

PRACTICAL GEOMETRY

In this unit the students will be able to:

- Construct a triangle when two sides and included angle are given.
- Construct a triangle when one side and two angles are given.
- Construct a triangle when two sides and angle opposite to one side are given.
- Draw angle bisectors, perpendicular bisectors, medians and altitudes of given triangle.
- Verify the concurrency of angle bisectors, perpendicular bisectors, medians and altitudes of given triangle.

Pyramids of ancient Egypt remain among the largest and most impressive structures constructed by any civilization. The building of the pyramids required a mastery of art, architecture, engineering and social organization at a level unknown before that time. Observe an image of those pyramids, shown in below and try to visualize their excellence in the field of construction.

Can you relate the geometrical shapes used in these pyramids with practical geometry?





Construction of Triangles

Construction of Triangle when Measures of Two Sides and Included Angle are Given

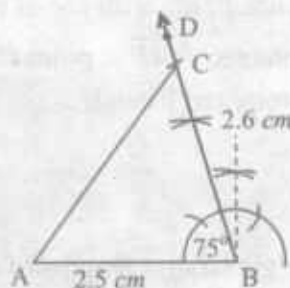
Example 1:

Construct a triangle ABC when $AB = 2.5 \text{ cm}$, $BC = 2.6 \text{ cm}$ and $\angle B = 75^\circ$.

Solution:

Steps of Construction:

- (i) Draw $AB = 2.5 \text{ cm}$.
- (ii) Construct an angle of 75° at point B with the help of ruler and compasses and draw $\overrightarrow{BD}$.
- (iii) With center B , draw an arc of radius 2.6 cm intersecting $\overrightarrow{BD}$ at C .
- (iv) Join C to A .
 ABC is required triangle.



Construction of a Triangle when Measures of One Side and Two Angles are Given

Example 2:

Construct a triangle PQR when $QR = 3.8 \text{ cm}$, $\angle P = 60^\circ$ and $\angle Q = 90^\circ$.

Solution: Angle R is required for construction. We know that:

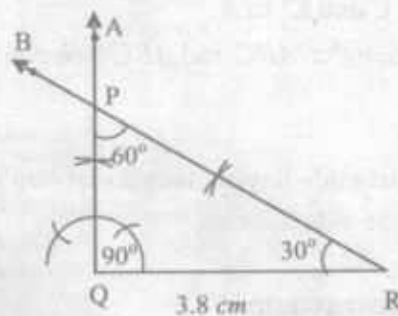
$$\angle P + \angle Q + \angle R = 180^\circ$$

$$60^\circ + 90^\circ + \angle R = 180^\circ$$

$$\angle R = 30^\circ$$

Steps of Construction:

- (i) Draw $QR = 3.8 \text{ cm}$.
- (ii) Construct an angle of 90° at Q and draw $\overrightarrow{QA}$.
- (iii) Construct an angle of 30° at R and draw $\overrightarrow{RB}$.
- (iv) Both $\overrightarrow{QA}$ and $\overrightarrow{RB}$ intersect at point P .
Thus, PQR is required triangle.



Construction of a Triangle when Measures of Two Sides and Angle Opposite to One of the Sides are Given (Ambiguous Case)

In such construction, we are not sure that how many triangles can be constructed.

$\therefore$ The above case is also called ambiguous case.

Let us solve some examples to explain this case.

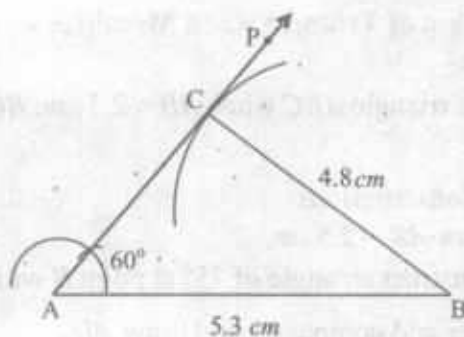
Example 3:

Construct a triangle ABC such that $AB = 5.3 \text{ cm}$ and $BC = 4.8 \text{ cm}$ and $\angle A = 60^\circ$.

Solution:

Steps of Construction:

- (i) Draw $AB = 5.3 \text{ cm}$.
- (ii) Construct $\angle BAP = 60^\circ$.
- (iii) With centre B , draw an arc of radius 4.8 cm which intersects $\overrightarrow{AP}$ at point C .
ABC is required triangle.

**Example 4:**

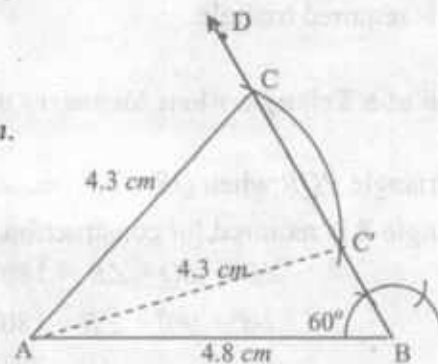
Construct a triangle having lengths of two sides 4.8 cm and 4.3 cm and angle of 60° opposite to the side 4.3 cm long.

Solution:

Steps of Construction:

- (i) Draw a line segment say AB of length 4.8 cm .
- (ii) Construct $\angle ABD = 60^\circ$.
- (iii) With center A , draw an arc of radius 4.3 cm intersecting $\overrightarrow{BD}$ at points C and C' .
- (iv) Join C and C' to A .

Thus, two triangles ABC and ABC' are constructed.

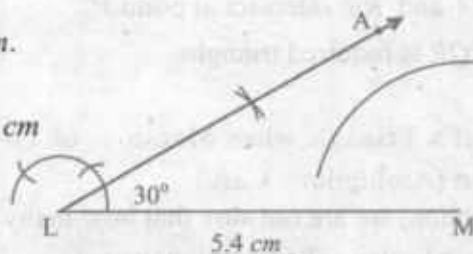
**Example 5:**

Construct a triangle having lengths of two sides as 2.3 cm and 5.4 cm and angle of 30° is opposite to the side 2.3 cm .

Solution:

Steps of Construction:

- (i) Draw a line segment LM of length 5.4 cm .
- (ii) Construct $\angle MLA = 30^\circ$.
- (iii) With centre M , draw an arc of radius 2.3 cm which does not intersect $\overrightarrow{LA}$.



$\therefore$ Construction of triangle is not possible.

EXERCISE 10.1

1. Construct the following triangles.
 - (a) $\triangle ABC$ when $AB = 5.8 \text{ cm}$, $AC = 4.2 \text{ cm}$, $\angle A = 90^\circ$
 - (b) $\triangle LMN$ when $LM = 4 \text{ cm}$, $MN = 4.7 \text{ cm}$, $\angle M = 120^\circ$
 - (c) $\triangle PQR$ when $PQ = 7.2 \text{ cm}$, $\angle P = 45^\circ$, $\angle Q = 75^\circ$
 - (d) $\triangle XYZ$ when $XY = 5 \text{ cm}$, $\angle X = 30^\circ$, $\angle Z = 105^\circ$
2. Construct the following triangles where possible.
 - (a) $AB = 6.8 \text{ cm}$, $BC = 8.1 \text{ cm}$, $\angle A = 90^\circ$
 - (b) $DE = 5.2 \text{ cm}$, $DF = 4 \text{ cm}$, $\angle E = 45^\circ$
 - (c) $QR = 7.0 \text{ cm}$, $PQ = 5.6 \text{ cm}$, $\angle R = 75^\circ$
 - (d) $XY = 3.8 \text{ cm}$, $XZ = 5 \text{ cm}$, $\angle Y = 60^\circ$
 - (e) If two sides of lengths 5.7 cm and 7.5 cm are given and angle of 105° is opposite to the side of length 7.5 cm .
 - (f) If two sides of length 6.1 cm and 3.8 cm are given and angle of 30° is opposite to the side of length 3.8 cm .

Bisectors of Angles of a Triangle

A ray which divides a given angle into two equal parts is called bisector of that angle. In the figure $\overline{BD}$ divides angle ABC into two equal angles ABD and CBD .

So $\overline{BD}$ is the bisector of angle ABC .

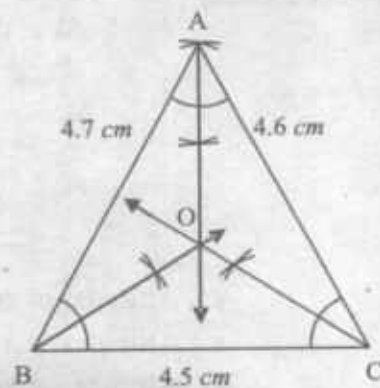
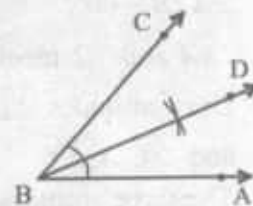
If we draw the bisectors of all the three angles of a triangle, they meet at a single point inside the triangle. We say that *the angle bisectors of a triangle are concurrent*.

Example 6: Construct a triangle ABC such that $AB = 4.7 \text{ cm}$, $BC = 4.5 \text{ cm}$ and $AC = 4.6 \text{ cm}$. Draw bisectors of three angles. Are they concurrent?

Solution: Steps of Construction:

- (i) Draw $BC = 4.5 \text{ cm}$.
- (ii) With centre B , draw an arc of radius 4.7 cm .
- (iii) With centre C , draw another arc of radius 4.6 cm intersecting first arc at A . Join A to B and C . ABC is the required triangle.
- (iv) Draw bisectors of $\angle A$, $\angle B$ and $\angle C$ which meet at point O .

$\therefore$ Angle bisectors of triangle are concurrent.





Altitudes of a Triangle

A perpendicular line segment from a vertex to opposite sides of triangle is called altitude of triangle.

In the adjoining figure, $\overline{CE} \perp \overline{AB}$. Also, CD is the distance between point C and $\overline{AB}$.

If we join C to A and B , a triangle ABC is formed. We say that CD is altitude of triangle ABC with respect to base AB .

Similarly, we can draw altitudes with respect to other sides. All the three altitudes meet at a single point.

$\therefore$ The altitudes of a triangle are concurrent.

Example 7:

Construct $\triangle XYZ$ when:

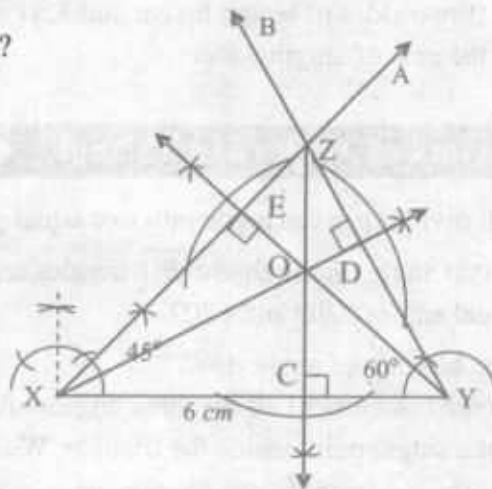
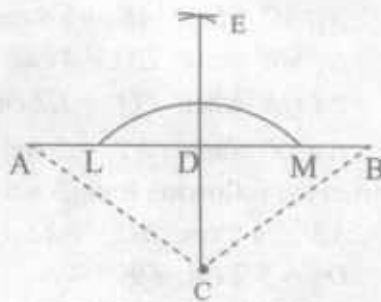
$$XY = 6 \text{ cm}, \angle X = 45^\circ, \angle Y = 60^\circ$$

Draw altitudes of triangle. Are they concurrent?

Solution:

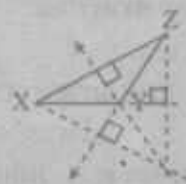
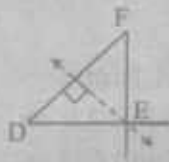
Steps of Construction:

- Draw $XY = 6 \text{ cm}$.
 - Construct $\angle YXA = 45^\circ$ and $\angle XYB = 60^\circ$.
 - $\overline{XA}$ and $\overline{YB}$ meet at Z .
 - Draw altitudes $\overline{XD} \perp \overline{YZ}$, $\overline{YE} \perp \overline{XZ}$ and $\overline{ZC} \perp \overline{XY}$.
 - The three altitudes meet at O .
- $\therefore$ Altitudes of a triangle are concurrent.



Key Fact

Look at the figures.

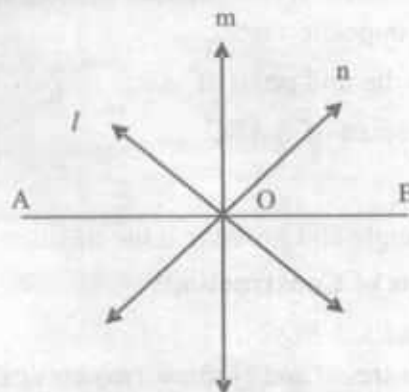


- Altitudes of an acute angled triangle meet inside the triangle.
- Altitudes of a right-angled triangle meet at the vertex of right angle.
- Altitudes of an obtuse angled triangle meet outside the triangle.



Perpendicular Bisectors of Sides of Triangle

Bisector is a line which divides a line segment into two equal parts. In the figure, lines l , m and n are the bisectors of $\overline{AB}$. Among the bisectors, the line m is perpendicular to $\overline{AB}$. So, m is a perpendicular bisector or right bisector of $\overline{AB}$. Point O is mid point of $\overline{AB}$.
i.e. $OA = OB$



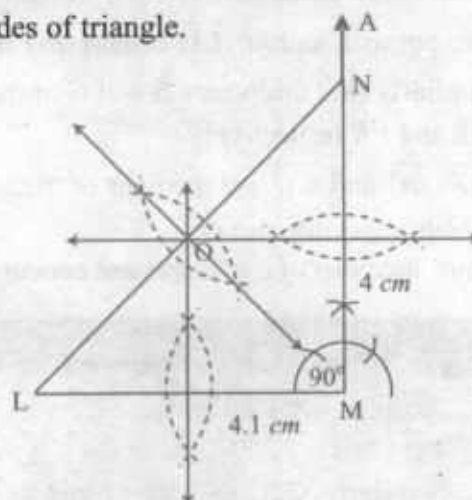
Example 8:

Construct a $\triangle LMN$ when $LM = 4.1$ cm, $MN = 4$ cm and $\angle M = 90^\circ$. Draw perpendicular bisectors of sides of triangle. Are they concurrent?

Solution:

Steps of Construction:

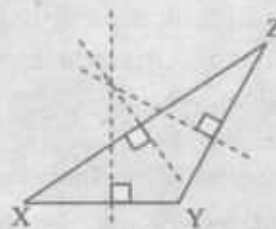
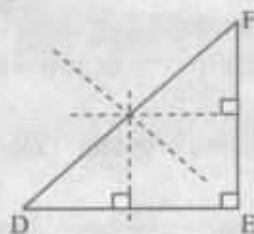
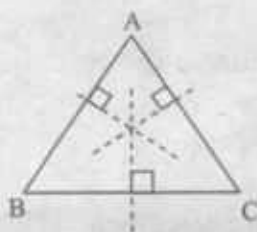
- Draw $LM = 4.1$ cm.
- Construct $\angle LMA = 90^\circ$.
- With centre M , draw an arc of radius 4 cm intersecting $\overline{MA}$ at N .
- Join N to L .
- Draw right bisectors of $\overline{LM}$, $\overline{MN}$ and $\overline{LN}$ which meet at point O .



$\therefore$ Right bisectors of sides of a triangle are concurrent.

Note: Look at the figures below.

- Right bisectors of an acute angled triangle meet inside the triangle.
- Right bisectors of a right-angled triangle meet at the mid point of the hypotenuse.
- Right bisectors of an obtuse angled triangle meet outside the triangle.

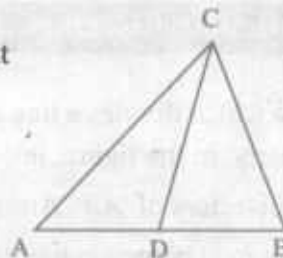


Medians of a Triangle

Median of a triangle is a line segment joining the mid point of a side to the opposite vertex.

In $\triangle ABC$, D is the mid point of $\overline{AB}$.

$\therefore \overline{CD}$ is the median of $\triangle ABC$.



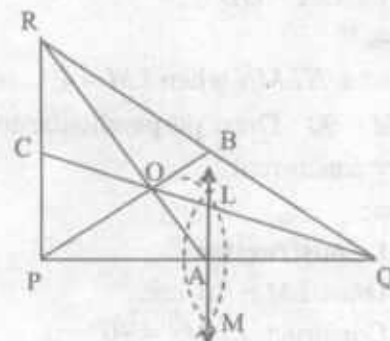
Example 9:

Construct a triangle and prove that the medians of a triangle are concurrent.

Solution: Steps of Construction:

- (i) Construct a $\triangle PQR$
- (ii) With centres P and Q , draw two arcs of equal radius (more than half of PQ), which meet at two points L and M . $\overline{LM}$ bisects $\overline{PQ}$ at A .
- (iii) Similarly find midpoints B and C of the sides QR and PR respectively.
- (iv) $\overline{AR}$, $\overline{BP}$ and $\overline{CQ}$ are medians of triangle which meet at point O .

Thus, medians of a triangle are concurrent.

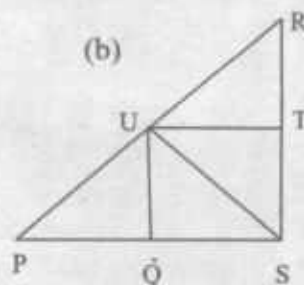
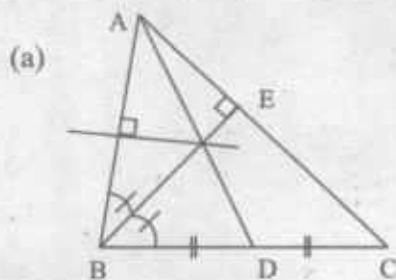


Key Fact

- In the above example, measure PO and OB . You will see that $PO : OB = 2 : 1$.
Similarly, $QO : OC = 2 : 1$ and $RO : OA = 2 : 1$.
 $\therefore$ Medians of a triangle trisect each other.
- The point where angle bisectors of triangle meet is called **inscribed centre** or **in-centre**.
- The point where altitudes of a triangle meet is called **orthocentre**.
- The point where perpendicular bisectors of a triangle meet is called **circumcentre**.
- The point where medians of a triangle meet is called **centroid**.
- In an equilateral triangle inscribed centre, circumcentre, orthocentre and centroid all lie at a single point (coincide).
- In any triangle circumcentre, orthocentre and centroid are **collinear** (lie on a same line).
- There are infinite of bisectors of any line segment. Only one of them is perpendicular to the line segment.
- Centre of gravity of a triangular shaped body lies on points of intersection of medians.

EXERCISE 10.2

1. Show that angle bisectors of the following triangles are concurrent.
 - (a) $\triangle ABC$ when $AB = 5.8 \text{ cm}$, $BC = 4.9 \text{ cm}$, $\angle B = 60^\circ$
 - (b) $\triangle XYZ$ when $XY = 5.5 \text{ cm}$, $\angle X = 45^\circ$, $\angle Y = 75^\circ$
2. Show that altitudes of the following triangles are concurrent.
 - (a) $\triangle ABC$ when $AB = 4 \text{ cm}$, $BC = 5 \text{ cm}$, $AC = 6 \text{ cm}$.
 - (b) $\triangle PQR$ when $PQ = 4.6 \text{ cm}$, $QR = 6.5 \text{ cm}$, $\angle P = 90^\circ$
 - (c) $\triangle LMN$ when $LM = 4.2 \text{ cm}$, $MN = 4 \text{ cm}$, $\angle M = 105^\circ$
3. Show that right bisectors of the following triangles are concurrent.
 - (a) $\triangle XYZ$ when $XY = 4.5 \text{ cm}$, $YZ = 5 \text{ cm}$, $ZX = 4.8 \text{ cm}$
 - (b) $\triangle PQR$ when $PQ = 4 \text{ cm}$, $QR = 5.8 \text{ cm}$, $\angle Q = 90^\circ$
 - (c) $\triangle DEF$ when $DE = 5 \text{ cm}$, $EF = 4 \text{ cm}$, $\angle E = 120^\circ$
4. Show that medians of the following triangles are concurrent.
 - (a) $\triangle ABC$ when $AB = 5.8 \text{ cm}$, $BC = 5 \text{ cm}$, $\angle B = 45^\circ$
 - (b) $\triangle DEF$ when $DE = 6 \text{ cm}$, $\angle D = 90^\circ$, $\angle E = 30^\circ$
5. Construct a right triangle ABC such that $\angle B = 90^\circ$
 - (a) Find its orthocentre, where does it lie?
 - (b) Find circumcentre M of the triangle. Is M the mid-point of hypotenuse AC . Is $BM = CM$?
6. Construct an obtuse angled triangle DEF . Find its (a) circumcentre
(b) orthocentre. Check whether they lie inside or outside the triangle?
7. Construct an isosceles triangle and find its
 - (a) centroid
 - (b) inscribed centre.
8. Recognize altitudes, perpendicular bisectors and medians in the following triangles.



KEY POINTS

- If two sides and their non included angle are given, more than one triangle can be constructed but sometimes construction of such triangles becomes impossible.
- Only one triangle can be constructed if:
 - (a) three sides are given.
 - (b) two sides and their included angle are given.
 - (c) two angles and any side are given.
- The case in which we are not sure about the number of triangles to be constructed is called an *ambiguous case*.
- Angle bisectors of a triangle are always concurrent inside the triangle.
- A line segment which joins vertex of a triangle to its opposite side perpendicularly is called its altitude.
- Altitudes of acute angled, right angled and obtuse angled triangles are concurrent inside, at the vertex of right angle and outside the triangle respectively.
- Perpendicular bisectors of acute angled, right angled and obtuse angled triangle are concurrent inside, at the mid point of hypotenuse and outside the triangle respectively.
- Medians of triangles are concurrent inside the triangle.
- The point of concurrency of
 - Angle bisectors of a triangle is called in-centre (inscribed centre)
 - Altitudes of a triangle is called orthocenter.
 - Perpendicular bisectors of a triangle is called circum-centre.
 - Medians of a triangle is called centroid.
 - Medians of a triangle divides each median in the ratio 2:1.

MISCELLANEOUS EXERCISE 10

1. Encircle the correct.

- (i) A line which bisects an angle into two equal parts is
(a) right bisector (b) angle bisector (c) altitude (d) median
- (ii) Which of the following is divided by a point called midpoint?
(a) an angle (b) a line (c) a line segment (d) a ray
- (iii) A line passing through mid point and perpendicular to a side of a triangle is called
(a) right bisector (b) angle bisector (c) mid point (d) altitude
- (iv) Two sides making arms of a right angle in right triangle are its
(a) altitudes (b) medians (c) vertices (d) bisectors
- (v) Right bisectors of a right triangle meet at mid point of
(a) medians (b) altitude (c) side (d) hypotenuse
- (vi) Medians of a triangle are
(a) collinear (b) concurrent (c) perpendicular (d) parallel
- (vii) In an equilateral triangle angle bisectors, medians, altitudes and right bisectors
(a) are parallel (b) are perpendicular (c) coincide (d) are collinear
- (viii) Right bisectors of equiangular triangle are its
(a) angle bisectors (b) altitudes (c) medians (d) All a, b, c
- (ix) If three lines meet at a point, they are called
(a) intersecting (b) perpendicular (c) collinear (d) concurrent
- (x) Medians of a triangle intersect each other in the ratio.
(a) 1 : 2 (b) 1 : 3 (c) 3 : 1 (d) 2 : 3

2. Construct the following triangles.

- (i) $\triangle PQR$ when $PQ = 6\text{ cm}$, $\angle P = 30^\circ$, $\angle R = 90^\circ$.
- (ii) $\triangle XYZ$ when $XY = 5.6\text{ cm}$, $XZ = 5.2\text{ cm}$, $\angle Y = 60^\circ$.
- (iii) $\triangle ABC$ when $BC = 7\text{ cm}$, $AC = 4.3\text{ cm}$, $\angle B = 45^\circ$.

3. Construct a right isosceles triangle whose hypotenuse is 6 cm.

4. Construct an equilateral triangle ABC . Find its

- (a) incentre (b) circumcentre (c) orthocentre (d) centroid

Do they coincide?

5. A student wants to find incentre of an equilateral triangle but he does not know how to bisect the angles. Can he find incentre by using any other method? Explain your answer by taking an example.

6. Construct a triangle and find its centre of gravity. (Hint: Find centroid)