

UNIT

8

PRACTICAL GEOMETRY

- ▶ **Construction of a Triangle**
- ▶ **Construction of a Quadrilateral**
- ▶ **Tangent to a Circle**

After completion of this unit, the students will be able to:

- ▶ construct a triangle having:
 - Two sides and the included angle.
 - One side and two of the angles,
 - Two of its sides and the angle opposite to one of them (with all the three possibilities).
- ▶ draw:
 - Angle bisectors.
 - Altitudes.
 - Medians, of a given triangle and verify their concurrency.
- ▶ construct a rectangle when.
 - Two sides are given.
 - Diagonal and one side are given.
- ▶ construct a square when its diagonal is given.
- ▶ construct a parallelogram when two adjacent sides and the angle included between them is given.
- ▶ locate the centre of given circle.
- ▶ draw a circle passing through three given non-collinear points.
- ▶ draw a tangent to a given circle from a point P when P lies.
 - On the circumference,
 - Outside the circle.
- ▶ draw:
 - Direct common tangent or external tangent.
 - Transverse common tangent or internal tangent to two equal circles.
- ▶ draw a tangent to.
 - Two unequal touching circles.
 - Two unequal intersecting circles.

8.1 CONSTRUCTION OF A TRIANGLE

When we are asked to construct a figure, we must use only the tools of geometry, namely, a ruler and a pair of compasses.

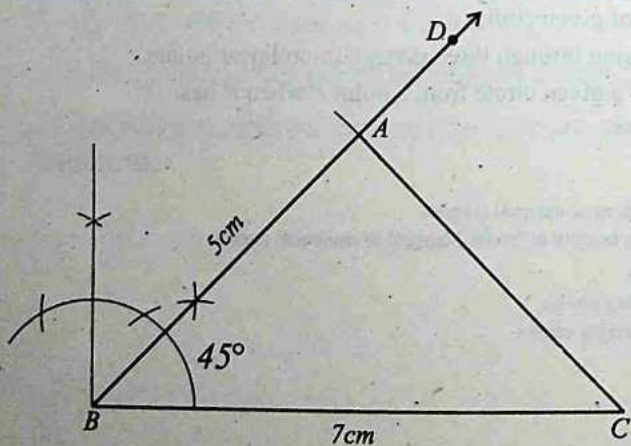
8.1.1 Construction

- Construct a triangle, when two sides and the included angle, are given.

Let the given two sides are of measure 7cm and 5cm and the included angle between them is of measure 45° .

Steps of Construction:-

- Draw a line segment $\overline{BC} = 7\text{cm}$
- At point B , draw $m\angle DBC = 45^\circ$ using compasses.
- With B as centre draw an arc of radius 5cm to cut $\overline{BD}$ at A .
- Join A to C .
- $\triangle ABC$ is the required triangle.

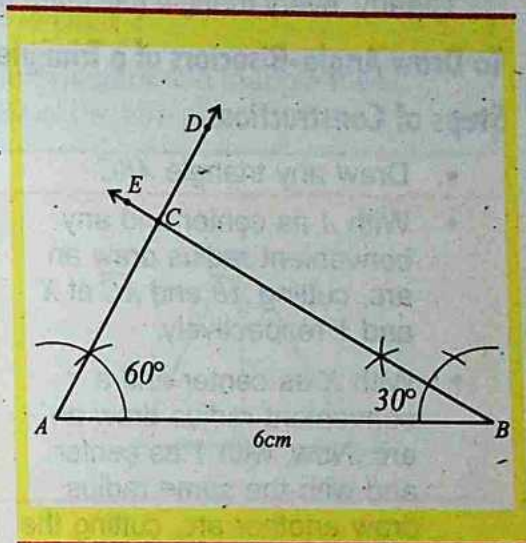


- **Construct a triangle, when two angles and their included side are given.**

Let the given two angles are $m\angle A = 60^\circ$ and $m\angle B = 30^\circ$ and the included side $m\overline{AB} = 6\text{cm}$

Steps of Construction:-

- Draw a line segment $AB = 6\text{cm}$.
- At point A draw $m\angle BAD = 60^\circ$ with the help of compasses.
- At point B draw $m\angle EBA = 30^\circ$ with the help of compasses.
- $\overrightarrow{AD}$ and $\overrightarrow{BE}$ intersect at C .
- $\triangle ABC$ is the required triangle.



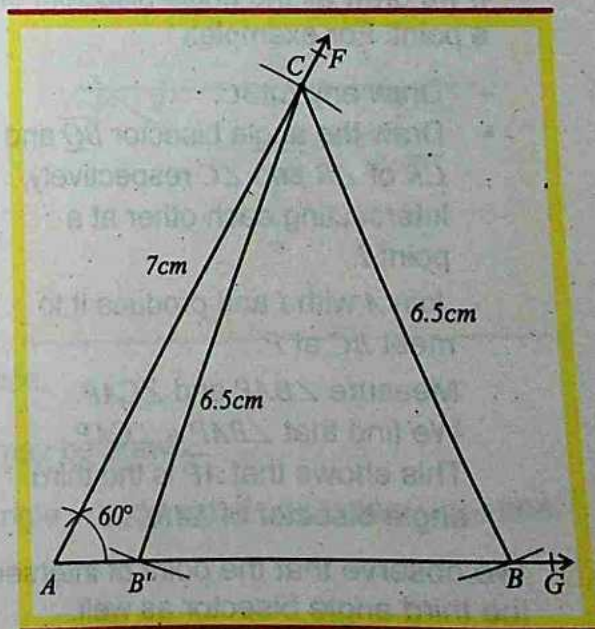
- **Construct a triangle when two sides and the angle opposite to one of them are given.**

Let $m\angle A = 60^\circ$, $m\overline{AC} = 7\text{cm}$, $m\overline{BC} = 6.5\text{cm}$

Steps of Construction:-

- On any line AG construct $\angle GAF = 60$ with the help of compasses.
- Draw $\overline{AC} = 7\text{cm}$
- With C as center draw an arc of radius 6.5cm cutting line AG in B and B' .
- Draw $\overline{CB}$ and $\overline{CB'}$.

$\triangle CAB$ and $\triangle CAB'$ are the two required triangles.



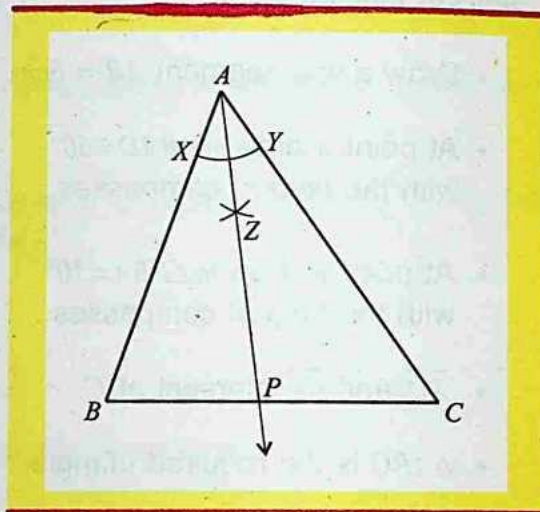
8.1.2 Angle Bisectors of a Triangle

An angle-bisector of a triangle is a line segment that bisects an angle of the triangle and has its other end on the side opposite to that angle. Clearly, every triangle has three angle bisectors, one for each angle.

To Draw Angle-Bisectors of a Triangle

Steps of Construction:-

- Draw any triangle ABC .
- With A as center and any convenient radius draw an arc, cutting $\overline{AB}$ and $\overline{AC}$ at X and Y respectively.
- With X as center and a convenient radius draw an arc. Now, with Y as center and with the same radius draw another arc, cutting the previously drawn arc at Z .
- Join AZ and produce it to meet $\overline{BC}$ at P . Then $\overline{AP}$ is the required angle bisector of $\angle A$.



Similarly the other angle bisectors may be drawn.

If we draw all the angle bisectors of a triangle, we find that they meet at a point. For example:

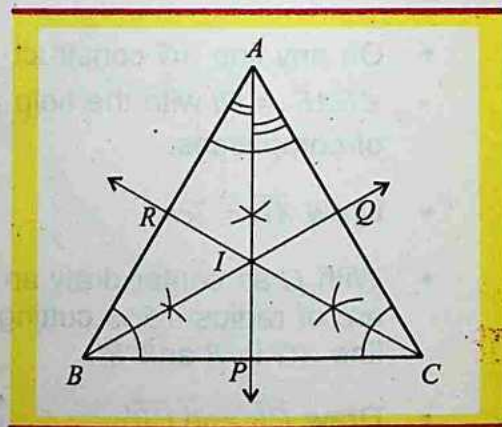
- Draw any $\triangle ABC$.
- Draw the angle bisector $\overline{BQ}$ and $\overline{CR}$ of $\angle B$ and $\angle C$ respectively, intersecting each other at a point I .

Join A with I and produce it to meet $\overline{BC}$ at P .

Measure $\angle BAP$ and $\angle CAP$.

We find that $\angle BAP = \angle CAP$.

This shows that $\overline{AP}$ is the third angle bisector of $\triangle ABC$.



We observe that the point of intersection of two angle bisectors lies on the third angle bisector as well.

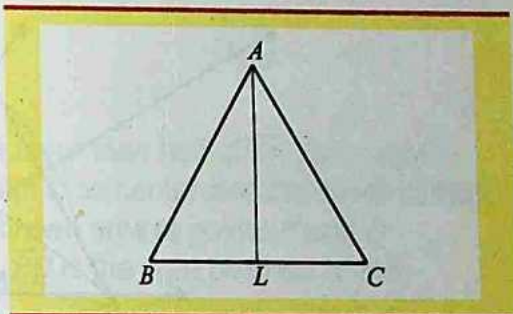
The angle bisectors of a triangle are concurrent, that is they meet at a point.

What we need to know ?

The point at which the three angle-bisectors of a triangle meet, is called the **incenter** of the triangle.

Altitudes of a Triangle:-

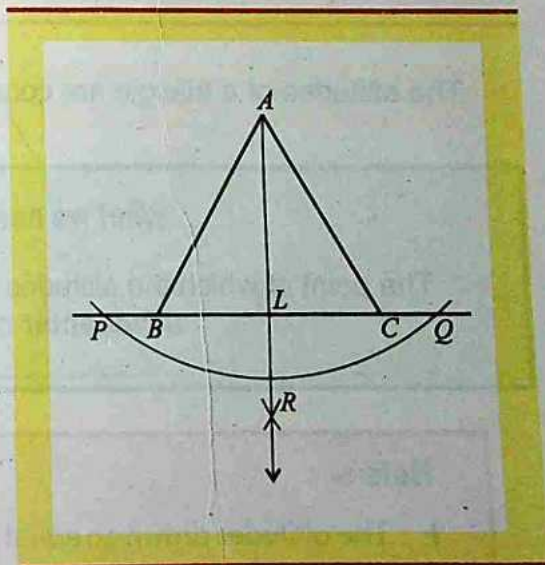
An altitude of a triangle is the line segment from a vertex of the triangle, perpendicular to the opposite side. Clearly, every triangle has three altitudes, one from each vertex.



Draw the altitudes of a triangle:-

Steps of Construction:-

- Draw any triangle ABC .
- With A as center and suitable radius, draw an arc cutting $\overline{BC}$ (or BC produced) at two points P and Q .
- With P as center and radius greater than half of $\overline{PQ}$ draw an arc. Now, with Q as center and the same radius, draw another arc, cutting the previously drawn arc at R .
- Join A with R , cutting $\overline{BC}$ at L . Then, $\overline{AL}$ is the required altitude.
- Similarly, the other altitudes may be drawn.



All the three altitudes of a triangle, (produced, if necessary) intersect at a point.

For example:

- Draw any triangle ABC .

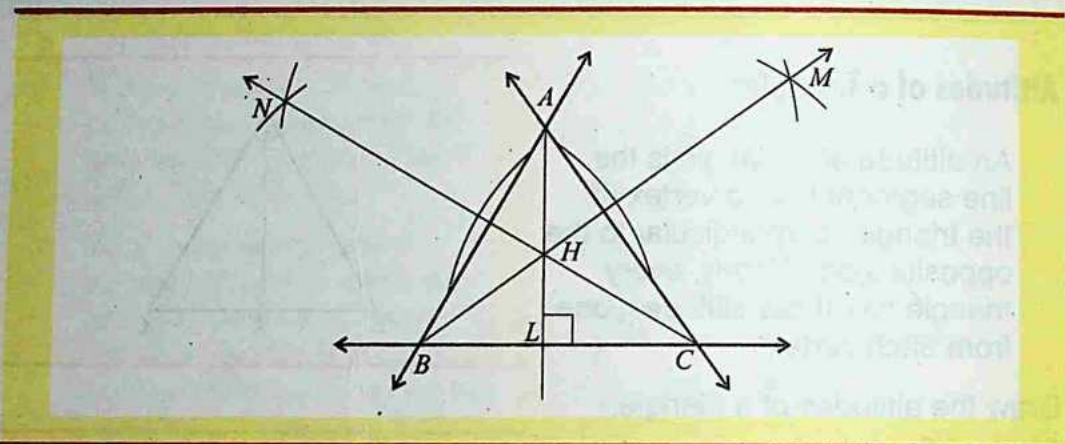
From B and C , draw the altitudes $\overline{BM}$ and $\overline{CN}$ respectively.

Let $\overline{BM}$ and $\overline{CN}$ meet in H (produced, if necessary).

Join A with H and produce it, if necessary, to meet $\overline{BC}$ in L .

Measure $\angle ALC$.

We find that $m\angle ALC = 90^\circ$ and, therefore, AL is also an altitude of $\triangle ABC$.



The altitudes of a triangle are concurrent i.e. they meet in one point.

What we need to know ?

The point at which the altitudes of a triangle meet, is called the **orthocenter** of the triangle.

Note :-

- ▶ The altitudes drawn on equal sides of an isosceles triangle are equal.
- ▶ The altitude bisects the base of an isosceles triangle.
- ▶ The altitudes of an equilateral triangle are equal.
- ▶ The altitudes of a triangle are concurrent, that is they meet at a point.

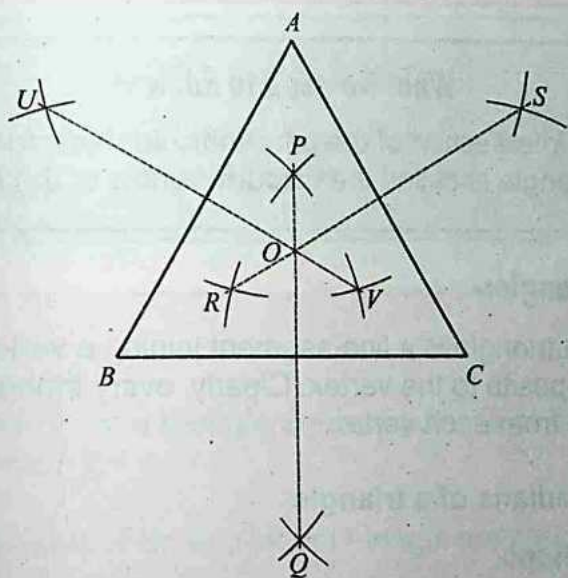
Perpendicular Bisectors of the Sides of a Triangle:-

A line segment which bisects any side of a triangle and makes a right angle with the side at its midpoint is called the perpendicular bisector or the right bisector of the side of the triangle. There are three perpendicular bisectors of a triangle, one of each side.

Construct the perpendicular bisectors of the sides of a triangle

Steps of Construction:-

- Draw any triangle ABC .
- With B as center and any radius more than half of $\overline{BC}$ draw arcs one on each side of $\overline{BC}$. Now, with C as center and the same radius draw arcs to cut the previously drawn arcs at points P and Q respectively. Join P with Q then, $\overline{PQ}$ is the right bisector of the side $\overline{BC}$.



- Also draw the perpendicular bisectors $\overline{RS}$ and $\overline{UV}$ of $\overline{AC}$ and $\overline{AB}$ respectively.
- Produce these right bisectors, if necessary, to meet at a point O .

We find that they meet at a point.
For example:

- Draw any triangle ABC .
- Draw the right bisectors $\overline{PL}$ and $\overline{RM}$ of $\overline{BC}$ and $\overline{AC}$ respectively.

Let $\overline{PL}$ and $\overline{RM}$ intersect at O .

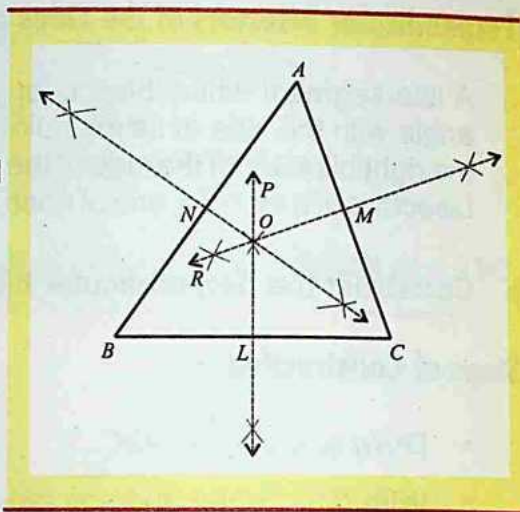
From O , draw $\overline{ON} \perp \overline{AB}$,
meeting $\overline{AB}$ at N .

Measure $\overline{AN}$ and $\overline{NB}$.

We find that $\overline{AN} = \overline{NB}$.

Thus, $\overline{ON}$ is the perpendicular
bisector of $\overline{AB}$. Thus, the point O

is common to the three perpendicular bisectors of the sides of $\triangle ABC$.



The perpendicular bisectors of the sides of a triangle are concurrent, that is, they meet at a point.

What we need to know ?

The point of intersection of the perpendicular bisectors of the sides of a triangle is called the **circum-center** of the triangle.

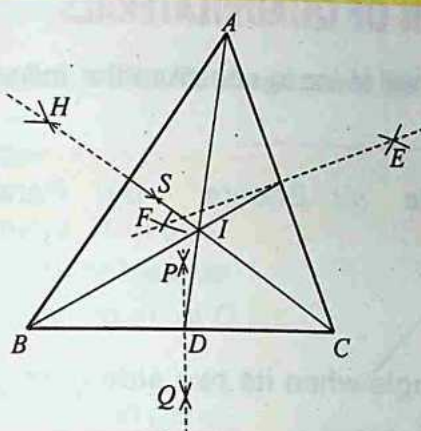
Medians Of A Triangle:-

A median of a triangle is a line-segment joining a vertex to the midpoint of the side opposite to the vertex. Clearly, every triangle has three medians, one from each vertex.

► Construct medians of a triangle

Steps of Construction:-

- Draw any triangle ABC .
- With B as center and any radius more than half of $\overline{BC}$ draw arcs one on each side of $\overline{BC}$. With C as center and the same radius draw two arcs, cutting the previous drawn arcs at points P and Q respectively.



- Join P with Q , meeting $\overline{BC}$ at D . Then, D is the midpoint of $\overline{BC}$.
- Join A with D , then, $\overline{AD}$ is the required median.
Similarly, draw the other medians from B and C .
We find that they meet at a point ' I '.

What we need to know ?

The point at which the medians of a triangle meet, is called the **centroid** of the triangle.

Note :-

- ▶ The centroid of a triangle divides each one of the medians in the ratio 2:1
- ▶ The medians of an equilateral triangle are equal.
- ▶ The medians to the equal sides of an isosceles triangle are equal.
- ▶ The medians of a triangle are concurrent.

8.2 CONSTRUCTION OF QUADRILATERALS

In this section, we will learn to construct the following types of quadrilaterals.

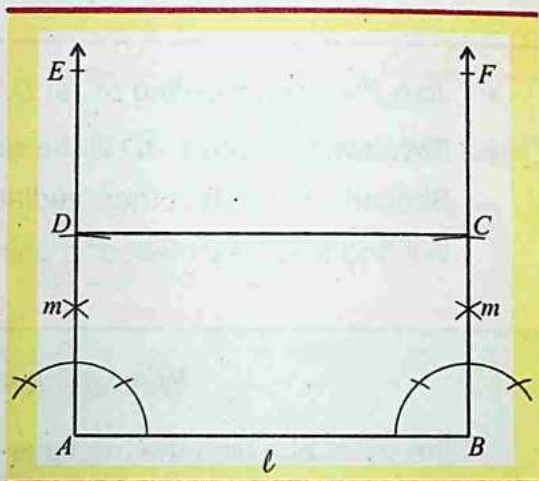
(i) Rectangle (ii) Square (iii) Parallelogram

8.2.1 Rectangle

- Construct a rectangle when its two sides are given.

Steps of Construction:-

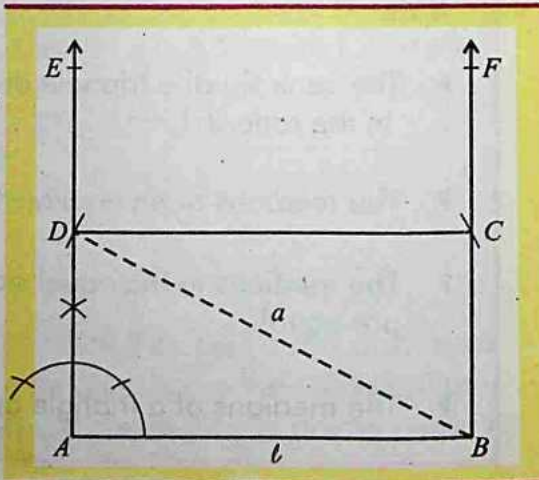
Draw a line-segment $\overline{AB} = \ell$.
 Construct $m\angle A = 90^\circ$ and $m\angle B = 90^\circ$. Taking "A" as center cut $\overline{AD} = m$ from $\overline{AE}$.
 Taking "B" as center cut $\overline{BC} = m$ from $\overline{BF}$.
 Join C with D.
 Thus ABCD is the required rectangle.



- Construct a rectangle when diagonal and one side are given.

Steps of Construction:-

Draw a line-segment $\overline{AB} = \ell$.
 Construct $m\angle A = 90^\circ$. Taking "B" as center and radius 'a' draw an arc cutting $\overline{AE}$ at D.
 With B as center and radius $\overline{AD}$, draw an arc. With D as center and radius $\overline{AB} = \ell$. Draw another arc $\overline{BF}$ cutting at C. Join C with D.
 ABCD is the required rectangle.



2.2 Square

Construct a square when its diagonal is given.

Steps of Construction:-

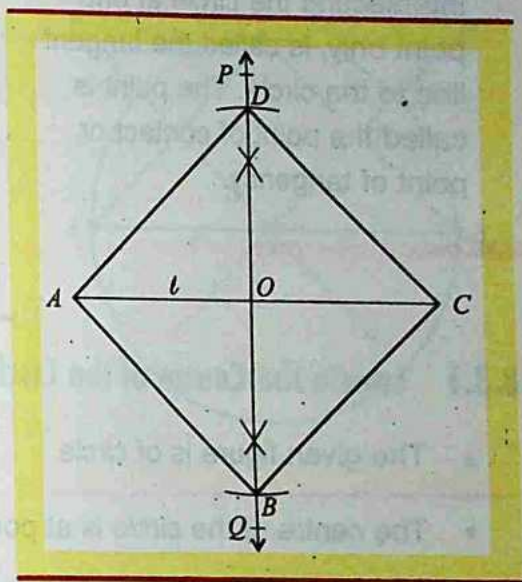
Draw a line-segment $\overline{AC} = l$.

Draw the perpendicular bisector $\overline{PQ}$ of " $\overline{AC}$ " intersecting $\overline{AC}$ at O .

From " O " cut $\overline{OD} = \frac{l}{2}$ and $\overline{OB} = \frac{l}{2}$ along $\overline{OP}$ and $\overline{OQ}$ respectively.

Join A with B ; B with C ; C with D and D with A .

$ABCD$ is a square.



8.2.3 Parallelogram

Construct a parallelogram when two adjacent sides and the angle between them is given.

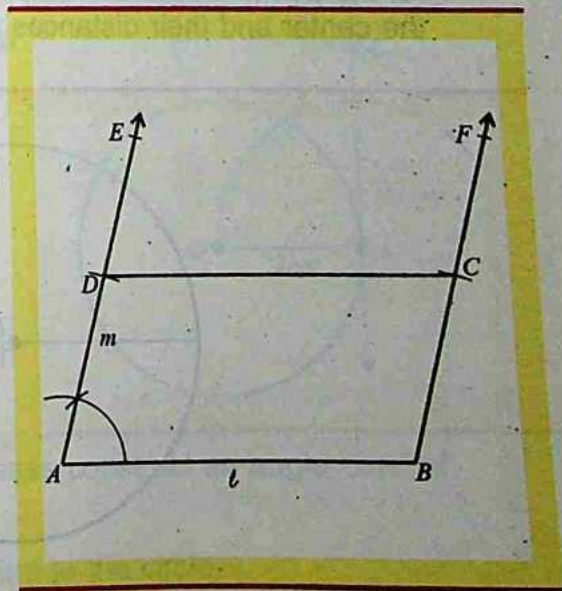
Draw a line-segment $\overline{AB} = l$.

Construct $\angle BAD = \angle A$.

Cut $\overline{AD} = m$ along AE .

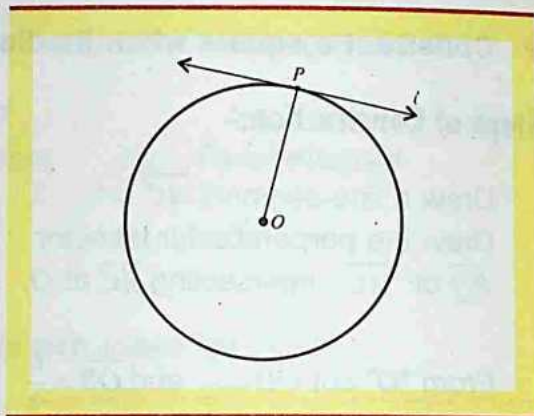
With B as center and radius " m " draw an arc cutting $\overline{BF}$ at C .

With D as center and radius " l " draw another arc cutting the previous arc at " C ". Join C with B and C with D . $ABCD$ is the required parallelogram.



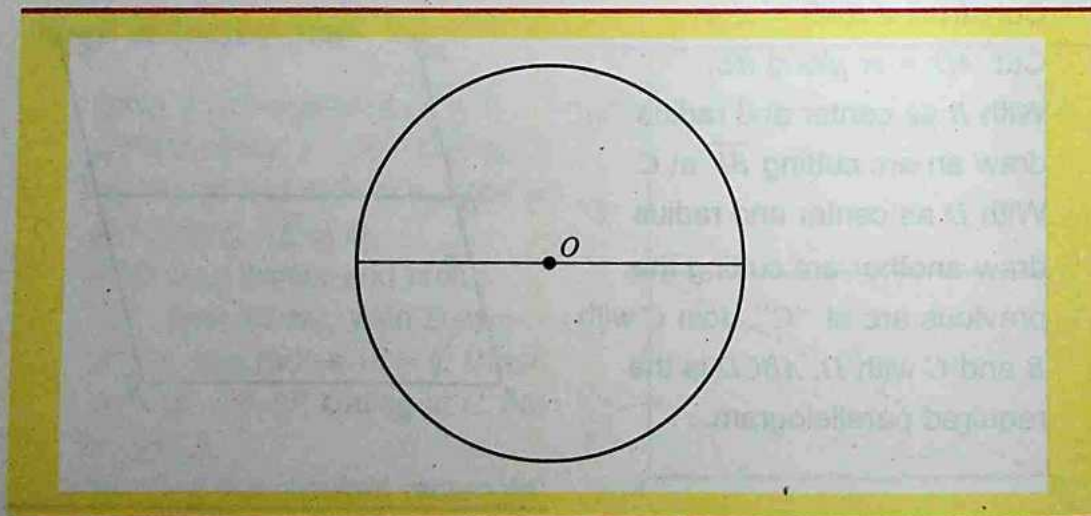
8.3 TANGENT TO THE CIRCLE

A line coplanar with a circle intersecting the circle at one point only, is called the tangent line to the circle. The point is called the point of contact or point of tangency.



8.3.1 Locate the Centre of the Circle

- The given figure is of circle.
- The centre of the circle is at point " O ".
- There is only one center of the circle.
- Center of the circle is not a point on the curve.
- Center of the circle is the mid point of the diameter.
- All the points on the curved path are at a constant distance from the center and their distances are called radii.



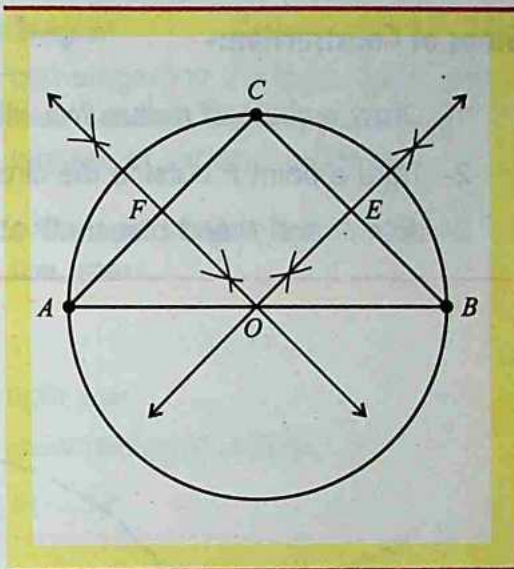
8.3.2 Draw a Circle Passing Through Three Non-Collinear Points

A, B and C are three non-collinear points. We are going to draw a circle through points A, B and C .

Steps of Construction:-

Take any three non-collinear points A, B and C .

- 1- Join A with B ; B with C and C with A , to make a triangle ABC as shown in the figure.
- 2- Draw the right bisectors of the sides AC and BC at points F and E respectively of $\triangle ABC$.
- 3- These bisectors meet at point " O ".
- 4- Taking " O " as the center and radius equal to the length $m \overline{OA} = m \overline{OB} = m \overline{OC}$, draw a circle passing through A, B and C .

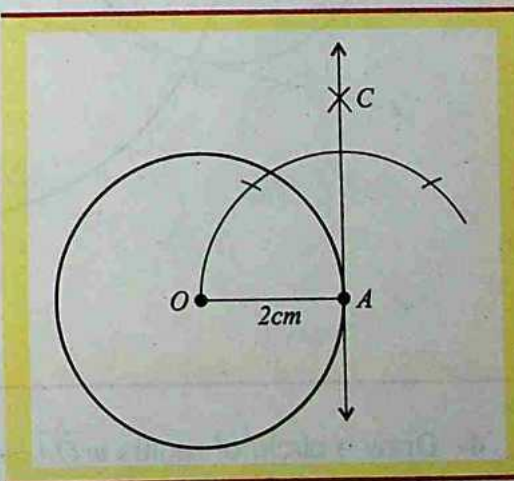


8.3.3 Tangent to a Circle

- Draw a tangent to a circle from a point on the circumference.

Steps of Construction:-

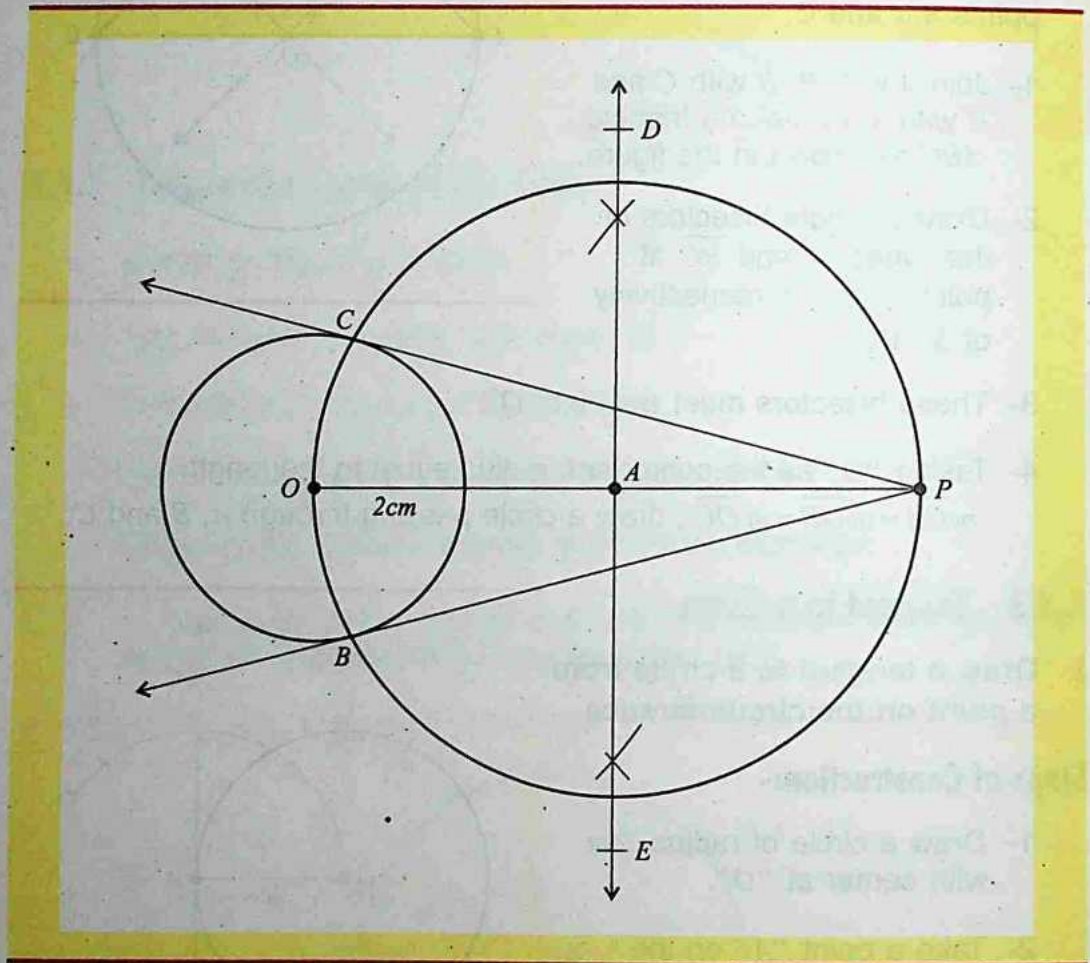
- 1- Draw a circle of radius 2cm with center at " O ".
- 2- Take a point " A " on the circumference of a circle with $m \overline{OA} = 2\text{cm}$.
- 3- With the help of the compasses construct an angle OAC of measure 90° at point A .
- 4- $\overrightarrow{AC}$ is the required tangent line to the circle.



- Draw a tangent to a circle from a point outside the circle.

Steps of Construction:-

- 1- Draw a circle of radius 2cm with center at " O ".
- 2- Take a point P outside the circle.
- 3- Join O and P and bisect OP at A .



- 4- Draw a circle of radius $m\overline{OA} = m\overline{AP}$ with center at " A ", intersecting the given circle at points B and C .
- 5- Join P with B and produce it.
- 6- $\overline{PB}$ and $\overline{PC}$ are the tangents from point P to the given circle.

8.3.4 Drawing Tangent to Two Equal Circles

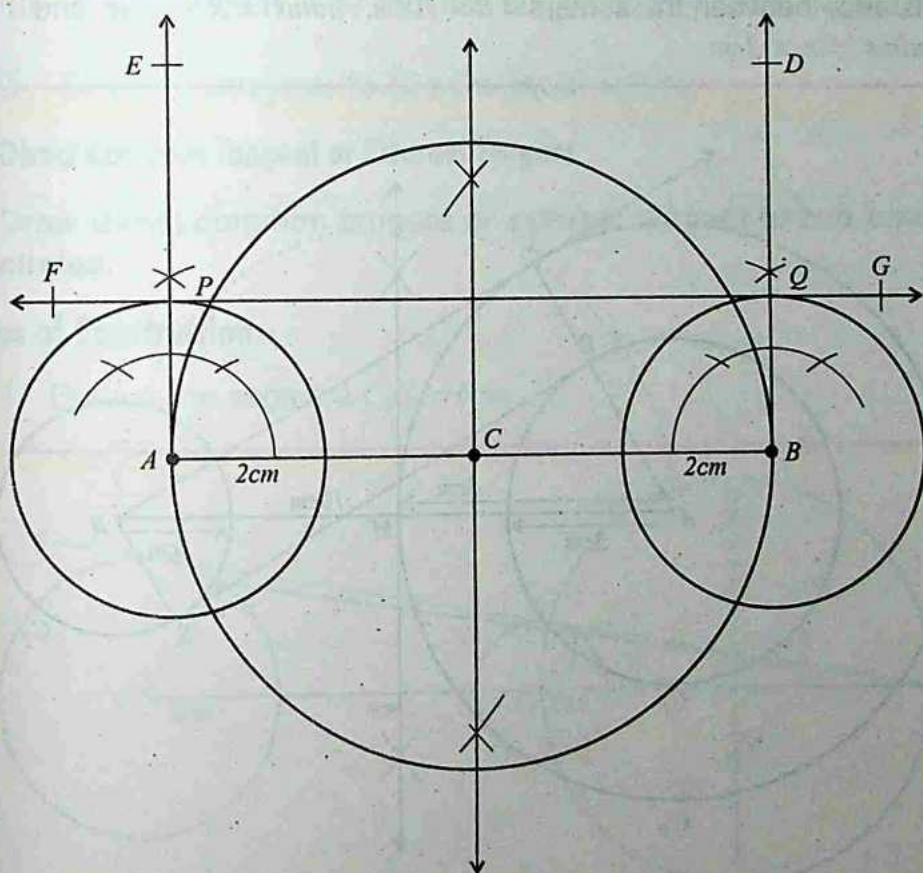
Direct Common Tangent or External Tangent

If the points of contact of a common tangent to the two circles are on the same side of the line joining their centers, then this common tangent is called direct common tangent or external tangent.

- Draw direct common tangent to the two circles having same radii 2cm having their centers 5cm apart.

Steps of Construction:-

- 1- Draw a line-segment AB of length 5cm .
- 2- With A and B as two centers draw circles of radius 2cm .
- 3- Draw $m\angle BAE = 90^\circ$ and $m\angle ABD = 90^\circ$.



- 1- Draw $\overline{AB} = 10\text{cm}$
- 2- Draw circle C_1 of radius 3cm with "A" as center.
- 3- Draw circle C_2 of radius 2cm with "B" as center.
- 4- Draw circle C_3 of radius 5cm with "A" as center.
- 5- Taking M as a mid point of $\overline{AB}$ draw a semicircle.
- 6- Draw tangent $\overrightarrow{BX}$ to the circle C_3 from point B .
- 7- Join A to X (AX intersects circle C_1 at point P).
- 8- From point B , draw $BS \parallel AP$ (by using set square).
- 9- BS intersects circle C_2 at S .
- 10- $\therefore \overrightarrow{PS}$ is a transverse common tangent to the circles C_1 and C_2 .

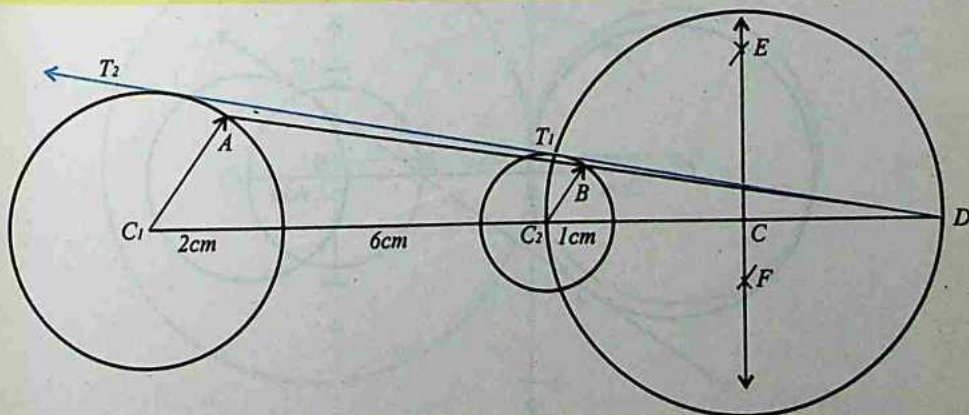
8.3.5 Drawing Tangents to Two Un-Equal Circles

Direct Common Tangent or External Tangent

- Draw direct common tangent or external tangent to two un-equal circles.

Steps of Construction:-

- 1- Draw a line segment $C_1C_2 = 6\text{cm}$.



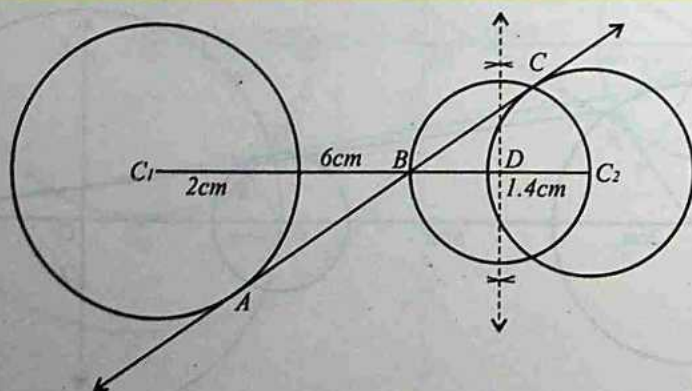
- 2- With centers at C_1 and C_2 draw circle of radius 2cm and 1cm respectively.
- 3- Extend the segment C_1C_2 to the right side.
- 4- From points C_1 and C_2 draw two parallel lines $\overline{C_1A}$ and $\overline{C_2B}$ such that $\angle C_2C_1A$ is an acute angle.
- 5- Join the points A and B extend it to D .
- 6- Draw a bisector of $\overline{C_1D}$ through C_2 .
- 7- Taking C_2 as center and $\overline{CC_2} = \overline{CD}$ radius, draw a circle intersecting the circle with center C_2 at T_1 .
- 8- Draw a line joining the points D and T_1 and touching the circle with center C_1 at T_2 .
- 9- The line $\overline{T_1T_2}$ is the direct common tangent to the given circles.

Transverse Common Tangent or Internal Tangent

- Draw common tangent or internal tangent to two unequal circles.

Steps of Construction:-

- 1- Draw a line-segment 6cm long with C_1 and C_2 as its end points ($m \overline{C_1C_2} = 6\text{cm}$).



- 2- Taking C_1 as center draw a circle of radius 2cm .
- 3- Taking C_2 as center draw a circle of radius 1.4cm .
- 4- Divide $\overline{C_1C_2}$ in the ratio $1.4:2$ (ratio of radius of the given circles) at point B .
- 5- Bisect the line-segment BC_2 at point D .
- 6- Taking D as center and $m\overline{BD} = m\overline{DC_2} = \text{radius}$, draw a circle intersecting the circle with center at C_2 at point C .
- 7- Draw a line through C and B and touching the second circle at A .
- 8- $\overline{AC}$ is the transverse tangent to the given circles.

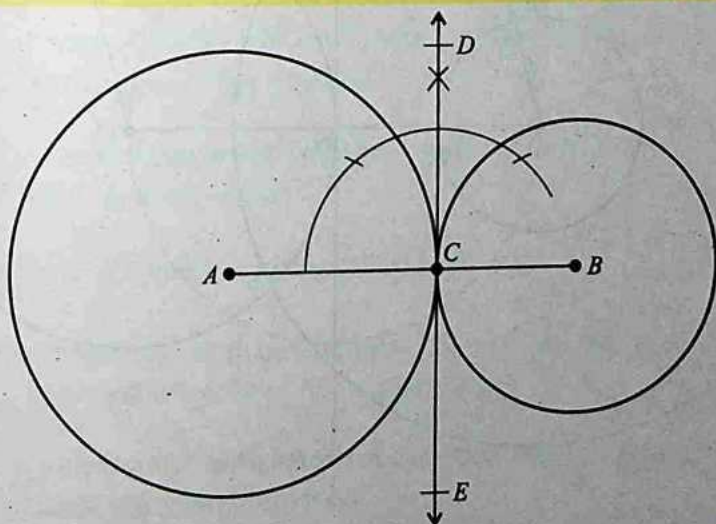
8.3.6 Drawing Tangents

Tangent to Two Unequal Touching Circles

- Draw a tangent to two unequal touching circles.

Steps of Construction:-

- 1- Draw two circles of radius 3cm and 2cm touching each other at point C .



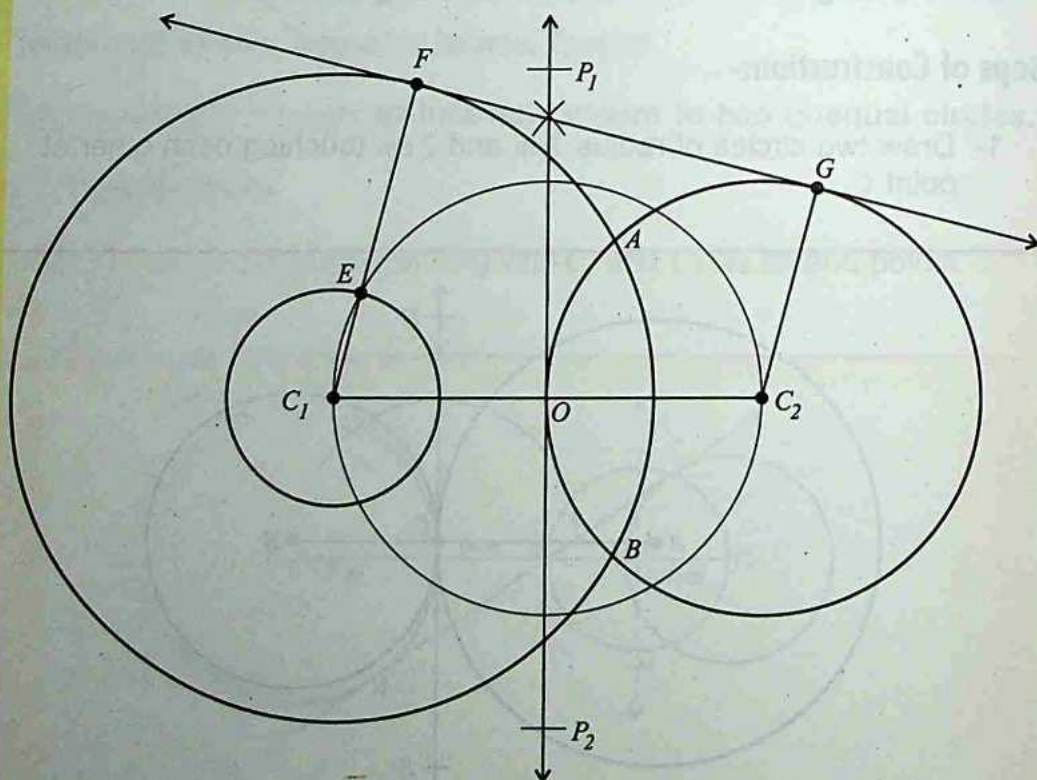
- 2- Draw $m \angle ACD = 90^\circ$ at point C .
- 3- Draw $\overline{DE}$ through C , which is perpendicular to AB .
- 4- $\overline{DE}$ is the required common tangent to the given two unequal touching circles.

Tangent to Two Unequal Intersecting Circles

- Draw a tangent to two unequal intersecting circles.

Construction:-

- 1- Draw a line-segment C_1C_2 of length 4cm .
- 2- Taking C_1 and C_2 as centers, draw two circles of radius 3cm and 2cm intersecting at points A and B respectively.
- 3- Taking C_1 as center draw a circle of radius $3\text{cm} - 2\text{cm} = 1\text{cm}$.



Bisect the line-segment C_1C_2 at O .

Taking O as center and $m\overline{C_1O} = m\overline{C_2O} = \text{radius}$, draw a circle intersecting the inner circle at point E .

Join the point C_1 to E and extend it to intersect the concentric circle at F .

Draw a line from C_2 , parallel to $\overline{C_1F}$ intersecting the circle with center C_2 at point G .

Draw a line joining the points F and G .

The line $\overline{FG}$ is the direct common tangent to two unequal intersecting circles.

EXERCISE - 8.1

Draw a triangle ABC in which $m\overline{BC} = 5.4\text{cm}$, $m\overline{AB} = 4.3\text{cm}$ and $m\overline{AC} = 3.9\text{cm}$. Find the in center.

Construct a ABC in which $m\overline{BC} = 4.6\text{cm}$, $\angle B = 110^\circ$ and $m\overline{AB} = 5\text{cm}$. Draw the perpendicular bisectors of its sides.

Draw an equilateral ΔABC in which $m\overline{AB} = m\overline{BC} = m\overline{AC} = 5\text{cm}$. Draw its altitudes and measure their lengths are they equal?

Construct a ΔABC in which $m\overline{BC} = 5.4\text{cm}$, $m\angle B = 65^\circ$ and $m\angle C = 55^\circ$. Find the centroid of the triangle.

Draw an equilateral triangle each of whose sides is 5.3 cm . Draw its medians. Are they equal?

Draw an equilateral triangle with length of each side 6 cm .

Construct a triangle ABC with base length 5cm and the angles at both ends of the base are 45° and 60° respectively.

Draw an isosceles triangle with length of the equal sides 5cm and the angle included between them is 60° .

- 9- Construct a rectangle whose adjacent sides are 4cm and 3cm .
- 10- Construct a rectangle whose one side is 6cm and an adjacent diagonal of 9cm .
- 11- Construct a square whose one side is 5cm .
- 12- Construct a square whose one side is 3.5cm .
- 13- Construct a rectangle whose two adjacent sides measure 5cm and 4cm and their included angle is 90° .
- 14- Draw a rectangle whose one side is 8cm and the length of each diagonal is 10cm .
- 15- Draw a rectangle $ABCD$ in which $m \overline{AB} = 6.5\text{cm}$ and $m \overline{AD} = 4.8\text{cm}$ and $m \angle BAD = 90^\circ$. Measure its diagonals.
- 16- Name the following quadrilaterals when:
 - (i) The diagonals are equal and the adjacent sides are unequal.
 - (ii) The diagonals are equal and the adjacent sides are equal.
 - (iii) All the sides are equal and one angle is 90° .
 - (iv) All the angles are equal and the adjacent sides are unequal.
- 17- Construct a rectangle with sides 10cm and 6cm .
- 18- Construct a square with side of length 6cm .
- 19- Name the following triangles.
 - (i) With all the three sides equal in length.
 - (ii) With two sides equal in length.
 - (iii) None of the sides is equal to the other.

- 20- Draw a circle with center O and radius 5cm . Explain the steps necessary to draw a segment of the circle.
- 21- Draw a circle with center O and any radius. Draw the diameter AB and shade one semicircular region.
- 22- Show four angles in a semi-circular region of question 21.
- 23- Draw a circle of radius 2cm with center O . Draw a chord and shade the portion showing major arc.
- 24- Draw a circle of radius 2.5cm with center at O . Draw a chord and shade the portion showing the minor arc of the circle.
- 25- Draw a semi-circle with diameter 4cm and center at O .
- 26- Draw a circle passing through the vertices of a square of side 3cm .
- 27- In a right triangle ABC , $\overline{AB} = 3\text{cm}$ and $\overline{BC} = 4\text{cm}$ with right angle at B . Draw a circle through A, B and C .
- 28- Draw a circle passing through the three vertices of an equilateral triangle with length of each side 4cm .

Review Exercise-8

I- Encircle the Correct Answer.

- The number of medians in a triangle is:
(a) 1 (b) 2
(c) 3 (d) 4
- The number of altitudes in a triangle is:
(a) 1 (b) 2
(c) 3 (d) 4
- The number of angle bisectors in a triangle is:
(a) 1 (b) 2
(c) 3 (d) 4
- The number of perpendicular bisectors of the side of a triangle is:
(a) 1 (b) 2
(c) 3 (d) 4
- The angle bisectors of a triangle are:
(a) concurrent (b) collinear
(c) perpendicular (d) non-concurrent
- The medians of a triangle are:
(a) concurrent (b) collinear
(c) non-concurrent (d) 4
- The altitudes of a triangle are:
(a) concurrent (b) collinear
(c) non-collinear (d) 5
- A line joining one vertex of a triangle to the mid point of its opposite sides is called:
(a) angle bisector (b) altitude
(c) median (d) side bisector

9. A line joining one vertex of a triangle and perpendicular to its opposite side is called:

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|---------------------------|--------------------------|
| (a) <i>angle bisector</i> | (b) <i>median</i> |
| (c) <i>altitude</i> | (d) <i>side bisector</i> |

10. A line coplanar with a circle and intersecting the circle at one point only is called:

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|-------------------------|------------------------|
| (a) <i>tangent line</i> | (b) <i>median</i> |
| (c) <i>altitude</i> | (d) <i>normal line</i> |

II- Fill in the blanks.

1. The altitudes of a triangle are _____
2. The medians of a triangle are _____
3. The angle bisector of a triangle are _____
4. The perpendicular bisector of the three sides of a triangle are _____
5. The line joining one vertex of a triangle and perpendicular to its opposite side is called _____ of a triangle.
6. A line joining one vertex of a triangle to the midpoint of its opposite side is called _____ of a triangle.
7. A line bisecting the angle of a triangle is called the _____
8. Every triangle has _____ altitudes.
9. Every triangle has _____ median.
10. Every triangle has _____ right bisectors.

SUMMARY

- 1- An angle bisector of a triangle is a line segment that bisects an angle of the triangle and has its other end on the side opposite to that angle.
- 2- Every triangle has three angle bisectors, one for each angle.
- 3- An altitude of a triangle is the line segment from one vertex, perpendicular to the opposite side.
- 4- Every triangle has three altitudes, one from each vertex.
- 5- A line-segment which bisects any side of a triangle and makes a right angle with the sides at its mid point is called the perpendicular bisector of the side of a triangle.
- 6- Every triangle has three perpendicular sides bisectors, one for each side.
- 7- The point at which the three angle bisectors of a triangle meet is called the incenter of the triangle.
- 8- The point at which the three altitudes of a triangle meet is called the orthocenter of the triangle.
- 9- The point of intersection of the three perpendicular bisectors of the sides of a triangle is called the circum-center of the triangle.
- 10- The point at which the three medians of a triangle meet is called the centroid of the triangle.
- 11- A line coplanar with a circle intersecting the circle at one point only is called the tangent line to the circle.