

UNIT 08

GEOMETRY OF STRAIGHT LINES

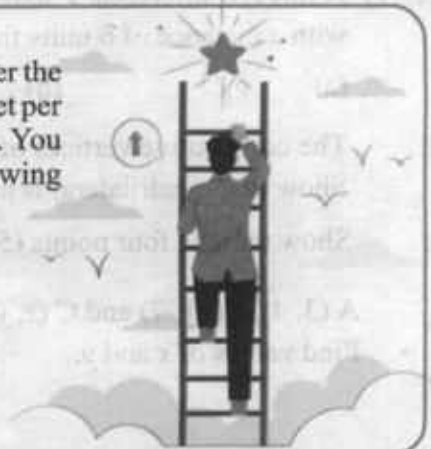
In this unit the students will be able to:

- Find the gradient of straight line when coordinates of two points in a plane are given.
- Find the gradient of parallel and perpendicular lines.
- Derive equations of straight line in slope-intercept form, point-slope form, two points form, intercepts-form, symmetric form and normal form.
- Show that the linear equation in two variables represents a straight line.
- Reduce the general form of the equation of a straight line to other standard forms.
- Find the angle between two coplanar intersecting lines.
- Find the equation of family of lines passing through the point of intersection of two given lines.
- Calculate angles of triangles when the slopes of sides are given.
- Apply concepts of coordinate geometry to real world problems.

To go from one floor to another at a library, you can take either the stairs or the escalator. You can climb stairs at a rate of 1.75 feet per second, and the escalator rises at a rate of 2 feet per second. You have to travel vertical distance of 28 feet. The equations showing the vertical distance d (in feet) traveled after t seconds are:

Stairs: $d = -1.75t + 28$ **Escalator:** $d = -2t + 28$

- a. Graph the equations in the same coordinate plane.
- b. How much time do you save by taking the escalator?





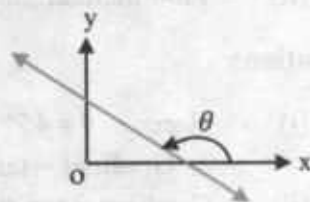
Gradient (slope) of Straight Line

Inclination of Straight Line

The inclination of a straight line is the angle θ which the line makes with positive x-axis measured in anti-clockwise direction.

Key Fact

- Any line parallel to x-axis is called horizontal line.
- Any line parallel to y-axis is called vertical line.
- Line which is neither horizontal nor vertical is called oblique line.
- Inclination of horizontal line is 0° .
- Inclination of vertical line is 90° .



Gradient

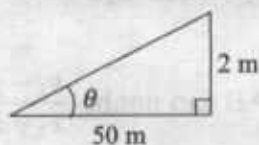
When a car rides up a hill, its speed reduces. The steeper the hill, the harder it is to climb.

The measure of the steepness of hill is called gradient (slope).

Gradient is ratio of the vertical distance to the horizontal distance.



$$\begin{aligned}\text{Gradient} &= \frac{\text{vertical distance}}{\text{horizontal distance}} = \frac{\text{rise}}{\text{run}} \\ &= \frac{2}{50} = \frac{1}{25} = \tan \theta\end{aligned}$$



Where the angle θ is inclination.

Key Fact

The gradient of a straight line is the tangent of the angle which the line makes with the positive direction of the x-axis.

Gradient of Straight Line

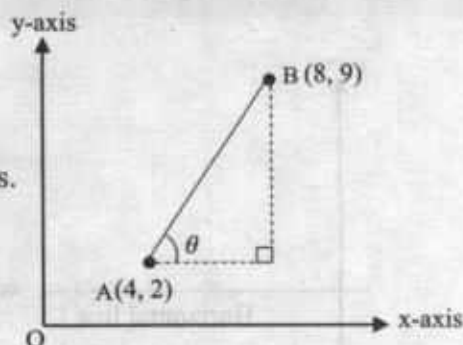
The gradient of straight line is defined as:

$$\tan \theta = \frac{\text{rise}}{\text{run}}$$

In the figure line AB is inclined at an angle θ with x-axis.

$$\begin{aligned}\text{Gradient of line AB} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{difference in y-coordinates of AB}}{\text{difference in x-coordinates of AB}} \\ &= \frac{9 - 2}{8 - 4} = \frac{7}{4}\end{aligned}$$

$$\text{Alternatively, gradient of AB} = \frac{2 - 9}{4 - 8} = \frac{-7}{-4} = \frac{7}{4}$$



Generally, gradient of a straight line is denoted by m .

Therefore, $m = \tan \theta$

Example 1:

- (i) Find the gradient (slope) of line whose inclination is 45° .
- (ii) Find inclination of line whose gradient is 1.732.

Solution:

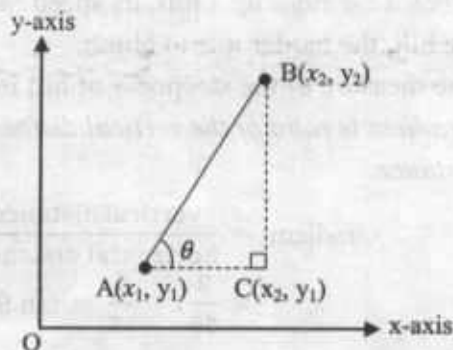
- (i) Here $\theta = 45^\circ$
 $\therefore$ Gradient $= \tan 45^\circ = 1$
- (ii) Gradient $= \tan \theta = 1.732$
 $\therefore$ Inclination $= \theta = \tan^{-1}(1.732) = 60^\circ$

Formula of Gradient

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points on a line AB, then gradient of AB is:

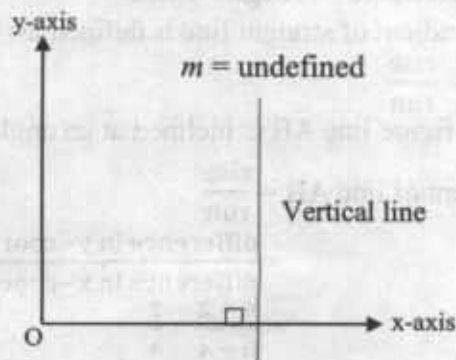
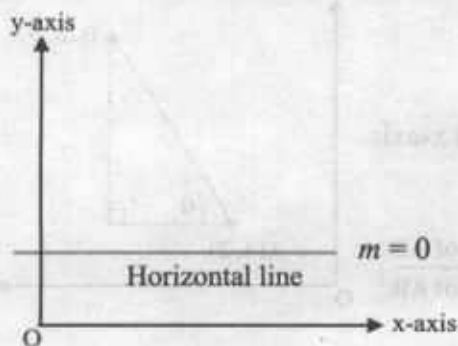
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

Gradient of AB can not be $\frac{y_1 - y_2}{x_2 - x_1}$ or $\frac{y_2 - y_1}{x_1 - x_2}$



Key Fact

- If the line is parallel to x-axis, then $\theta = 0^\circ$, and we have $m = \tan 0^\circ = 0$. Thus, gradient of horizontal line is 0.
- If the line is perpendicular to x-axis, then $\theta = 90^\circ$, and we have $m = \tan 90^\circ$ which is undefined. Thus, gradient of vertical line is undefined.



Sign of Gradient

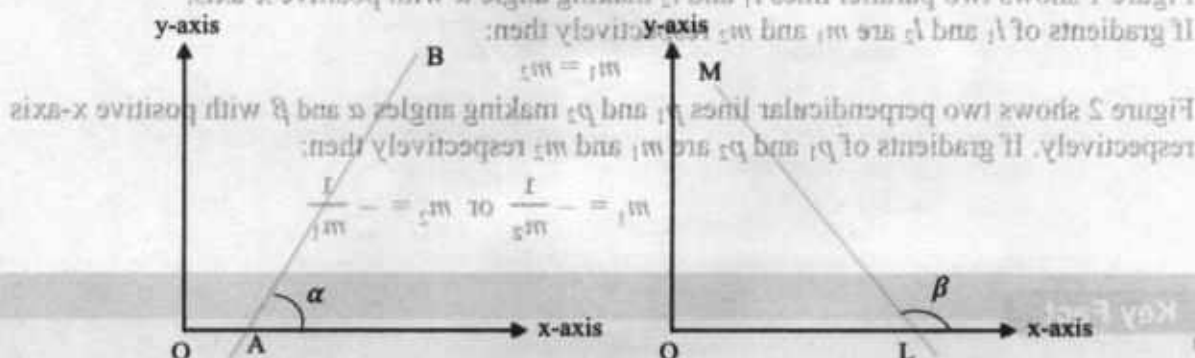


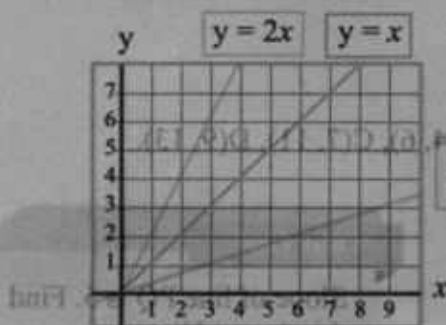
Figure 1 Figure 2

- In the figure 1, line AB makes an angle α ($0^\circ < \alpha < 90^\circ$) with the positive x-axis. As the value of $\tan \alpha$ is positive in first quadrant, therefore gradient of line AB is positive for $0^\circ < \alpha < 90^\circ$.
- In the figure 2, line LM makes an angle β ($90^\circ < \beta < 180^\circ$) with the positive x-axis. As the value of $\tan \beta$ is negative in second quadrant, therefore gradient of line LM is negative for $90^\circ < \beta < 180^\circ$.

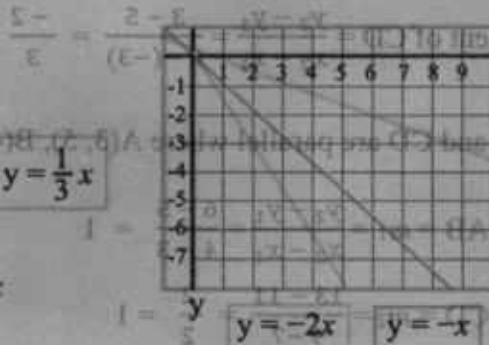
Key Fact

Gradients can be positive or negative but are always observed from left to right.

Positive Gradients



Negative Gradients



Gradient of Parallel and Perpendicular Lines

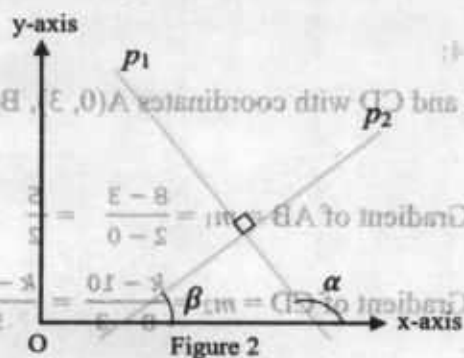
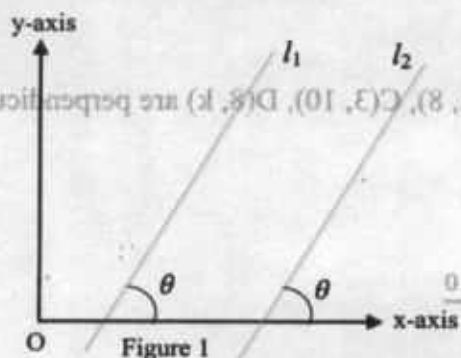


Figure 1 shows two parallel lines l_1 and l_2 making angle α with positive x-axis. If gradients of l_1 and l_2 are m_1 and m_2 respectively then:

$$m_1 = m_2$$

Figure 2 shows two perpendicular lines p_1 and p_2 making angles α and β with positive x-axis respectively. If gradients of p_1 and p_2 are m_1 and m_2 respectively then:

$$m_1 = -\frac{1}{m_2} \text{ or } m_2 = -\frac{1}{m_1}$$

Key Fact

- If two non-vertical lines are parallel, their gradients are equal.
- If two lines have same gradients, they are parallel.
- If m_1 and m_2 are gradients of two perpendicular lines, then:

$$m_1 \times m_2 = -1$$

- Three points A, B and C are collinear if gradients of AB, BC and AC are equal.

Example 2:

Find gradients of lines joining: (i) A(1, 4), B(3, 6) (ii) C(-3, 5), D(0, 3)

Solution:

$$(i) \quad \text{Gradient of AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 4}{3 - 1} = 1$$

$$(ii) \quad \text{Gradient of CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{0 - (-3)} = -\frac{2}{3}$$

Example 3:

Prove that lines AB and CD are parallel where A(3, 5), B(4, 6), C(7, 11), D(9, 13).

Solution:

$$\text{Gradient of AB} = m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 5}{4 - 3} = 1$$

$$\text{Gradient of CD} = m_2 = \frac{13 - 11}{9 - 7} = \frac{2}{2} = 1$$

$$\text{As, } m_1 = m_2$$

Therefore, AB and CD are parallel lines.

Check Point

Slope of line PQ is 5. Find the slope of line perpendicular to PQ.

Example 4:

Lines AB and CD with coordinates A(0, 3), B(2, 8), C(3, 10), D(8, k) are perpendicular. Find k.

Solution:

$$\text{Gradient of AB} = m_1 = \frac{8 - 3}{2 - 0} = \frac{5}{2}$$

$$\text{Gradient of CD} = m_2 = \frac{k - 10}{8 - 3} = \frac{k - 10}{5}$$

As, AB and CD are perpendicular,
Therefore,

$$m_1 \times m_2 = -1$$

$$\Rightarrow \frac{5}{2} \times \frac{k-10}{5} = -1$$

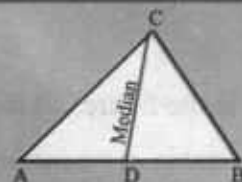
$$\Rightarrow \frac{5k-50}{10} = -1$$

$$\Rightarrow 5k-50 = -10$$

$$\Rightarrow 5k = 40$$

Key Fact

Line segments joining mid point of any side of a triangle with opposite vertex is called median of triangle.



EXERCISE 8.1

- Find the gradient (slope) of line whose inclination is:
 - 0°
 - 30°
 - 60°
 - 90°
 - 120°
 - 150°
 - 170°
 - 45.5°
- Find inclination of the line whose slope is:
 - 0
 - 0.577
 - 1.732
 - 0.364
- Find gradient and inclination of lines joining:
 - A(2, 6), B(5, 8)
 - C(-2, 4), D(1, -3)
 - E(5, -2), F(-2, -3)
- If A(-2, 6) and B(7, -3), find the slope of line:
 - parallel to AB.
 - perpendicular to AB.
- Find x if the slope of line passing through A(3, x), B(5, 8) is 4.
- Find k if lines passing through A(k, 2), B(3, 5) and C(5, -1), D(8, 7) are parallel.
- Find k if lines passing through P(-1, 2), Q(4, 7) and R(2, k), S(7, 10) are perpendicular.
- Using slopes, prove that points X(0, -3), Y(4, 7) and Z(6, 12) are collinear.
- Find value of y if points P(4, y), Q(5, 2) and R(6, 2y + 1) are collinear.
- Prove by using slopes that points A(3, -1), B(-5, -5) and C(1, 3) are vertices of a right angled triangle.
- Using slope, prove that A(-2, 1), B(6, 3), C(10, 5) and D(2, 3) are vertices of parallelogram.
- P(x, y), Q(-2, 2), R(1, 4) and S(10, 1) are vertices of a parallelogram. Find P(x, y).
- Three vertices of a rhombus are A(2, -1), B(3, 4) and C(-2, 3). Find fourth vertex.
- If (5, 0), (0, 5) and (8, 8) are vertices of a triangle, find:
 - slopes of sides.
 - slopes of medians.
 - slopes of altitudes.

Equation of Straight Line Parallel to x-axis

In the figure, all the points on the line parallel to x-axis remain at a constant distance b from x-axis.

Therefore, all points on the line satisfy the equation:

- If $b > 0$, then the line is above the x-axis (line l).
- If $b < 0$, then the line is below the x-axis (line m).
- If $b = 0$, then the line becomes x-axis and equation of x-axis become $y = 0$.

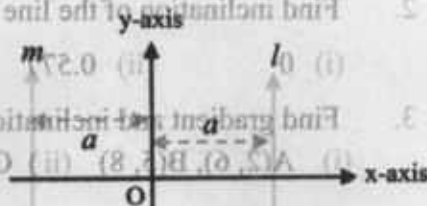


Equation of Straight Line Parallel to y-axis

In the figure, all the points on the line parallel to y-axis remain at a constant distance a from y-axis.

Therefore, all points on the line satisfy the equation:

- If $a > 0$, then the line is on the right of the y-axis (line l).
- If $a < 0$, then the line is on the left of the y-axis (line m).
- If $a = 0$, then the line becomes y-axis and equation of y-axis become $x = 0$.



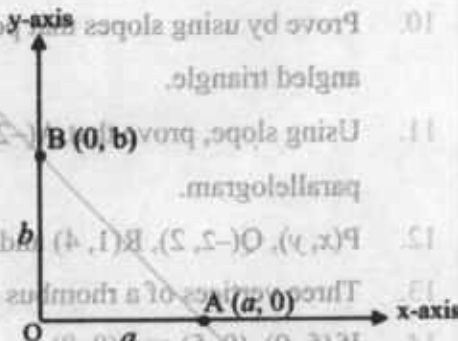
Intercepts

If a line AB intersects x-axis at $(a, 0)$, then a is

called **x-intercept** of the line AB.

If a line AB intersects y-axis at $(0, b)$, then b is

called **y-intercept** of the line AB.





Derivations of Standard Forms of Equations of Straight Lines

1. Slope-Intercept Form

Theorem: Equation of straight line (non-vertical) with slope (gradient) m and y-intercept c is:

$$y = mx + c$$

Proof:

Let $P(x, y)$ be any point on the straight line l with slope m and y-intercept c .

y-intercept c means that line intersects y-axis at point $A(0, c)$.

As $P(x, y)$ and $A(0, c)$ lie on the line l , therefore slope of line is:

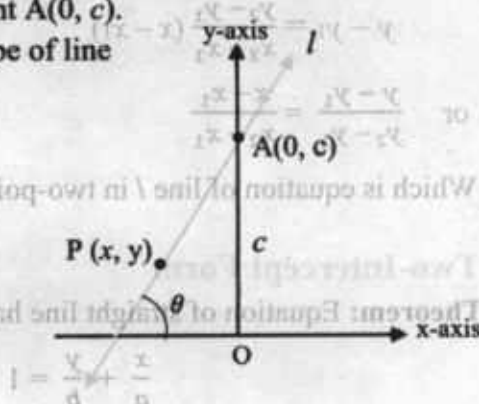
$$m = \frac{y - c}{x - 0}$$

$$\Rightarrow y - c = mx$$

$$\Rightarrow y = mx + c$$

Which is equation of line l in slope intercept form.

If $c = 0$, then the line $y = mx$ passes through origin.



2. Point Slope Form

Theorem: Equation of non-vertical straight line passing through point $B(x_1, y_1)$ with slope m is:

$$y - y_1 = m(x - x_1)$$

Proof:

Let $P(x, y)$ be any point on the straight line l with slope m

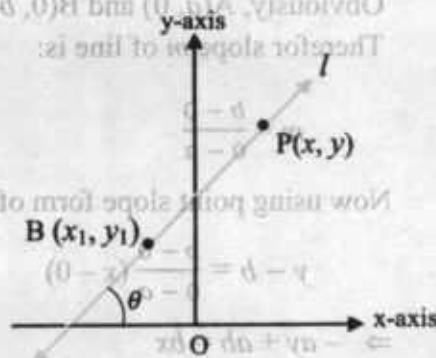
and passing through point $B(x_1, y_1)$.

As $P(x, y)$ and $B(x_1, y_1)$ lie on the line l , therefore slope m of the line is:

$$m = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow y - y_1 = m(x - x_1)$$

Which is equation of line l in point slope form.



3. Two-Point Form

Theorem: Equation of non-vertical straight line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \text{ or } y - y_2 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_2)$$

Proof:

Let $P(x, y)$ be any point on the straight line l passing through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

As $A(x_1, y_1)$ and $B(x_2, y_2)$ lie on the line l , therefore slope m of the line is:

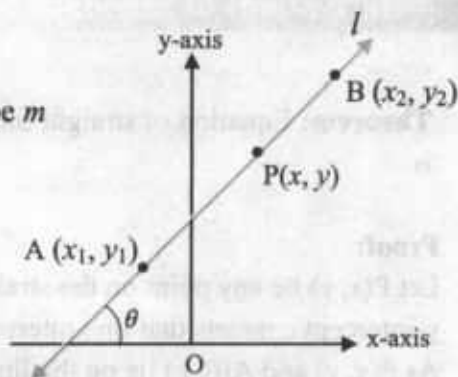
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now using point slope form of line, we have:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

$$\text{or } \frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Which is equation of line l in two-point form.

**4. Two-Intercept Form**

Theorem: Equation of straight line having x-intercept a and y-intercept b is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Proof:

Let $P(x, y)$ be any point on the straight line l with x-intercept a and y-intercept b .

Obviously, $A(a, 0)$ and $B(0, b)$ lie on the line l .

Therefore slope m of line is:

$$m = \frac{b - 0}{0 - a}$$

Now using point slope form of line, we have:

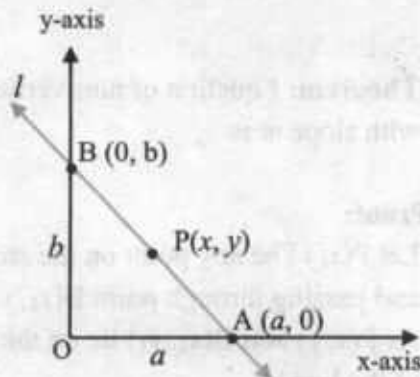
$$y - b = \frac{b - 0}{0 - a} (x - 0)$$

$$\Rightarrow -ay + ab = bx$$

$$\Rightarrow bx + ay = ab$$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = 1 \quad (\text{dividing both sides by } ab)$$

Which is equation of line l in 2 intercept form.



5. Symmetric Form

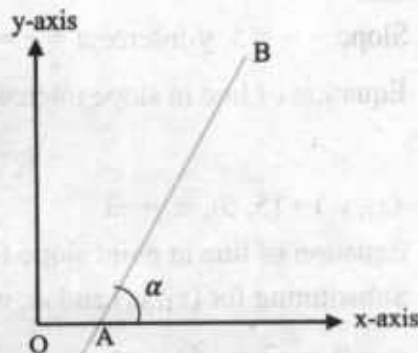
If α is inclination of a line l , then slope of line is:

$$m = \tan \alpha$$

$$\Rightarrow \tan \alpha = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow \frac{\sin \alpha}{\cos \alpha} = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow \frac{x - x_1}{\cos \alpha} = \frac{y - y_1}{\sin \alpha}$$



Which is symmetric form of equation.

6. Normal Form

Theorem: If p is perpendicular from line l to the origin and α is the inclination of this perpendicular then:

$$x \cos \alpha + y \sin \alpha = p$$

Proof:

Let the line l intersects x -axis and y -axis at points A and B respectively. Let $P(x, y)$ be any point on the line l and OC be perpendicular to line from origin, then $OC = p$

From right angled triangle OCA :

$$\cos \alpha = \frac{OC}{OA} = \frac{p}{OA}$$

$$\text{or } OA = \frac{p}{\cos \alpha}$$

From right angled triangle OCB :

$$\cos (90^\circ - \alpha) = \frac{OC}{OB} = \frac{p}{OB}$$

$$\sin \alpha = \frac{p}{OB} \text{ or } OB = \frac{p}{\sin \alpha}$$

As, OA and OB are x -intercept and y -intercept of AB , therefore using two-intercept form, we have:

$$\frac{x}{OA} + \frac{y}{OB} = 1 \Rightarrow \frac{x}{p/\cos \alpha} + \frac{y}{p/\sin \alpha} = 1 \Rightarrow x \cos \alpha + y \sin \alpha = p$$

Which is normal form of equation of line l .

Example 4:

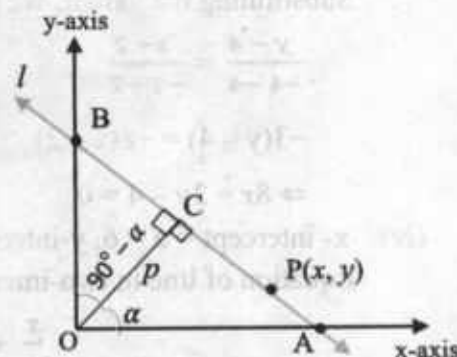
Find the equation of straight line with the following information:

(i) slope = 5, y -intercept = 3

(ii) through $(5, 0)$ with slope -3

(iii) through $(2, 4)$ and $(-1, -4)$

(iv) x -intercept = 6, y -intercept = -4



Solution:

- (i) Slope = $m = 5$, y-intercept = $c = 3$

Equation of line in slope intercept form is: $y = mx + c$

$$\Rightarrow y = 5x + 3 \text{ or } 5x - y + 3 = 0$$

- (ii) $(x_1, y_1) = (5, 0)$, $m = -3$

Equation of line in point slope form is: $y - y_1 = m(x - x_1)$

Substituting for (x_1, y_1) and m , we have:

$$y - 0 = -3(x - 5) \Rightarrow y = -3x + 15 \text{ or } 3x + y - 15 = 0$$

- (iii) $(x_1, y_1) = (2, 4)$, $(x_2, y_2) = (-1, -4)$

Equation of line in two-point form is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Substituting the values, we have:

$$\frac{y - 4}{-4 - 4} = \frac{x - 2}{-1 - 2} \Rightarrow \frac{y - 4}{-8} = \frac{x - 2}{-3}$$

$$-3(y - 4) = -8(x - 2) \Rightarrow -3y + 12 = -8x + 16$$

$$\Rightarrow 8x - 3y - 4 = 0$$

- (iv) x-intercept = $a = 6$, y-intercept = $b = -4$

Equation of line in two-intercepts form is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Substituting the values, we have:

$$\frac{x}{6} + \frac{y}{-4} = 1 \Rightarrow -4x + 6y = -24$$

$$\text{or } 2x - 3y - 12 = 0$$

Example 5:

Find the equation of straight line:

- through $(3, -2)$ and perpendicular to the line with slope 3.
- having y-intercept = -7 and parallel to the line with slope 4.
- through $(-5, 1)$ and perpendicular to the line passing through $(0, 9)$ and $(-2, 6)$.
- if length of perpendicular from the origin is 6 units inclined at 60° .

Solution:

- (i) $(x_1, y_1) = (3, -2)$

Slope of given line = 3

As, the required line is perpendicular to the given line, therefore, slope of required line is:

$$m = -\frac{1}{3}$$

∴ Equation of required line is:

$$y + 2 = -\frac{1}{3}(x - 3) \quad (\text{point-slope form})$$

$$\Rightarrow 3y + 6 = -x + 3 \Rightarrow x + 3y + 9 = 0$$

(ii) y-intercept = -7

Slope of given line = 4

As, the required line is parallel to the given line, therefore, slope of required line is:

$$m = 4$$

∴ Equation of required line is:

$$y = 4x - 7 \quad (\text{slope intercept form})$$

$$\Rightarrow 4x - y - 7 = 0$$

(iii) $(x_1, y_1) = (-5, 1)$ and perpendicular the line passing through $(0, 9)$ and $(-2, 6)$.

$$\text{Slope of given line} = \frac{6-9}{-2-0} = \frac{-3}{-2} = \frac{3}{2}$$

As, the required line is perpendicular to the given line, therefore, slope of required line is:

$$m = -\frac{2}{3}$$

∴ Equation of required line is:

$$y - 1 = -\frac{2}{3}(x + 5) \quad (\text{point-slope form})$$

$$\Rightarrow 3y - 3 = -2x - 10 \Rightarrow 2x + 3y + 7 = 0$$

(iv) $p = 6$ and $\theta = 60^\circ$

Equation of line in normal form is:

$$x \cos 60^\circ + y \sin 60^\circ = 6$$

$$\Rightarrow x \times \frac{1}{2} + y \times \frac{\sqrt{3}}{2} = 6$$

$$\Rightarrow x + \sqrt{3}y = 12 \quad \text{or} \quad x + \sqrt{3}y - 12 = 0$$

General Form of Equation of Straight Line

General form of a linear equation in two variables x and y is:

$$ax + by + c = 0$$

where a , b and c are real numbers and a and b cannot be both zero.

Theorem:

A linear equation $ax + by + c = 0$, in two variables x and y represents a straight line. Here a, b, c are constants, and a and b are not simultaneously zero.

Proof:

$$ax + by + c = 0 \quad (1)$$

Case 1: When $a = 0$ and $b \neq 0$ then equation (1) takes the form.

$$by + c = 0 \quad \text{or} \quad y = -\frac{c}{b}$$

which represents equation of line parallel to x -axis.

Case 2: When $a \neq 0$ and $b = 0$ then equation (1) takes the form.

$$ax + c = 0 \quad \text{or} \quad x = -\frac{c}{a}$$

which represents equation of line parallel to y -axis.

Case 3: When $a \neq 0$ and $b \neq 0$ then equation (1) takes the form.

$$by = -ax - c = 0 \quad \text{or} \quad y = -\frac{a}{b}x - \frac{c}{b} = \left(-\frac{a}{b}x\right) + \left(-\frac{c}{b}\right)$$

which represents slope-intercept form of equation of line with:

$$\text{slope} = -\frac{a}{b} \quad \text{and} \quad y\text{-intercept} = -\frac{c}{b}$$

Key Fact

From case 3, we can have a rule for finding slope of equation of line $ax + by + c = 0$ as follows:

$$\text{slope} = -\frac{a}{b} = -\frac{\text{coefficient of } x}{\text{coefficient of } y}$$

Reduction of General Form of Equation of a Straight Line to Other Standard Forms

We know that general form of equation of straight line in two variable is:

$$ax + by + c = 0 \quad (1)$$

We can reduce equation (1) into various standard forms as follows:

1. Slope-Intercept Form

From equation (1) we can write.

$$by = -ax - c = 0 \quad \text{or} \quad y = -\frac{a}{b}x - \frac{c}{b} = \left(-\frac{a}{b}x\right) + \left(-\frac{c}{b}\right)$$

which represents slope-intercept form of equation of line with:

$$\text{slope} = -\frac{a}{b} \quad \text{and} \quad y\text{-intercept} = -\frac{c}{b}$$

2. Point-Slope Form

$$\text{Slope of equation (1) is } m = -\frac{a}{b}$$

Check Point

Find equation of sides of triangle with vertices $P(1, 2)$, $Q(5, 6)$ and $R(11, 12)$.

A point on equation (1) is $(x_1, y_1) = (0, -\frac{c}{b})$.

∴ Equation of line in point-slope form becomes:

$$y - \frac{c}{b} = -\frac{a}{b}(x - (-\frac{c}{b})) \quad \text{or} \quad y - \frac{c}{b} = -\frac{a}{b}(x + \frac{c}{b})$$

3. Two-Point Form

Two points on equation (1) can be taken as $(x_1, y_1) = (0, -\frac{c}{b})$ and $(x_2, y_2) = (-\frac{c}{a}, 0)$.

∴ Equation of line in two-point form is:

$$\frac{y - \frac{c}{b}}{0 - \frac{c}{b}} = \frac{x - 0}{-\frac{c}{a} - 0} \quad \text{or} \quad y - \frac{c}{b} = -\frac{a}{b}(x - 0)$$

4. Two-Intercept Form

From equation (1)

$$ax + by = -c \quad \text{or} \quad \frac{ax}{-c} + \frac{by}{-c} = 1 \quad \text{or} \quad \frac{x}{-c/a} + \frac{y}{-c/b} = 1$$

which is equation in two-intercept form with:

$$x\text{-intercept} = -\frac{c}{a} \quad \text{and} \quad y\text{-intercept} = -\frac{c}{b}$$

5. Symmetric Form

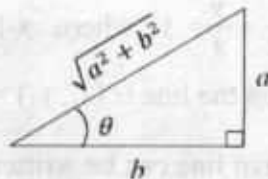
Slope of equation (1) is $m = \tan \theta = -\frac{a}{b}$

$$\sin \theta = \frac{a}{\sqrt{a^2 + b^2}} \quad \text{and} \quad \cos \theta = \frac{b}{\sqrt{a^2 + b^2}}$$

A point on equation (1) is $(x_1, y_1) = (0, -\frac{c}{b})$.

∴ Equation of line in symmetric form is:

$$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} \quad \text{or} \quad \frac{x - 0}{\frac{b}{\sqrt{a^2 + b^2}}} = \frac{y + \frac{c}{b}}{\frac{a}{\sqrt{a^2 + b^2}}}$$



6. Normal Form

Equation in normal form is:

$$x \cos \theta + y \sin \theta = p \quad (2)$$

From equation (1), we have:

$$ax + by = -c \quad (3)$$

Equations (2) and (3) are identical, therefore:

$$\frac{a}{\cos \alpha} = \frac{b}{\sin \alpha} = \frac{-c}{p}$$

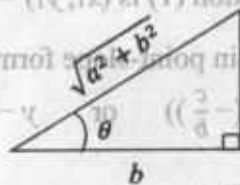
$$\Rightarrow \frac{p}{-c} = \frac{\cos \theta}{a} = \frac{\sin \theta}{b} = \frac{\sqrt{\cos^2 \theta + \sin^2 \theta}}{\pm \sqrt{a^2 + b^2}} = \frac{1}{\pm \sqrt{a^2 + b^2}}$$

$$\Rightarrow p = \frac{-c}{\pm \sqrt{a^2 + b^2}}$$

Now, dividing both sides by $\pm\sqrt{a^2+b^2}$, we get

$$\frac{ax}{\pm\sqrt{a^2+b^2}} + \frac{by}{\pm\sqrt{a^2+b^2}} = \frac{-c}{\pm\sqrt{a^2+b^2}}$$

$$\Rightarrow x\left(\frac{a}{\pm\sqrt{a^2+b^2}}\right) + y\left(\frac{b}{\pm\sqrt{a^2+b^2}}\right) = \frac{-c}{\pm\sqrt{a^2+b^2}}$$



Which is required normal form of equation of line.

The sign of radical is chosen in this way that right hand side becomes positive.

Example 6:

Reduce the equation $6x - 5y + 15 = 0$ into:

- (i) slope-intercept form (ii) two-intercept form (iii) point-slope form
(iv) two-point form (v) normal form (vi) symmetric form

Solution:

(i) $6x - 5y + 15 = 0 \Rightarrow 5y = 6x + 15 \Rightarrow y = \frac{6}{5}x + 3$

where $m = \frac{6}{5}$ and $c = 3$

(ii) $6x - 5y + 15 = 0 \Rightarrow 6x - 5y = -15$

$$\Rightarrow \left(\frac{6x}{-15}\right) - \left(\frac{5y}{-15}\right) = 1 \Rightarrow \frac{x}{-15/6} + \frac{y}{15/5} = 1$$

$$\Rightarrow \frac{x}{-5/2} + \frac{y}{3} = 1 \text{ where } x\text{-intercept} = -\frac{5}{2} \text{ and } y\text{-intercept} = 3$$

(iii) A point on the line is $(x_1, y_1) = (0, 3)$. Also slope of line is $m = -\left(\frac{6}{-5}\right) = \frac{6}{5}$
Equation of line can be written as $y - 3 = \frac{6}{5}(x - 0)$

(iv) Two points on the line are $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = \left(-\frac{5}{2}, 0\right)$
Equation of line passing through both points is:

$$\frac{y - 3}{0 - 3} = \frac{x - 0}{-\frac{5}{2} - 0}$$

(v) Given equation can be written as $6x - 5y = -15$

Or $-6x + 5y = 15$ (Make right side positive)

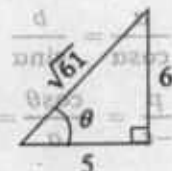
Dividing both sides of $-6x + 5y = 15$ by $\sqrt{(-6)^2 + 5^2} = \sqrt{36 + 25} = \sqrt{61}$

$$\frac{-6}{\sqrt{61}}x + \frac{5}{\sqrt{61}}y = \frac{15}{\sqrt{61}} \quad \text{or} \quad x\left(\frac{-6}{\sqrt{61}}\right) + y\left(\frac{5}{\sqrt{61}}\right) = \frac{15}{\sqrt{61}}$$

Which is normal form of equation.

(vi) Given equation is $6x - 5y + 15 = 0$.

$$m = \tan\theta = \frac{6}{5} \text{ which implies } \sin\theta = \frac{6}{\sqrt{61}} \text{ and } \cos\theta = \frac{5}{\sqrt{61}}$$



A point on the line is $(x_1, y_1) = (0, 3)$.

Thus equation in symmetric form is:

$$\frac{x-0}{\frac{5}{\sqrt{61}}} = \frac{y-3}{\frac{6}{\sqrt{61}}}$$

EXERCISE 8.2

1. Find the equation of horizontal line passing through:

(i) $(2, 3)$

(ii) $(8, 0)$

(iii) $(-5, -9)$

(iv) $(-\frac{3}{2}, -\frac{5}{2})$

2. Find the equation of vertical line passing through:

(i) $(1, 5)$

(ii) $(9, 6)$

(iii) $(-4, -7)$

(iv) $(\frac{1}{4}, \frac{3}{4})$

3. Find equation of the line with the following information:

(i) slope = 2, y-intercept = -3

(ii) through $(-5, 7)$ with slope 4

(iii) through $(4, -5)$ with slope 0

(iv) through $(-2, 9)$ with slope undefined

(v) through $(-6, 1)$ and $(2, -4)$

(vi) through $(2, -4)$ and $(8, 4)$

(vii) x-intercept = -6, y-intercept = 5

(viii) slope = -1, x-intercept = 11

4. Find equation of line in symmetric form when:

(i) $(x_1, y_1) = (-4, 2)$ and $\tan \theta = \frac{3}{4}$

(ii) $(x_1, y_1) = (6, -6)$ and $\theta = 30^\circ$

5. Find equation of line in normal form when:

(i) $p = 5$ and $\theta = 120^\circ$

(ii) $p = 10$ and $\tan \theta = 1$

6. Find the equation of straight line:

(i) through $(-4, -4)$ and parallel to the line with slope -5.

(ii) through $(5, -1)$ and perpendicular to the line with slope $\frac{1}{4}$.

(iii) having y-intercept = 4 and parallel to the line with slope $\frac{1}{2}$.

(iv) having x-intercept = -2 and perpendicular to the line with slope 4.

(v) through $(-1, 4)$ and perpendicular to the line passing through $(3, 0)$ and $(1, -2)$.

(vi) through $(6, -4)$ and parallel to the line passing through $(-5, 2)$ and $(3, 6)$.

7. Find equation of the line through $(3, 7)$ and parallel to the line $4x - 3y + 1 = 0$.

8. Find equation of the line through $(-2, -1)$ and perpendicular to the line $x - 2y = 0$.

9. Find equation of perpendicular bisector of line segment joining $(0, 6)$ and $(2, -2)$.

10. Find equation of medians and altitudes of triangle with vertices

$A(0, 4)$, $B(4, 6)$ and $C(-2, -2)$.

11. Reduce the equations (a) $6x + 8y - 11 = 0$ (b) $4x - 3y + 9 = 0$ into:

(i) slope-intercept form

(ii) two-intercept form

(iii) point-slope form

(iv) two-point form

(v) normal form

(vi) symmetric form



Angle Between Two Coplanar Intersecting Lines

Theorem:

Let l_1 and l_2 be two non-vertical non-perpendicular coplanar lines making angles α and β with positive x-axis respectively as shown in figure.

If m_1 and m_2 are the slopes of l_1 and l_2 respectively, then the angle θ from l_1 to l_2 is:

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Proof: In $\triangle ABC$, β and θ are two interior angles and α is non adjacent exterior angle.

$$\therefore \alpha = \beta + \theta$$

$$\Rightarrow \theta = \alpha - \beta$$

$$\Rightarrow \tan \theta = \tan (\alpha - \beta)$$

$$\Rightarrow \tan \theta = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

As $\tan \alpha = m_2$ and $\tan \beta = m_1$,

$$\therefore \tan \theta = \frac{m_2 - m_1}{1 + m_2 m_1}$$

Example 7:

Find the measure of angle from l_1 to l_2 if: slope of $l_1 = -1$ and slope of $l_2 = 2$

Solution:

slope of $l_1 = m_1 = -1$ and slope of $l_2 = m_2 = 2$

If θ be the angle from l_1 to l_2 then:

$$\tan \theta = \frac{m_2 - m_1}{1 + m_2 m_1} = \frac{2 - (-1)}{1 + 2(-1)} = -3$$

$$\theta = \tan^{-1}(-3) = 108.43^\circ$$

Example 8:

Find the interior angles of triangle whose vertices are $A(-2, 0)$, $B(3, 0)$ and $C(6, 5)$.

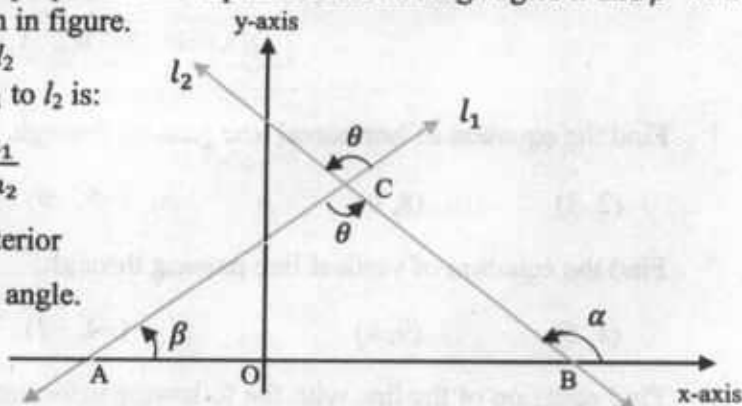
Solution:

Let the slopes of sides AB, BC and AC be denoted by m_1 , m_2 and m_3 respectively.

$$\therefore m_1 = \frac{0-0}{3+2} = 0, \quad m_2 = \frac{5-0}{6-3} = \frac{5}{3}, \quad m_3 = \frac{5-0}{6+2} = \frac{5}{8}$$

Let α , β and γ be interior angles of $\triangle ABC$ at vertex A, B and C respectively, then:

Angle is measured from AB to AC.



Key Fact

- If $l_1 \parallel l_2$, then $\tan \theta = 0$
 $\Rightarrow \frac{m_2 - m_1}{1 + m_2 m_1} = 0 \Rightarrow m_1 = m_2$
- If $l_1 \perp l_2$, then $\tan \theta = \tan 90^\circ = \infty$
 $\Rightarrow \frac{m_2 - m_1}{1 + m_2 m_1} = \infty$
 $\Rightarrow 1 + m_2 m_1 = 0 \Rightarrow m_2 m_1 = -1$
- Angles are measured from positive x-axis in counter clock-wise direction.

$$\tan \alpha = \frac{m_3 - m_1}{1 + m_3 m_1} = \frac{\frac{5}{8} - 0}{1 + (\frac{5}{8})(0)} = \frac{5}{8}$$

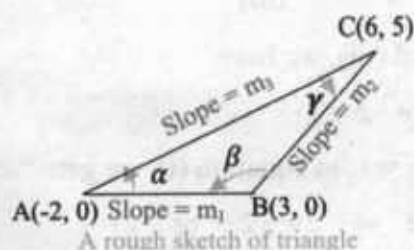
$$\Rightarrow \alpha = \tan^{-1} \left(\frac{5}{8} \right) = 32^\circ$$

$$\tan \beta = \frac{m_1 - m_2}{1 + m_1 m_2} = \frac{0 - \frac{5}{3}}{1 + (0)(\frac{5}{3})} = -\frac{5}{3}$$

$$\Rightarrow \beta = \tan^{-1} \left(-\frac{5}{3} \right) = 121^\circ$$

$$\tan \gamma = \frac{m_2 - m_3}{1 + m_2 m_3} = \frac{\frac{5}{3} - \frac{5}{8}}{1 + (\frac{5}{3})(\frac{5}{8})} = \frac{\frac{25}{24}}{\frac{49}{24}} = \frac{25}{49}$$

$$\Rightarrow \gamma = \tan^{-1} \left(-\frac{9}{7} \right) = 27^\circ$$



Key Fact

It is better to find third angle of triangle by subtracting sum of measures of first two angles from 180° .

Point of Intersection of Two Straight Lines

Let l_1 and l_2 be two non-parallel straight lines such that:

$$l_1: a_1x + b_1y + c_1 = 0 \quad \text{and} \quad l_2: a_2x + b_2y + c_2 = 0$$

where $a_1b_2 - a_2b_1 \neq 0$ otherwise $l_1 \parallel l_2$.

Let $P(x_1, y_1)$ be the point of intersection of l_1 and l_2 , then:

$$a_1x_1 + b_1y_1 + c_1 = 0 \quad (1)$$

$$a_2x_1 + b_2y_1 + c_2 = 0 \quad (2)$$

Solving (1) and (2) simultaneously, we get:

$$\frac{x_1}{b_1c_2 - b_2c_1} = \frac{y_1}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow x_1 = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \quad \text{and} \quad y_1 = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

Therefore, $\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1} \right)$ is required point of intersection.

Example 9:

Find point of intersection of following lines.

$$x + 2y = 3 \quad (i)$$

$$3x - y = 2 \quad (ii)$$

Solution:

Multiplying equation (ii) by 2 we get:

Key Fact

- Solution of two parallel lines does not exist.
- Two non-parallel lines intersect at one and only one point.
- Infinite number of lines can pass through a point.

$$6x - 2y = 4 \quad (\text{iii})$$

Adding (i) and (iii), we have:

$$7x = 7 \quad \text{or} \quad x = 1$$

Substituting $x = 1$, in equation (i), we get:

$$1 + 2y = 3 \Rightarrow 2y = 2 \quad \text{or} \quad y = 1$$

$\therefore$ Required point of intersection is (1, 1).

Equation of Family of Lines

Let l_1 and l_2 be two non-parallel straight lines:

$$l_1: a_1x + b_1y + c_1 = 0 \quad (\text{i})$$

$$l_2: a_2x + b_2y + c_2 = 0 \quad (\text{ii})$$

with point of intersection $P(x_1, y_1) = P\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$ where $a_1b_2 - a_2b_1 \neq 0$.

We can find a family of lines through the point of intersection P.

For a non-zero k, the equation:

$$a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0 \quad (\text{iii})$$

is also linear and represents a straight line.

Equation (iii) represents infinite number of lines for different values of k and therefore it is

known as family of lines.

It can be easily proved that if (x_1, y_1) is point of intersection of (i) and (ii), then equation (iii) also passes through (x_1, y_1) .

Thus equation (iii) is the required family of lines passing through point of intersection of equations (i) and (ii). As mentioned above that equation (iii) represents infinite number of lines by changing values of k. For particular value of k, a particular line of family from (iii) can be determined by imposing one more condition.

Example 10:

Find equation of line passing through point of intersection of lines $x + 2y = 0$, $x - y = 3$

(a) passing through point (1, 1) (b) parallel to $3x - 4y + 7 = 0$

Solution:

$$x + 2y = 0 \quad (\text{i})$$

$$x - y - 3 = 0 \quad (\text{ii})$$

A family of lines through the point of intersection of lines (i) and (ii) is:

$$x + 2y + k(x - y - 3) = 0$$

$$\Rightarrow (1 + k)x + (2 - k)y - 3k = 0 \quad \text{(iii)}$$

- (a) As equation (iii) passes through (1, 1), therefore:

$$(1 + k)(1) + (2 - k)(1) - 3k = 0$$

$$\Rightarrow 1 + k + 2 - k - 3k = 0 \Rightarrow 3 - 3k = 0 \Rightarrow k = 1$$

Substituting the value of k in equation (iii), we get the required line as follows:

$$(1 + 1)x + (2 - 1)y - 3 \times 1 = 0 \Rightarrow 2x + y - 3 = 0$$

- (b) Slope of line (iii) is:

$$m_1 = -\left(\frac{1+k}{2-k}\right)$$

$$3x - 4y + 7 = 0 \quad \text{(iv)}$$

Slope of line (iv) is:

$$m_2 = -\left(\frac{3}{-4}\right) = \frac{3}{4}$$

It is given that lines (iii) and (iv) are parallel, therefore:

$$-\left(\frac{1+k}{2-k}\right) = \frac{3}{4} \Rightarrow \frac{-1-k}{-2+k} = \frac{3}{4} \Rightarrow \frac{-1-k}{-2+k} = \frac{3}{4}$$

$$4(-1 - k) = 3(-2 + k) \Rightarrow -4 - 4k = -6 + 3k$$

$$7k = 2 \text{ or } k = \frac{2}{7}$$

Substituting the value of k in $x + 2y + k(x - y - 3) = 0$.

$$x + 2y + \frac{2}{7}(x - y - 3) = 0 \Rightarrow 7x + 14y + 2x - 2y - 6 = 0$$

$$\Rightarrow 9x + 12y - 6 = 0$$

Which is required line.

EXERCISE 8.3

- Find the measure of angle from l_1 to l_2 if:
 - slope of $l_1 = 0$ and slope of $l_2 = 1$
 - slope of $l_1 = -0.5$ and slope of $l_2 = 4.5$
 - slope of $l_1 = \tan 45^\circ$ and slope of $l_2 = \tan 135^\circ$
- Find the measure of angle from l_1 to l_2 if:

(i) l_1 : joining (2, 0) and (5, 0)	l_2 : joining (2, 0) and (5, 5)
(ii) l_1 : joining (-2, 1) and (3, 4)	l_2 : joining (-1, 3) and (4, 8)
(iii) l_1 : joining (-5, -4) and (5, 1)	l_2 : joining (-3, 2) and (0, 5)
(iv) l_1 : joining (2, -6) and (5, -9)	l_2 : joining (5, -5) and (-10, -5)

- (v) l_1 : joining $(0, -3)$ and $(7, -9)$ l_2 : joining $(2, -2)$ and $(-8, -12)$
3. Find the interior angles of triangle ABC when:
- (i) Slope of AB = 0, Slope of BC = -1, Slope of AC = 1
(ii) Slope of AB = 0.25, Slope of BC = 1.25, Slope of AC = 1
(iii) Slope of AB = 0.4, Slope of BC = -1.5, Slope of AC = 1.667
(iv) Slope of AB = -1, Slope of BC = 0.8, Slope of AC = 0
4. Find angle between lines:
- (i) $3x + 2y + 5 = 0$ and $2x - 3y + 8 = 0$
(ii) $x + 2y - 6 = 0$ and $2x - 4y + 9 = 0$
(iii) $6x - y + 1 = 0$ and $x - 7y + 12 = 0$
5. Find the interior angles of triangle XYZ whose vertices are:
- (i) $X(-2, 3), Y(-3, -4), Z(5, 2)$ (ii) $X(-3, 2), Y(0, -1), Z(3, 3)$
(iii) $X(-2, 0), Y(1, -4), Z(6, 6)$ (iv) $X(-4, 1), Y(0, -3), Z(4, 3)$
6. Find the point of intersection of lines:
- (i) $2x + y + 1 = 0$ and $x - y - 4 = 0$
(ii) $x + y + 3 = 0$ and $2x - 5y + 8 = 0$
(iii) $2x + 5y + 3 = 0$ and $3x - 4y - 5 = 0$
7. Find equation of line passing through point of intersection of lines
 $3x + 2y + 1 = 0, x - 2y + 3 = 0$ and
(a) passing through point $(-1, 0)$. (b) parallel to $3x - 4y + 3 = 0$.
8. Find the equation of family of lines passing through point of intersection of
 $6x + 5y + 3 = 0$ and $2x - 5y + 13 = 0$ with slope 3.
9. Find equation of line passing through point of intersection of lines
 $2x - 5y + 4 = 0, 6x - 4y + 5 = 0$ and
(a) parallel to x-axis. (b) parallel to y-axis.
10. Find equation of line passing through point of intersection of lines
 $2x - y + 2 = 0, x - 2y + 1 = 0$ and
(a) parallel to $x - 2y + 11 = 0$ (b) perpendicular to $2x + 5y + 2 = 0$.
11. Find equation of line passing through point of intersection of lines
 $x - 2y + 4 = 0, 3x - y - 3 = 0$ and
(a) parallel to line passing through $(2, -3)$ and $(0, 4)$.
(b) perpendicular to line passing through $(2, -3)$ and $(0, 4)$.



Real World Problems of Coordinate Geometry

Example 11: In the linear equation $y = 1.12x + 8$ if “ x ” represents the number of kilometers and “ y ” represents the cost of the bus fare.

- (i) What will be the cost of bus fare after travelling 50 km?
- (ii) Asif paid a bus fare of Rs. 480 in a journey. Find number of kilometers travelled by him?

Solution:

Given: $y = 1.12x + 8$ (i)

- (i) Substituting $x = 50$ in equation (i), we have:

$$y = 1.12 \times 50 + 8 = 64$$

$\therefore$ Cost of bus fare = Rs.64

- (ii) Substituting $y = 480$ in equation (i), we have:

$$480 = 1.12x + 8 \Rightarrow 1.12x = 480 - 8 = 472$$

$$x = 472 \div 1.12 = 421.43$$

$\therefore$ Numbers of kilometers = 421.43

Example 12:

Saadia buys mangoes @ Rs. 150 per kilogram and melon @ Rs. 80 per kilogram. She has Rs. 620 to spend on fruit. Write an equation in standard form that describes the situation. If she buys 2 kilograms of mangoes, how many kilograms of melon can she buy?

Solution:

Suppose x denotes number of kilograms of mangoes and y denotes number of kilograms of melon.

The equation that describes this situation is:

$$150x + 80y = 620 \quad (i)$$

As she buys 2 kilograms of oranges, substituting $x = 2$ into the equation (i) we get:

$$150(2) + 80y = 620 \quad \text{or} \quad 300 + 80y = 620$$

$$\Rightarrow 80y = 620 - 300 = 320 \Rightarrow y = 320 \div 80 = 4$$

$\therefore$ Saadia can buy 4 kilograms of melon.

Example 13:

The pollution index in a large city increases in an approximately linear fashion from 8 am until

3 pm in autumn season. On 24th November, the reading at 10:00 hours is around 80 parts per million (ppm) and at 14:00 hours, the reading is 110 ppm.

- Write an equation for this situation.
- What does the gradient (slope) mean in terms of pollution?
- What does the y-intercept mean in terms of pollution?
- What will be the pollution index at 12 pm?

Solution:

- If t denotes time and p denotes pollution index then:

$$(t_1, p_1) = (10, 80) \text{ and } (t_2, p_2) = (14, 110)$$

Using equation of line in two-point form, we have:

$$\frac{t-t_1}{t_2-t_1} = \frac{p-p_1}{p_2-p_1}$$

Substituting the values, we have:

$$\begin{aligned} \frac{t-10}{14-10} &= \frac{p-80}{110-80} \Rightarrow \frac{t-10}{4} = \frac{p-80}{30} \\ \Rightarrow 30(t-10) &= 4(p-80) \Rightarrow 30t-300 = 4p-320 \\ \Rightarrow 30t-4p+20 &= 0 \Rightarrow 15t-2p+10=0 \dots\dots (i) \end{aligned}$$

Which is required equation.

- From (i), we have:

$$\begin{aligned} 2p &= 15t + 10 \Rightarrow p = 7.5t + 5 \dots\dots (ii) \\ \Rightarrow m &= 7.5 \text{ and } c = 5 \end{aligned}$$

Here the gradient 7.5 means that pollution index is increasing @ 7.5 ppm per hour.

- y-intercept = $c = 5$ means that pollution index at 00:00 hours (12am) is 5 ppm.

You can plot graph to understand in a better way.

- Substituting $t = 12$ in equation (ii), we get:

$$p = 7.5 \times 12 + 5 = 95 \text{ ppm}$$

EXERCISE 8.4

- Nasir sold 'fruiters' @ Rs.120 per dozen and 'Shakri Malta' @ Rs.150 per dozen and earned Rs. 1200. Write an equation in standard form that describes the situation. If he sold 4 dozen of 'Shakri Malta', how many dozens of 'fruiters' he sold?

2. The linear equation $y = 1450x + 2000$ describes the total cost for staying in a hotel for one day, where Rs. 2000 is the rent of room for maximum 8 people and Rs. 1450 is the food cost per person.
 - (i) Find the cost paid to hotel if a group of 7 people stays for one day.
 - (ii) How many people can stay in the hotel for Rs. 13,600 for one day?
3. If one company provides Rs. 5500 per week along with extra bonus of Rs. 700 and the other offers Rs. 800 per day. Convert the data into linear equations and tell which is better deal for two weeks?
4. A man earns Rs. 120 per hour. He has Rs. 500 in addition with him.
 - (i) Write a linear equation for the situation and tell how much will he get after 12 hours?
 - (ii) What does slope show in this situation?
 - (iii) What does y-intercept represent here?
5. Ali shifted in a rented house on first September. The electric meter of house was showing 44 units on that day. If the average electricity consumed is 18 units per day:
 - (i) Represent the situation through linear equation.
 - (ii) How many units are consumed till 30 September?
 - (iii) What will be the bill after one month @ Rs. 20 per unit?
 - (iv) After how many days, the meter shows 404 units?
6. Alia hired a taxi with a fixed charge of Rs. 1500 plus Rs. 450 per 30 minutes.
 - (i) Represent the relation as a linear equation.
 - (ii) What will be the cost of taxi fare after 5 hours?
 - (iii) What is the slope of equation in the case?
7.
 - (i) Derive the relation between Fahrenheit and Celsius scales in slope-intercept form.
 - (ii) What does y-intercept and slope show in the equation?
 - (iii) What is temperature in Fahrenheit when temperature in Celsius is 5°C ?
8. A cricket team scores 96 runs in 16 overs and 180 runs in 30 overs.
 - (i) Write an equation of line for this situation.
 - (ii) What does the gradient mean in terms of scores?
 - (iii) What does the y-intercept mean in terms of scores?
 - (iv) What will be the predicted score after 45 overs?
 - (v) After how many overs, the predicted score will be 240?

9. Abdullah rented a truck for one day. The truck company charged Rs. 5000 per day and some additional money per kilometre. He drives 125 kilometres and paid Rs. 30,000.
 - (i) Write an equation in point-slope form that describes this situation.
 - (ii) Find the amount per kilometre the truck rental company charges and relate it with slope.
 - (iii) How much would it cost if Abdullah drove 180 km?
10. A ship starts travelling from Karachi with latitude of 25° N and longitude 67° E. If the ship travels in a straight line and reaches a destination with latitude of 32° N and longitude 54° E, then derive the equation of line in point-slope form. If the ship moves to another location with a latitude of 39° N, what is longitude of that location?
11. Length and width of a plot are in the ratio 2 : 1.
 - (i) Write equation of line and find the length of plot if the width of plot is 30 feet.
 - (ii) What is slope in this case and what does it mean?

KEY POINTS

- The inclination of a straight line is the angle θ which the line makes with positive x-axis measured in anti-clockwise direction.
- Line which is neither horizontal nor vertical is called an oblique line.
- The gradient of a straight line is the tangent of the angle which the line makes with the positive direction of the x-axis.
- Gradient of horizontal line is 0 and that of vertical line is undefined.
- If two lines have same gradients, they are parallel.
- If m_1 and m_2 are gradients of two perpendicular lines, then:

$$m_1 \times m_2 = -1$$
- Three points A, B and C are collinear if gradients of AB, BC and AC are equal.
- In $y = b$, if $b > 0$, then the line is above the x-axis, if $b < 0$, then the line is below the x-axis and $b = 0$, then the line becomes x-axis.
- In $x = a$, if $a > 0$, then the line is on the right of the y-axis, if $a < 0$, then the line is on the left of the y-axis and if $a = 0$, then the line becomes y-axis.
- If a line AB intersects x-axis at $(a, 0)$, then a is called x-intercept of the line AB and if a line AB intersects y-axis at $(0, b)$, then b is called y-intercept of the line AB.

- Equation of straight line with slope m and y-intercept c is $y = mx + c$.
- Equation of straight line passing through point $B(x_1, y_1)$ with slope m is $y - y_1 = m(x - x_1)$.
- Equation of straight line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$
- Equation of straight-line having x-intercept a and y-intercept b is:

$$\frac{x}{a} + \frac{y}{b} = 1$$
- If p is perpendicular from line l to the origin and α is the inclination of this perpendicular then $x \cos \alpha + y \sin \alpha = p$
- A linear equation $ax + by + c = 0$, in two variables x and y represents a straight line where a, b, c are constants, and a and b are not simultaneously zero.
- Angle between two lines with slopes m_1 and m_2 is defined by:

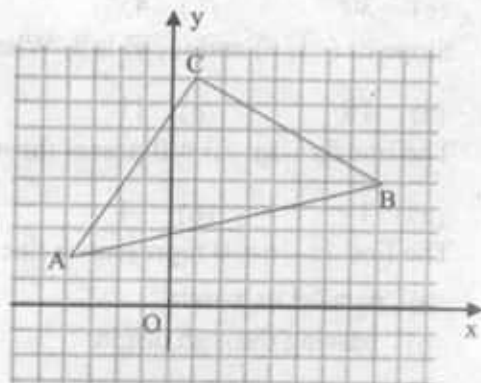
$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$
- For a non-zero k , the equation $a_1 x + b_1 y + c_1 + k(a_2 x + b_2 y + c_2) = 0$ is also linear and represents family of lines passing through (x_1, y_1) .

MISCELLANEOUS EXERCISE 8

1. Encircle the correct option in the following.

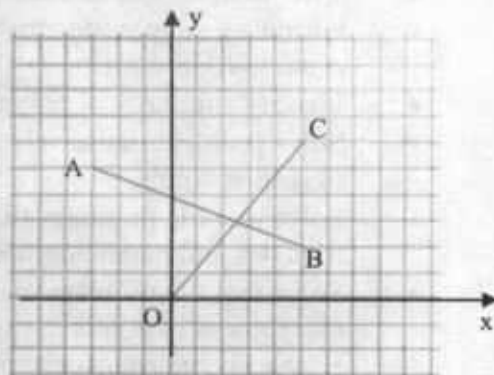
- Slope of two lines are 0 and ∞ . What is angle between both lines?
 (a) 0° (b) 60° (c) 90° (d) 180°
- One angle of right triangle ABC is determined by the line joining $A(1, 2)$ and $B(2, 3)$. Find the third angle.
 (a) 30° (b) 45° (c) 60° (d) 80°
- Slope of $(-1, 6)$ and $(1, y)$ is 3. What is y ?
 (a) 10 (b) 11 (c) -12 (d) 12
- The line $5x - ky - 3 = 0$ passes through $(1, 2)$. What is k ?
 (a) 1 (b) -1 (c) -2 (d) 2
- The line $5x - 6 = 0$ represents a line:
 (a) parallel to x-axis (b) parallel to y-axis
 (c) passing through origin (d) touching both axis
- Line $y = 0$ represents:
 (a) x-axis (b) y-axis (c) a plane (d) a point

- (vii) The line $y = b$, is above the x -axis, if
 (a) $b = 0$ (b) $b < 0$ (c) $b > 0$ (d) $b \neq 0$
- (viii) The line $x = a$, is left to the y -axis, if
 (a) $a = 0$ (b) $a \neq 0$ (c) $a > 0$ (d) $a < 0$
- (ix) Slope of a line l is -4 . What is slope of a line perpendicular to l ?
 (a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) 4 (d) -4
- (x) Which of the following line has slope $\frac{2}{3}$?
 (a) $2x + 3y = 0$ (b) $2x - 3y = 2$ (c) $3x - 2y = 1$ (d) $3x + 2y = 3$
- (xi) The line $y = 5x - 3$ is written in the form:
 (a) point-slope (b) two-intercept (c) slope-intercept (d) two-point
- (xii) x -intercept of the line $x + y = 5$ is:
 (a) 1 (b) -1 (c) 5 (d) -5
- (xiii) A line intersects both axis at $(2, 0)$ and $(0, 7)$ respectively. Its y -intercept is:
 (a) -2 (b) 2 (c) -7 (d) 7
- (xiv) Point of intersection of lines $9x - 7y = 0$ and $8x - 11y = 0$, is:
 (a) $(0, 0)$ (b) $(0, 7)$ (c) $(9, 0)$ (d) $(7, 9)$
2. Prove that $A(3, -10)$, $B(1, 4)$ and $C(2, -3)$ are collinear points.
3. x -intercept of a line is double of y -intercept. Find equation of line if it passes through $(2, 1)$.
4. Reduce $5x - 2y + 1 = 0$ into slope intercept form and two intercept form.
5. (i) Reduce $x - 3y = 3$ into intercept form and find x - and y -intercepts.
 (ii) Find its slope and transform the equation into point slope form.
6. Normal form of an equation of line is $x \cos 150^\circ + y \sin 150^\circ = 10$. Transform the equation into slope-intercept form and find its slope and y -intercept.
7. (i) Find vertices of triangle ABC .
 (ii) Calculate slopes of its sides.
 (iii) Find interior angles of triangle.



8. Two points $P(4, -1)$ and $Q(8, 3)$ lie on a line. Find:
- coordinates of mid-point M of PQ .
 - slope of PM .
 - the equation of line parallel to PQ through $(-2, 2)$.
 - the equation of line perpendicular to PQ through $(-2, 2)$.
9. Two points $A(2, -2)$ and $B(4, 6)$ lie on a line. Find:
- length of AB .
 - slope of BA .
 - values of a and b when the line AB is $ax + by - 10 = 0$.
 - the equation of line parallel to AB passing through $(0, 3)$.
10. Find equation of line passing through mid-point of $(4, 4)$ and $(8, 0)$ parallel to the line having slope $\frac{2}{5}$.

11. Lines OC and AB are shown in the graph. Find
- coordinates of end points of OC and AB .
 - slopes of both lines.
 - equations of both lines.
 - coordinates of point of intersection of both lines.



12. Locate two points on the coordinate plane that satisfy the equation $x - 2y = 2$. Find:
- the slope of segment l connecting two points.
 - slope of segment p perpendicular to l .
 - mid-point of the segment l .
 - equation of line passing through mid-point of segment l and slope of p .