7x-490$ 3. $x^3+5x^2-2x-24$ 4. $y+4$
 5. x^4+x^3-x-1 6. $(3x^3+x^2+x-2)(4x^3-10x^2+4x+2)$

Exercise 4.6

- 1.(i) $\frac{1}{x^3+7x-8}$ (ii) $\frac{x^2}{x^2+y^2}$ (iii) $\frac{n}{m^2+n^2}$ (iv) 1
 2. $\frac{7}{12}$ 3. $\frac{ab}{2a-1}$ 4. 1 5. $\frac{2xy}{x-y}$ 6. $\frac{4a^2+2a+1}{6a}$ 7. $\frac{x-5}{x-1}$

$$8. \frac{x+2y}{4x^2-9y^2} \quad 9. \frac{x^2+y^2}{xy(x^2-y^2)} \quad 10. \frac{4}{3} \quad 11. \frac{12a^2-4a+7}{3(4a^2-9)} \quad 12. \frac{23x}{(1+2x)(2+x)(5-9x)} \quad 13. \frac{1-q}{p}$$

Exercise 4.7

$$\begin{array}{llll} 1. \pm (4y-7) & 2. \pm a(5a-3) & 3. \pm (x^2 - \frac{1}{x^2} + 2) & 4. \pm (a - \frac{1}{a} - 4) \\ 5. \pm (a^2 + 10a + 20) & 6. \pm (x^2 + 4x + 2) & 7. \pm (x^2 + 5x + 3) & 8. \pm (7b^2 + 2ab + a^2) \\ 9. \pm (2x^2 - 3x + 5) & 10. \pm (1 - 5x + x^2) & & \\ 11. \pm (x^2 + 2 - \frac{1}{x^2}) & 12. \pm (a - 4 - \frac{7}{a}) & 13. \pm (x^2 - x + \frac{1}{4}) & 14. \pm (2x^2 + 8 + \frac{8}{x^2}) \\ 15. (i) -a-1 & (ii) a+1 & (iii) a=-1 & 16. p=216, q=324 \quad 17. k=8 \end{array}$$

Exercise 4.8

$$\begin{array}{llll} 1. a) (x-3)m, (x+1)m & b) P=(4x-4)m & c) Rs. (80x-80) & d) Rs. 250(x^2-2x-3) \\ 2. a) (5x-3)m & b) P(20x-12)m & c) Rs. 75 & d) Rs. 75(25x^2-30x+9) \\ 3. a) (5x-2)m & b) 6(5x-2)^2 & c) Rs. 192(5x-2)^2 & \\ 4. radius 1=x-3, radius 2=x-5 & 5. 5m-6n, 5m+6n & 6. if speed=x+2 then distance=x+3 and vice versa \end{array}$$

Miscellaneous Exercise 4

$$\begin{array}{llllllllll} 1. & (i) & (d) & (ii) & (d) & (iii) & (c) & (iv) & (a) & (v) & (a) & (vi) & (c) & (vii) & (d) \\ & (viii) & (c) & (ix) & (d) & (x) & (a) & (xi) & (b) & (xii) & (b) & (xiii) & (b) & (xiv) & (b) \\ & (xv) & (d) & (xvi) & (a) & (xvii) & (c) & (xviii) & (c) & (xix) & (a) & (xx) & (c) & & \\ 2. & (a+3)(a-3)(a^2-14) & 3. & (2x+3)(3x+5) & 4. & (x^2-8x-7)(x^2-8x+13) \\ 5. & (x^2+2x-4)(x^2-2x-4) & 6. & (3x^2+4x-1)(3x^2+4x-6) & 7. & (m+n)(m-n)(2m^2+3n^2) \\ 8. & (x+2y)(x-2y)(x^2+14y^2) & 9. & (x+2a)(x-b) & & \\ 10. & (a+b)(a-b)(a^2+b^2)(a^2+ab+b^2)(a^2-ab+b^2)(a^4-a^2b^2+b^4) & & & & \\ 11. & 4x^2y(x+3)(7x-5) & 12. & (x-y)(2x-2y+1)(2x-2y-1) & & \\ 13. & (p+2q)(p-2q)(x-2y)(x^2+2xy+4y^2) & 14. & (x^2+y^2+3xy)(x^2+y^2-3xy) & & \\ 15. & (i) x+3 & (ii) & 3x^2+1 & & \\ 16. & (i) (x+1)(x+2)(2x+1) & (ii) & (x+2)(x+3)(3x+2) & & \\ 17. & a+b+c, (a+b+c)(a-b)(b-c)(c-a) & & & & \\ 18. & \pm (\frac{3a}{x} - \frac{1}{5} + \frac{2x}{3a}) & 19. & (i) x & (ii) & \frac{x+2}{(x+1)(x+3)} & 20. & a=-2, b=5 \end{array}$$

Exercise 5.1

$$\begin{array}{llllllll} 1. \{-3\} & 2. \{1\} & 3. \{1\} & 4. \left\{-\frac{2}{5}\right\} & 5. \{8\} & 6. \{7\} & 7. \{10\} & 8. \left\{-\frac{1}{4}\right\} \\ 9. \{24\} & 10. \left\{-\frac{3}{2}\right\} & 11. \{11\} & 12. \{9\} & 13. \{-1.3\} & & & \end{array}$$

Exercise 5.2

1. {8} 2. {7} 3. {14} 4. {40} 5. {40} 6. ϕ 7. {21}
8. {13} 9. {3} 10. {3} 11. {3} 12. ϕ 13. {64} 14. {10} 15. {6} 16. {-3}

Exercise 5.3

1. $\left\{\frac{5}{3}, -\frac{5}{3}\right\}$ 2. {4, -8} 3. $\left\{2, -\frac{8}{5}\right\}$ 4. {1, -3} 5. {2} 6. $\left\{\frac{4}{3}, \frac{8}{3}\right\}$
7. {3, -2} 8. $\left\{-3, \frac{5}{3}\right\}$ 9. {9, -9} 10. ϕ 11. $\left\{\frac{13}{2}, -\frac{11}{2}\right\}$ 12. $\left\{5, -\frac{1}{7}\right\}$
13. $\left\{\frac{5}{2}, -\frac{2}{3}\right\}$ 14. {1, -7}

Exercise 5.4

- 1(a). (i) Yes (ii) No (iii) Yes (iv) Yes (v) Yes (b) (i) {2, 3, 4, 5} (ii) {1, 2, 3, 4, 5, 6}
(iii) $\{z | z \in \mathbb{R} \wedge z \leq -3\}$ (iv) {0, 1, 2, 3, 4} (v) $\{x | x \in \mathbb{R} \wedge -4 < x < -\frac{3}{2}\}$
2. {1, 2} 3. {0, 1, 2, 3, 4, 5} 4. $\{y | y \in \mathbb{R} \wedge y > 3\}$ 5. $\{y | y \in \mathbb{R} \wedge y \leq 6\}$
6. $\{x | x \in \mathbb{R} \wedge x > -2 \text{ or } x < -6\}$ 7. $\{x | x \in \mathbb{R} \wedge -1 < x \leq 3\}$
8. $\{x | x \in \mathbb{R} \wedge -5 \leq x \leq 2\}$ 9. $\{x | x \in \mathbb{R} \wedge x < -5 \text{ or } x > -1\}$
10. $\{x | x \in \mathbb{R} \wedge -4 \leq x < -3\}$ 11. $\{x | x \in \mathbb{R} \wedge x < -9 \text{ or } x > -3\}$
12. $\{x | x \in \mathbb{R} \wedge x \geq 13\}$

Miscellaneous Exercise 5

1. (i) (d) (ii) (a) (iii) (c) (iv) (c) (v) (b) (vi) (c)
(vii) (d) (viii) (c) (ix) (a) (x) (b) (xi) (b) (xii) (b)
(xiii) (c) (xiv) (d) (xv) (b) (xvi) (b)
2. $x = 10$ 3. $a = \frac{3}{2}$ 4. ϕ 5. $\left\{\frac{13}{5}, -3\right\}$ 6. $\{x | x \in \mathbb{R} \wedge x \leq -8\}$
7. $\{y | y \in \mathbb{R} \wedge y > \frac{-3}{2}\}$ 8. $\{x | x \in \mathbb{R} \wedge x \leq -2 \text{ or } x \geq 6\}$ 9. $\{y | y \in \mathbb{R} \wedge y < 10 \text{ or } y > 19\}$

Exercise 6.1

1. (i) 18,000" (ii) 18,00" (iii) 37,800" (iv) 73,220"
2. (i) 4,500' (ii) 2' (iii) 3,040' (iv) 1,830.5'
3. (i) 2.25° (ii) 0.0417° (iii) 61° (iv) 45.7625°
4. (i) 60°7'30" (ii) 0°2'15" (iii) 1°0'51" (iv) 255°27'

5. (i) $\frac{\pi}{4}$ rad (ii) $\frac{5\pi}{6}$ rad (iii) $\frac{121\pi}{360}$ rad (iv) $\frac{2\pi}{3}$ rad
 6. (i) 4.71 rad (ii) 1.05 rad (iii) 3.15 rad (iv) 1.32 rad
 7. (i) 45° (ii) 150° (iii) 15° (iv) $31^\circ 30'$
 (v) $57^\circ 17' 45''$ (vi) 180° (vii) 2160° (viii) $286^\circ 28' 44''$

Exercise 6.2

1. (i) 5.24cm, 13.09cm² (ii) 25.13m, 150.77m² (iii) 6.36dm, 19.08dm²
 2. 5.24cm, 10.48cm²
 3. (i) 2 rad (ii) 3.78 rad (iii) 0.79 rad (iv) 0.5 rad
 4. (i) 1.27cm (ii) 22.9m (iii) 22.57cm (iv) 20dm
 5. 15.71inch 6. 84° 7. 30.85cm, 56.55cm²
 8. (i) $\frac{\pi}{2}$ (ii) $\frac{7\pi}{12}$ (iii) $\frac{3\pi}{8}$

Exercise 6.3

1. Complete yourself.
 2. (i) 0 (ii) $\frac{7}{4}$ (iii) $\frac{1}{\sqrt{3}}$ (iv) $-\frac{1}{2}$ (v) $\frac{1}{2}$
 4. (i) 45° (ii) 270° (iii) 45° (iv) 30° (v) 90°
 5. $\sin \theta = \frac{\sqrt{3}}{2}$, $\tan \theta = \sqrt{3}$, $\cot \theta = \frac{1}{\sqrt{3}}$, $\sec \theta = 2$, $\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$ 6. $\frac{\sqrt{3}}{4}$
 7. $\sin \theta = -\frac{1}{\sqrt{2}}$, $\cos \theta = \frac{1}{\sqrt{2}}$, $\cot \theta = -1$, $\sec \theta = \sqrt{2}$, $\operatorname{cosec} \theta = -\sqrt{2}$

Exercise 6.4

3. (i) 30° (ii) 45° (iii) 45°
 4. (i) $\angle A = \angle B = 45^\circ$, $b = 4\text{cm}$ (ii) $\angle R = 30^\circ$, $QR = 12\text{cm}$, $PR = 8\sqrt{3}\text{cm}$
 (iii) $\angle P = \angle Q = 45^\circ$, $PQ = 8\sqrt{2}\text{m}$ (iv) $\angle Y = 30^\circ$, $\angle Z = 60^\circ$, $XY = 8\sqrt{3}\text{m}$
 (v) $\angle N = 90^\circ$, $\angle L = 36.87^\circ$, $\angle M = 53.13^\circ$ 5. r^2

Exercise 6.5

1. 11.55 m 2. 30° 3. 1.39 m 4. 10 m
 5. 52 m 6. 50.7 m 7. 34.7 m 8. 18.93 m, 12 m
 9. 80m, 33.69° 10. 0.73m/s

Exercise 6.6

1. measure yourself 2. find yourself 3. 070° 4. 7.07km, 270°
 5. 4km, 103° 6. 15.6 m 7. 13km 8. 9.96km, 8.36km

Miscellaneous Exercise 6

1. (i)(a) (ii)(b) (iii)(d) (iv)(c) (v)(b) (vi)(a) (vii)(c) (viii)(d) (ix)(d)
 (x)(b) (xi)(c) (xii)(a) (xiii)(c) (xiv)(c) (xv)(d)
 2. $\sin \alpha = \frac{12}{13}$, $\tan \alpha = \frac{-12}{5}$, $\sec \alpha = \frac{-13}{5}$, 3. 5.77 m/s 4. 2.69
 5. 10cm, 10cm, $10\sqrt{2}$ cm 6. 28.39 m 7. 163 m 8. 050°

Exercise 7.1

1. (i) 5 (ii) 10 (iii) $2\sqrt{10}$ (iv) $10\sqrt{2}$ (v) $\sqrt{5}$
 (vi) $\sqrt{17}$ (vii) $\frac{\sqrt{29}}{2}$ (viii) $\sqrt{\frac{193}{2}}$ (ix) $\frac{5\sqrt{10}}{12}$ (x) $\frac{5\sqrt{29}}{4}$
 2. (i) non-collinear (ii) non-collinear (iii) collinear (iv) collinear (v) collinear
 (vi) non-collinear
 3. (i) right triangle; perimeter = $5 + \sqrt{13}$ units
 (ii) equilateral triangle; perimeter = 6 units
 (iii) scalene triangle; perimeter ≈ 24.49 units
 (iv) right triangle; perimeter ≈ 27.21 units
 (v) isosceles triangle; perimeter = $6 + 2\sqrt{34}$ units
 (vi) right isosceles triangle; perimeter = $12 + 6\sqrt{2}$ units 8. Yes

Exercise 7.2

1. (i) (4, 7) (ii) $(\frac{1}{2}, 1)$ (iii) (2, 1)
 2. (i) E(3, 1); F($\frac{7}{2}$, 4); G($-\frac{1}{2}$, 4); H(-1, 1) (ii) Draw yourself.
 (iii) EF = GH = $\frac{\sqrt{37}}{2}$ units; FG = HE = 4 units (iv) rectangle
 3. Diagonals do not bisect each other. 4. Length of diagonals are $4\sqrt{2}$ units, $2\sqrt{2}$ units.
 5. (i) D(-0.5, 3.5); E(3.5, 3.5) (ii) DE = 4 units; BC = 8 units
 (iii) $\frac{1}{2}$ BC = 4 units (iv) DE = $\frac{1}{2}$ BC
 6. (i) Hassan's starting point is (6, 0) and Ali's starting point is (0, 6) (ii) C($3\sqrt{2}$, $3\sqrt{2}$)
 7. (i) M(3, 3); N(0, 3); O(0, 0) (ii) A(0, 0); C(-4, 2)

Miscellaneous Exercise 7

1. (i) b (ii) a (iii) a (iv) c (v) c (vi) d
 (vii) c (viii) b (ix) d (x) c (xi) b (xii) a
 (xiii) b (xiv) a (xv) c (xvi) d (xvii) d (xviii) b
 (xix) c (xx) c (xxi) c (xxii) b (xxiii) d (xxiv) a

Exercise 8.1

- (i) 0 (ii) 0.577 (iii) 1.732 (iv) undefined (v) -1.732 (vi) -0.577
(vii) -0.176 (viii) 1.018
- (i) 0° (ii) 30° (iii) 120° (iv) 20°
- (i) $m = \frac{2}{3}, \theta = 33.69^\circ$ (ii) $m = -\frac{7}{3}, \theta = 113.20^\circ$ (iii) $m = \frac{1}{7}, \theta = 8.13^\circ$
- (i) $m = -1$ (ii) $m = 1$ 5. $x = 0$ 6. $k = \frac{15}{8}$ 7. $k = 15$ 9. $y = 1$
12. $P(x, y) = P(7, -1)$ 13. Fourth vertex: $(-3, -2)$
14. Slopes of sides: $-1, \frac{3}{8}, \frac{8}{3}$; Slopes of medians: $-\frac{13}{2}, -\frac{2}{13}, 1$; Slopes of altitudes: $1, -\frac{8}{3}, -\frac{3}{8}$

Exercise 8.2

- (i) $y = 3$ (ii) $y = 0$ (iii) $y = -9$ (iv) $y = -\frac{5}{2}$
- (i) $x = 1$ (ii) $x = 9$ (iii) $x = -4$ (iv) $x = \frac{1}{4}$
- (i) $2x - y - 3 = 0$ (ii) $4x - y + 27 = 0$ (iii) $y + 5 = 0$
(iv) $x + 2 = 0$ (v) $5x + 8y + 22 = 0$ (vi) $4x - 3y - 20 = 0$
(vii) $5x - 6y + 30 = 0$ (viii) $x + y - 11 = 0$
- (i) $\frac{x+4}{4/5} = \frac{y-2}{3/5}$ (ii) $\frac{x-6}{\cos 30^\circ} = \frac{y+6}{\sin 30^\circ}$ 5. (i) $x - \sqrt{3}y + 10 = 0$ (ii) $x + y - 2\sqrt{10} = 0$
- (i) $5x + y + 24 = 0$ (ii) $4x + y - 19 = 0$ (iii) $x - 2y + 8 = 0$
(iv) $x + 4y + 2 = 0$ (v) $x + y - 3 = 0$ (vi) $x - 2y - 14 = 0$
- $4x - 3y + 9 = 0$ 8. $2x + y + 5 = 0$ 9. $x - 4y + 7 = 0$
- Equations of medians: $2x + y - 4 = 0, x - y + 2 = 0, 7x - 4y + 6 = 0$
Equations of altitudes: $3x + 4y - 16 = 0, yx + 3 - 22 = 0, 2x + y + 6 = 0$
- (a) Slope-intercept form: $y = -\frac{3}{4}x + \frac{11}{8}$; Two-intercept form: $\frac{x}{11/6} + \frac{y}{11/8} = 1$
Point-slope form: $y - \frac{11}{8} = -\frac{3}{4}(x - 0)$; Two-point form: $\frac{y - 11/8}{0 - 11/8} = \frac{x - 0}{11/6 - 0}$
Normal form: $x\left(\frac{3}{5}\right) + y\left(\frac{4}{5}\right) = -\frac{3}{4}(x - 0)$; Symmetric form: $\frac{x - 0}{-4/5} = \frac{y - 11/8}{3/5}$ (b) Do yourself.

Exercise 8.3

- (i) 45° (ii) 104° (iii) 90° 2. (i) 59° (ii) 14° (iii) 18.4° (iv) 45° (v) 85.6°
- (i) $45^\circ, 45^\circ, 90^\circ$ (ii) $31^\circ, 143^\circ, 6^\circ$ (iii) $37^\circ, 78^\circ, 65^\circ$ (iv) $45^\circ, 96^\circ, 39^\circ$
- (i) 90° (ii) 53° (iii) 108°
- (i) $90^\circ, 45^\circ, 45^\circ$ (ii) $54^\circ, 82^\circ, 44^\circ$ (iii) $63^\circ, 90^\circ, 27^\circ$ (iv) $59^\circ, 79^\circ, 42^\circ$
- (i) $(1, -3)$ (ii) $\left(-\frac{23}{7}, \frac{2}{7}\right)$ (iii) $\left(\frac{13}{23}, -\frac{19}{23}\right)$ 7. (a) $x + 1 = 0$ (b) $3x - 4y + 7 = 0$
- $15x - 5y + 39 = 0$ 9. (a) $11y - 7 = 0$ (b) $22x + 9 = 0$
- (a) $x - 2y + 1 = 0$ (b) $5x - 2y + 5 = 0$ 11. (a) $7x + 2y - 20 = 0$ (b) $2x - 7y + 17 = 0$

Exercise 8.4

- $4x + 5y = 40$, 5 dozen
- (i) Rs.12150 (ii) 8 people
- $y = 5500x + 700$, $b = 800$ a ; First deal is better.
- (i) $y = 120x + 500$, total amount = Rs.1940 (ii) earning per hour (iii) additional amount
- (i) $y = 18x + 44$ (ii) 584 units (iii) Rs.11680 (iv) 20 days
- (i) $y = 900x + 1500$ (ii) Rs.6000 (iii) charge per hour
- (i) $F = \frac{9}{5}C + 32$ (ii) 41°F (iii) y-intercept shows that temperature in F starts from 32 and slope shows rate of increase in temperature in F.
- (i) $y = 6x$ (ii) run rate per over (iii) scores start from 0 (iv) 270 runs (v) 40 overs
- (i) $(y - 5000) = 200(x - 0)$ (ii) Rs.200, slope shows the amount per kilometre (iii) Rs.41000
- (i) $(y - 67) = -\frac{13}{7}(x - 25)$, 41°E
- (i) $y = 2x$, Length of plot = 60 feet (ii) Slope is 2 which shows relation between length and width.

Miscellaneous Exercise 8

- (i) c (ii) b (iii) d (iv) a (v) b (vi) a (vii) c
(viii) d (ix) a (x) b (xi) c (xii) c (xiii) d (xiv) a
- $y + \frac{x}{2} = 1$
- $y = \frac{5}{2}x + \frac{1}{2}$, $\frac{x}{-1/5} + \frac{y}{1/2} = 1$
- (i) $\frac{x}{3} + \frac{y}{-1} = 1$, $a = 3$, $b = -1$ (ii) $m = \frac{1}{3}$, $y + 1 = \frac{1}{3}(x - 0)$ 6. $y = \sqrt{3}x + 20$, $m = \sqrt{3}$, $c = 20$
- (i) $A(-4, 2)$, $B(8, 5)$, $C(1, 9)$ (ii) $m_1 = \frac{1}{4}$, $m_2 = -\frac{4}{7}$, $m_3 = \frac{7}{5}$ (iii) Interior angles: 40° , 44° , 96°
- (i) $M(6, 1)$ (ii) Slope of PM = 1 (iii) $x - y + 4 = 0$ (iv) $x + y = 0$
- (i) $\sqrt{68}$ units (ii) Slope of BA = 4 (iii) $a = 4$, $b = -1$ (iv) $4x - y + 3 = 0$
- $2x - 5y - 2 = 0$ 11. (i) $O(0, 0)$, $C(5, 6)$, $A(-3, 5)$, $B(5, 2)$
(ii) Slope of OC = $\frac{6}{5}$, Slope of AB = $-\frac{3}{8}$ (iii) $6x - 5y = 0$, $3x + 8y - 31 = 0$ (iv) $(\frac{155}{63}, \frac{62}{21})$
- $l: A(0, -1)$, $B(2, 0)$ (i) Slope of $l = \frac{1}{2}$ (ii) Slope of $p = -2$
(iii) mid-point of $l = (1, -\frac{1}{2})$ (iv) $4x + 2y - 3 = 0$

Exercise 9.1

- (i), (ii), (iii), (v), (vi), (viii), (ix), (xi)
- No, by assigning particular values to a and b .
- 63, 'the next number is $2 \times 31 + 1 = 63$ '
- Axioms: (i), (ii), (v), (vii), (viii), (x)
Neither axioms nor postulate: (iv).
- Postulates: (iii), (vi), (ix)

Exercise 9.2

- Similar pairs: (i), (ii), (iv), (vi), (vii)
- (i) $x = 10$ cm, $y = 6$ cm (ii) $a = 30^\circ$ (iii) $x = 7.5$ cm (iv) $x = 2$ cm, $y = 3.6$ cm, $z = 8$ cm
(v) $a = 4$ cm, $b = 4.2$ cm (vi) $a = 4$ cm, $b = 6$ cm (vii) $a = 12$ cm
- DE = 3 cm, EF = 4 cm, FD = 2 cm 4. DE = 18 m 5. DE = 15 m 6. 20 cm 7. 10 m
- (a) $a = 3$ cm (b) $x = 2.4$ cm, $y = 5.25$ cm (c) $a = 16$ cm (d) $x = 7.2$ cm, $y = 4.5$ cm
- $\angle C = \angle AED = 50^\circ$, yes 10. $\angle A = \angle B = 60^\circ$, equilateral and acute angled 11. 10 m

Exercise 9.3

- (i) 16 cm² (ii) 32 cm² (iii) 249 cm² (iv) 45 cm²
- (i) 6 cm (ii) 5 cm (iii) 30 cm (iv) 19.6 cm
- (i) $9 : 16$ (ii) 640 cm² (iii) 900 cm² 4. $4 : 5$, 10 cm 5. $XB : XZ = AB : YZ = 5 : 6$
- 5600 m² 7. (i) $9 : 1$ (ii) 3150 cm²
- (i) $8 : 13$ each (ii) $64 : 169$ (iii) 192 cm² (iv) 315 cm², trapezium

Exercise 9.4

- (i) yes (ii) no (iii) yes (iv) no
- (i) $d = 20$ cm, $V_1 : V_2 = 125 : 1$ (ii) $h = 9$ m, $c = 9.5$ m, $b = 18$ m, $V_1 : V_2 = 27 : 8$
- (i) $V_2 = 333.33$ mm³ (ii) $V_1 = 208.33$ ft³ (iii) $V_1 = 29.44$ cm³ (iv) $V_2 = 121.5$ m³
- $8 : 27$ 5. 5.8 cups approx 6. 10000 ft³ 7. Area = 292.5π cm², Volume = 945π cm³
- $2 : 5$ 9. 15 cm 10. 625 cm³ 11. (i) $4 : 5$ (ii) $16 : 25$ (iii) $64 : 125$
- 6 cm 13. 3 m

Exercise 9.5

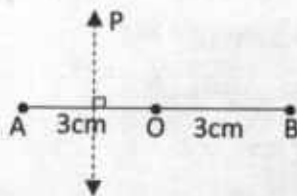
- (a) 8 (b) 6 (c) 3 (d) 9 2. (a) $k = 108^\circ$ (b) $p = q = r = s = t = 72^\circ$
- 12 sides 4. Minimum interior angle = 60° , maximum interior angle = $179.9999\dots (< 180^\circ)$
If maximum interior angle is 180° , then adjacent sides of polygon become collinear.
- (a) 72° (b) 40° (c) 24° (d) 18° 6. (a) 60° (b) 120° (c) 6
- No, because 360° is not a multiple of 50° . 8. square 9. decagon 10. 5040° 12. 5 sides
- $28^\circ, 49^\circ, 71^\circ, 93^\circ, 119^\circ$ 14. 7 15. 1980° 16. 18 sides

Exercise 9.6

- $P = 15$ m, cost = Rs.3,300 2. Rs.27,300 3. $P = 30$ m, $A = 60$ m²
- (i) Rs.9,600 (ii) 259.81 m² (iii) Rs.129,900
- (i) Rs.50,400 (ii) 120 ft² (iii) Rs.4,800 6. $P = 14$ ft

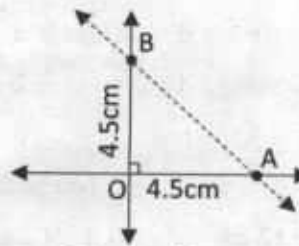
Exercise 9.7

1.



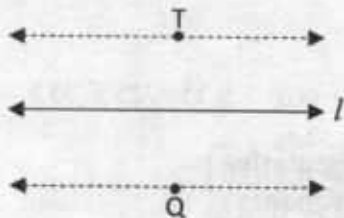
Locus is perpendicular bisector of $\overline{AO}$.

2.



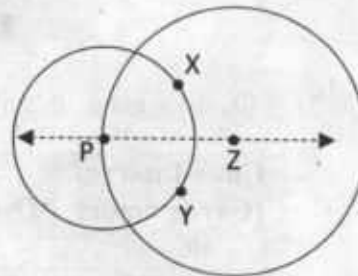
Locus: Line making equal intercepts

3.



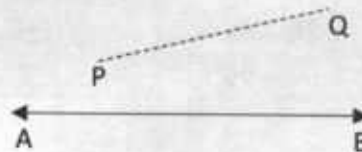
(iii) 6.2 cm

4.

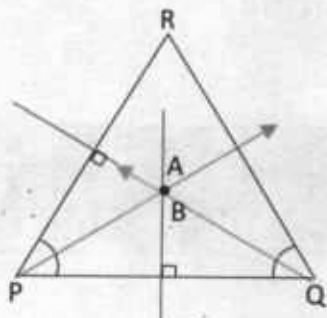


(iii) Second circle passes through the centre

5. (i) line segment (ii) yes
(iii) by drawing $\overline{PQ}$ parallel to $\overline{AB}$.
(iv) by taking P and Q on $\overline{AB}$.

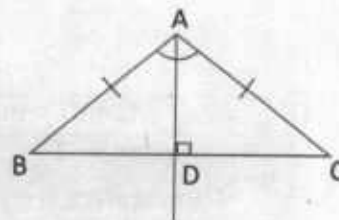


6.



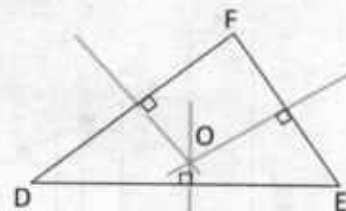
(iii) Points A and B coincide.

7.

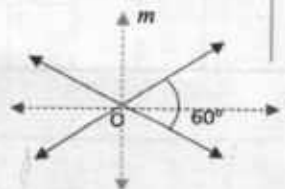


By measuring angle ADC, and sides BD and CD, we see that AD is right bisector of BC.

8. Join non collinear points D, E and F.
Find point of intersection of right bisector of sides.
Locus of point equidistant from given points is O.
Only one such point exists.



9. (iii) Locus l and angle bisector are same.
(iv) Both angle bisectors are perpendicular.



Miscellaneous Exercise 9

1. (i) c (ii) d (iii) a (iv) b (v) c (vi) c
 (vii) d (viii) d (ix) b (x) d (xi) a (xii) b
 (xiii) c (xiv) b (xv) d (xvi) a (xvii) a
2. 60° 3. No 4. yes 5. 140 6. (ii) 5 cm 7. 32 cm
9. $BC = 8$ cm, $DE = 12$ cm 10. 1 : 9 11. 200 cm^2 12. 135 ft^3
13. (i) 756 m^2 (ii) 1012.5 in^2

Exercise 11.1

1. (i) 30 (ii) 31 – 40 (iii) none (iv) 51 (v) 50.5 (vi) 10 (vii) 5.5, 15.5, 25.5, 35.5, 45.5, 55.5

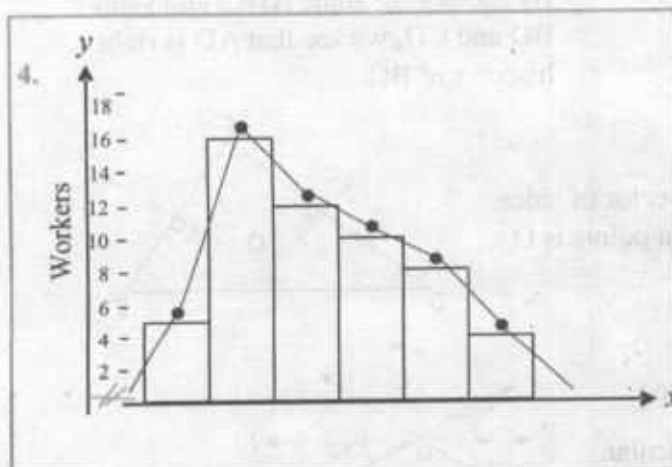
2. (i), (ii)

Class Interval (Cards made)	Frequency (No. of students)	Cumulative frequency
40 – 49	7	7
50 – 59	13	20
60 – 69	22	42
70 – 79	17	59
80 – 89	13	72
90 – 99	8	80

- (iii). 42 (iv). 8.75% (v). 60-69 (vi). 10

3.

Medals won	0	1	2	3	4	5	6
No. of players Frequency(f)	5	8	10	8	5	5	4
Cumulative frequency	5	13	23	31	36	41	45



5. Do yourself

6. Do Yourself 7. (i) 48 (ii) 8 (iii) 1 (iv) 48% app (v) 8

(vi)	C.I	9-11	11-13	13-15	15-17	17-19	19-21	21-23
f	8	9	13	10	5	2	1	

8. Do Yourself 9. Do Yourself

8

Exercise 11.2

1. (i) 7 (ii) 0 (iii) 14 (iv) 6.95 (v) 5 2. $\bar{x}_A = 48$, $\bar{x}_B = 33.6$, A is better
 3. 78, 67.5, 69, 77 4. 11.33 5. 23.51 6. (i) 4 (ii) 0 (iii) 2 (iv) 2 (v) 4
 7. 1.0 8. 64.09 9. (i) no (ii) 2 (iii) 120, 225 (iv) no
 10. mean = median = mode = 2 11. (a) 9.83 (b) 14.38

Exercise 11.3

1. (i) $\frac{2}{5}$ (ii) $\frac{1}{5}$ (iii) $\frac{2}{5}$ (iv) $\frac{3}{5}$ 2. (i) $\frac{1}{4}$ (ii) $\frac{5}{12}$ (iii) 0.00175
 3. (i) $\frac{1}{8}$ (ii) $\frac{1}{4}$ (iii) $\frac{7}{8}$ (iv) 0 (v) -1
 4. (i) $\frac{2}{5}$ (ii) $\frac{7}{10}$ (iii) $\frac{3}{5}$ (iv) $\frac{9}{10}$ (v) $\frac{3}{10}$ (vi) $\frac{3}{10}$ (vii) $\frac{1}{5}$ (viii) $\frac{2}{5}$ (ix) $\frac{1}{2}$
 5. (i) $\frac{1}{2}$ (ii) 0 (iii) $\frac{9}{10}$ (iv) $\frac{1}{10}$ (v) $\frac{1}{10}$ (vi) $\frac{1}{10}$ (vii) $\frac{3}{5}$ (viii) $\frac{3}{10}$
 6. relative frequencies $\frac{3}{5}$, $\frac{2}{5}$ (i) 312 (ii) 122 7. relative frequencies $\frac{1}{8}$, $\frac{5}{24}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{8}$
 (i) 750 (ii) 375 (iii) 83 app 8. (a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (d) 1 (e) 10

Miscellaneous Exercise 11

1. (i). (b) (ii). (a) (iii). (c) (iv). (d) (v). (c) (vi). (a) (vii). (b)
 (viii). (c) (ix). (b) (x). (b) (xi). (c) (xii). (b) (xiii). (c) (xiv). (b)
 (xv). (d) (xvi). (b) (xvii). (d) (xviii). (d) (xix). (c) (xx). (a)
 2. a. mean 23.44, median = mode 25 3. 53.594, 35 5. a. mean = median = mode = 25 6. (a)
 50 hits (b) 450 misses 7. 8 days 8 16 assessments 9. (a) $\frac{13}{50}$ (b) $\frac{37}{50}$ (c) $\frac{11}{50}$

TABLES OF LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	16	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	24
16	2041	2068	2095	2122	2148						3	5	8	11	14	17	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
											2	4	6	8	10	12	14	16	18
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	15	17
						3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
22	3424	3444	3464	3483	3502						2	4	6	7	9	11	13	15	17
						3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
23	3617	3636	3655	3674	3692						2	4	5	7	9	11	12	14	16
						3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
24	3802	3820	3838	3856	3874	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
						4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
25	3979	3997	4014	4031	4048						2	3	5	6	8	9	11	13	14
						4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
26	4150	4166	4183	4200	4216	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
						4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
27	4314	4330	4346	4362	4378						1	3	4	6	7	9	10	11	13
						4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
28	4472	4487	4502	4518	4533	5011	5024	5038	5051	5065	1	3	4	6	7	8	10	11	12
						5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
29	4624	4639	4654	4669	4683	5237	5250	5263	5276	5289	1	3	4	5	6	8	9	10	12
						5391	5403	5416	5428	5440	1	3	4	5	6	8	9	10	11
30	4771	4786	4800	4814	4829	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
						5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
31	4914	4928	4942	4955	4969	5752	5763	5775	5786	5798	1	2	3	5	6	7	8	9	10
						5866	5877	5888	5899	5910	1	2	3	5	6	7	8	9	10
32	5051	5065	5079	5092	5105	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
						6064	6075	6085	6096	6107	1	2	3	4	5	6	8	9	10
33	5185	5198	5211	5224	5237	6170	6180	6191	6201	6212	1	2	3	4	5	6	7	8	9
						6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
34	5315	5328	5340	5353	5366	6375	6385	6395	6405	6415	1	2	3	4	5	6	7	8	9
						6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
35	5441	5453	5465	5478	5490	6571	6580	6590	6599	6609	1	2	3	4	5	6	7	8	9
						6665	6675	6684	6693	6702	1	2	3	4	5	6	7	7	8
36	5563	5575	5587	5599	5611	6758	6767	6776	6785	6794	1	2	3	4	5	5	6	7	8
						6848	6857	6866	6875	6884	1	2	3	4	4	5	6	7	8
37	5682	5694	5705	5717	5729	6937	6946	6955	6964	6972	1	2	3	4	4	5	6	7	8
						6981	6990	6999	7008	7017	1	2	3	4	4	5	6	7	8
38	5798	5809	5821	5832	5843														
39	5911	5922	5933	5944	5955														
40	6021	6031	6042	6053	6064														
41	6128	6138	6149	6160	6170														
42	6232	6243	6253	6263	6274														
43	6335	6345	6355	6365	6375														
44	6435	6444	6454	6464	6474														
45	6532	6542	6551	6561	6571														
46	6628	6637	6646	6656	6665														
47	6721	6730	6739	6749	6758														
48	6812	6821	6830	6839	6848														
49	6902	6911	6920	6928	6937														

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	7
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	6	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	5	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	4	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	6
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	3	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	5	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	4	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	2	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	5
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8696	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	3	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	4	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4

TABLES OF ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1027	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1100	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	2
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	1	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	1	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	1	2	2	2	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	1	2	2	2	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	1	2	2	2	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	1	2	2	2	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	1	2	2	2	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	1	2	2	2	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	1	2	2	2	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	1	2	2	2	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	1	1	2	2	2	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	1	1	2	2	2	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	1	1	2	2	2	3
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	1	1	2	2	2	3
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	1	1	2	2	2	3
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	1	1	2	2	2	3
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	1	1	2	2	2	3
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	1	1	2	2	2	3
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	1	1	2	2	2	3
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	1	1	2	2	2	3
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	1	1	2	2	2	3
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	1	1	2	2	2	3
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	1	1	2	2	2	3
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	2	3	3	3	4
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	2	3	3	3	4
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	2	3	3	3	4
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	2	3	3	3	4
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	2	3	3	3	4
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	2	3	3	3	4
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	2	3	3	3	4
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	2	3	3	3	4
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2686	1	1	2	2	2	3	3	3	4
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	2	2	3	3	3	4
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	2	2	3	3	3	4
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	2	2	3	3	3	4
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	2	2	3	3	3	4
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	2	2	3	3	3	4
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	2	2	3	3	3	4
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	5	6	7	8
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	6	7	8
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	6	7	8
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	7	8
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	7	8
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	8
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	4	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	5	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	6	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	7	8	9
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	9	10
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	7	8	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	6	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	11
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	10	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	9	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	3	4	5	6	8	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	12
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	7	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	11	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	10	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	13
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	9	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	12	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	11	12	14
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	13	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	10	11	13	15
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	7	8	10	12	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	9	10	12	14	16
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	11	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	13	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	15	17
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	8	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	10	12	14	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	16	18
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

G L O S S A R Y

- Absolute value:** The absolute value of a number is its distance from 0 on the number line.
- Absolute value equation:** An equation that contains an absolute value symbol.
- Additive identity:** Additive identity of real numbers is zero but additive identity of matrices are null matrices.
- Additive inverse:** The element which is inverse to a given element with respect to addition.
- Algebraic expression:** An expression consisting of variables and constants, connected by algebraic operations.
- Altitude:** A line segment between a vertex and a side of a polygon, that is perpendicular to the side.
- Angle bisector:** A ray that divides an angle into two equal parts.
- Anti logarithm:** Inverse operation of logarithm.
- Arithmetic mean:** It is ratio of sum of quantities and number of quantities.
- Axiom:** It is a mathematical statement which is assumed to be true without any proof.
- Base of logarithm:** The number in terms of which a given number is expressed as a logarithm or exponential.
- Bearings:** Bearings are the angles measured clockwise from due north and are expressed as three digit numbers.
- Binary relation:** A binary relation R is a subset of the Cartesian product of two sets.
- Cartesian plane:** the two dimensional, the points of which are identify by their coordinates.
- Cartesian product of sets:** Cartesian product of sets A and B is the set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.
- Characteristic:** The integral part of logarithm of a number.
- Circular region:** The union of a circle and its interior region is the circular region.
- Circular system:** The system in which angle is measured in radians.
- Coefficient of variation:** The percentage ratio between standard deviation and mean of a data.
- Collinear points:** The points lying on same straight line.
- Common logarithm:** Logarithm having base 10.
- Complement of set:** Let $A \subset U$, then the set of all elements of U , which are not in A , is called complement of A .
- Complementary events:** Two events in which one shows non-occurrence of other.
- Compound inequalities:** Two inequalities that are joined by the word "and" or the word "or".
- Conjecture:** A statement that is believed to be true but its truth has not been proved. It is a true statement that needs proof.
- Converse:** A proposition derived from another by interchanging its subject and predicate terms.
- Corollary:** A proposition that follows directly from the statement of another proposition.
- Coterminal angles:** Two angles having same terminal ray in standard position.
- Deductive reasoning:** In deductive reasoning we work from some general result to a specific conclusion.
- Diagonal:** A line joining any two vertices of a polygon that are not joined by any of its edges.
- Difference of sets:** Difference of sets A and B is a collection of those elements of A which are not present in B .
- Disjoint sets:** Two sets are said to be disjoint if they have no common elements.
- Distance formula:** It provides distance between two points in a plane or line.
- Domain of binary relation:** Set of the first elements of ordered pairs in a binary relation.
- Equality:** A mathematical statement with the equality sign is called an equality.
- Equivalent equations:** Two or more equations, having the same solution.
- Equivalent inequalities:** Two or more inequalities that have the same solution.
- Expected frequency:** The expected frequency of an event is the frequency we expect to see based on probabilities.
- Exterior angle of polygon:** Angle formed by any side of the polygon and the extension of its adjacent side.
- Extraneous root:** A solution of equation which does not satisfy solution of equation.

Factor: Any integer or polynomial that exactly divides a given integer or polynomial.

Factor theorem: A polynomial $p(x)$ has a factor $x - c$, if and only if $p(c) = 0$.

Factorization of algebraic expression: The process in which an algebraic expression can be expressed as the product of its factors.

Family of lines: For a non-zero k , the equation $l_1 + k l_2 = 0$, is also linear and represents family of lines where l_1 and l_2 are lines.

Frequency: Number of repetitions of an individual value in an ungrouped data.

Frequency table: A table consisting of values of a data along with their frequencies is called a frequency table.

Fundamental assumptions: Fundamental assumptions are statements which are regarded true without any proof.

Geometric mean: It is a measure of central tendency which is usually used to find the average rates of change in different intervals.

Gradient: It is the tangent of angle which the line makes with positive direction of the x-axis.

Grouped data: A tabular arrangement of data in which various items are arranged or distributed into some classes.

Harmonic mean: It is ratio of number of values to the sum of reciprocal of values of data.

Histogram: The representation of data by a chart having adjacent rectangular bars.

Identity: It is an equation which remains true for all possible replacements of variable involved in it.

Inclination of straight line: Inclination of a straight line is the angle

SYMBOLS AND ABBREVIATIONS USED IN MATH

$=$	$\rightarrow$	is equal to
$\neq$	$\rightarrow$	is not equal to
$\in$	$\rightarrow$	is member of
$\notin$	$\rightarrow$	is not member of
$\emptyset$	$\rightarrow$	empty set
$\cup$	$\rightarrow$	union of sets
$\cap$	$\rightarrow$	intersection of sets
$\Leftrightarrow$	$\rightarrow$	if and only if
$\overline{AB}$	$\rightarrow$	line Segment AB
AB	$\rightarrow$	measurement of side AB
$\angle A$	$\rightarrow$	measurement of angle A
$\cong$	$\rightarrow$	is congruent to
$\perp$	$\rightarrow$	is perpendicular to
Δ	$\rightarrow$	triangle
$\Rightarrow$	$\rightarrow$	implies that
$\wedge, \&$	$\rightarrow$	and
$\vee$	$\rightarrow$	or
$<$	$\rightarrow$	is less than
$>$	$\rightarrow$	is greater than
$\leq$	$\rightarrow$	is less than or equal to
$\geq$	$\rightarrow$	is greater than or equal to
@	$\rightarrow$	at the rate of
%	$\rightarrow$	percent
π	$\rightarrow$	Pie
:	$\rightarrow$	ratio
::	$\rightarrow$	proportion
$\therefore$	$\rightarrow$	therefore, hence
$\because$	$\rightarrow$	because, since
<i>i.e.</i>	$\rightarrow$	that is
$\approx$	$\rightarrow$	approximately equal to
$\sqrt{\quad}$	$\rightarrow$	square root / radical
<i>e.g.</i>	$\rightarrow$	for example
$\mid$	$\rightarrow$	such that
$\leftrightarrow$	$\rightarrow$	corresponding to
$\parallel$	$\rightarrow$	is parallel to

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About Authors

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Dr. Saleem Ullah Satti possesses Ph.D (Maths) along with different professional teaching degrees. Presently, he is working in mathematics department Air University, Islamabad. He has experience of two decades in teaching of mathematics at various levels. He authored and reviewed many books. He has been involved in the development of curriculum for mathematics upto higher level. As a resource person, he has conducted training programmes, workshops and seminars on the subject.
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Ms. Hafsa Jabeen

She has been teaching mathematics at the high school and college levels for the past 20 years. She holds a master's degree in mathematics from the University of the Punjab-Lahore. She has worked as a secondary and higher secondary subject coordinator. She has made a substantial contribution to the design and review of elementary level mathematics books under the supervision of Mr. Obaid Ur Rehman (Late), who was known for his expertise and tremendous input in writing many books on mathematics at various levels.