

How a
human eye
works?

Unit 12 GEOMETRICAL OPTICS

After studying this chapter you should be able to

- ✓ describe the terms used in reflection including normal, angle of incidence, angle of reflection and state laws of reflection.
- ✓ solve problems of image location by spherical mirrors by using mirror formula.
- ✓ define the terminology for the angle of incidence θ , and angle of refraction θ , and describe the passage of light through parallel-sided transparent material.
- ✓ solve problems by using the equation $\sin \theta_i / \sin \theta_r = n$ (refractive index).
- ✓ state the conditions for total internal reflection.
- ✓ describe the passage of light through a glass prism.
- ✓ describe how total internal reflection is used in light propagation through optical fibres.
- ✓ describe how light is refracted through lenses.
- ✓ define power of a lens and its unit.
- ✓ solve problems of image location by lenses using lens formula.
- ✓ define the terms resolving power and magnifying power.
- ✓ draw ray diagram of simple microscope and mention its magnifying power.
- ✓ draw ray diagram of compound microscope and mention its magnifying power.
- ✓ draw ray diagram of a telescope and mention its magnifying power
- ✓ draw ray diagrams to show the formation of images in the normal eye, a shortsighted eye and a long-sighted eye.
- ✓ describe the correction of short-sight and long-sight.

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Unit 12 Geometrical Optics

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- 12.6 Refraction through a prism
- 12.7 Lenses
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- 12.9 The human eye
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- 12.11 Compound microscope
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Key Points and Projects

Exercise

The study of light is called optics. Geometrical optics is concerned with tracing the geometrical paths of light rays. Instruments that can explore previously inaccessible domains often open new doors to understanding nature. For example, astronomy owes its progress to the invention of the telescope, and modern biology could not have been created without the microscope. In this unit we shall discuss the ideas that govern the construction of optical instruments such as these. The direction of the path in which light is traveling is represented as ray by a straight line with an arrow on it.

12.1 REFLECTION OF LIGHT

We see objects if light from it enters our eyes. Some objects such as the Sun, electric lamps and candles make their own light. We call these luminous sources. Most things that we see do not make their own light but reflect it from a luminous source. They are non-luminous objects for example this page, teaching board and the Moon.

Reflection of light is same as for other types of waves, such as reflection of water waves in ripple tank. Most objects reflect a certain portion of the light falling on them, mirrors reflect more light and in a regular manner. An ordinary mirror is made by depositing a thin layer of silver on one side of a piece of glass and protecting it with paint. The silver at the back of the glass - acts as the reflecting surface.

Suppose that a ray of light is incident on a flat, shiny surface, such as the mirror in Figure 12.1. The following terms are used in describing the reflection of light

- i. **Incident ray**: the approaching ray of light.
- ii. **Reflected ray**: the ray of light reflected from a reflecting surface.

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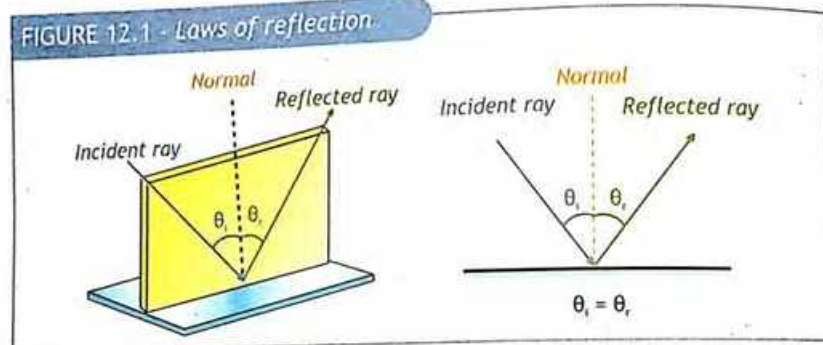
iii. **Point of incidence:** the point at which the incident ray strikes the reflecting surface.

iv. **Normal:** the line drawn at right angles to the reflecting surface at the point of incidence.

v. **Angle of incidence (θ_i):** the angle between the incident ray and the normal.

vi. **Angle of reflection (θ_r):** the angle between the reflected ray and the normal.

FIGURE 12.1 - Laws of reflection



12.1.1 Laws of Reflection: The laws of reflection describes the behavior of the incident and reflected rays.

- **First Law:** The incident ray, the reflected ray and the normal to the surface all lie in the same plane
- **Second Law:** the angle of reflection θ_r equals the angle of incidence θ_i :

$$\theta_i = \theta_r$$

12.1

12.1.2 Image characteristics: We can completely describe any image by defining four characteristics.

- The **magnification** is the ratio of the image size to the object size. If the magnification is greater than one, the image is larger than the object. When it is equal to one, the object and image are the same size. If the magnification is less than one, the image is smaller than the object.
- The **attitude** of an image shows whether the image is oriented the same way as the object (upright) or upside down (inverted) with respect to the object.
- The **image location (or position)** is the distance between the image and the optical device – mirror or lens.

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- The **type of image**, indicates whether the image is real or virtual. An image is real if light rays are actually converging at a point then continuing on beyond that point and diverging. In other words, if you place a screen at the image position, the image would appear on the screen (and would be perfectly in focus). If an image is not real, it is virtual. If you place a screen at the position of a virtual image, nothing would appear on the screen. There are no light rays actually converging on the image position. Light rays only appear as though they are diverging from the image location. This will become more clear as you recall and practice drawing ray diagrams you learned in grade 6, grade 7 and grade 8.

12.2 SPHERICAL MIRRORS

Reflecting surfaces can also be curved, usually spherical, which means they form a section of a sphere. As Figure 12.2 shows, a spherical mirror has the shape of a section sliced from the surface of a sphere. If the inside surface of the mirror is polished, it is a concave mirror. If the outside surface is polished, it is a convex mirror.

FIGURE 12.2 - Spherical mirror

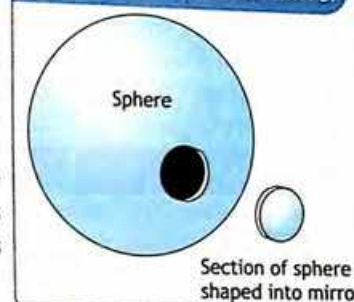
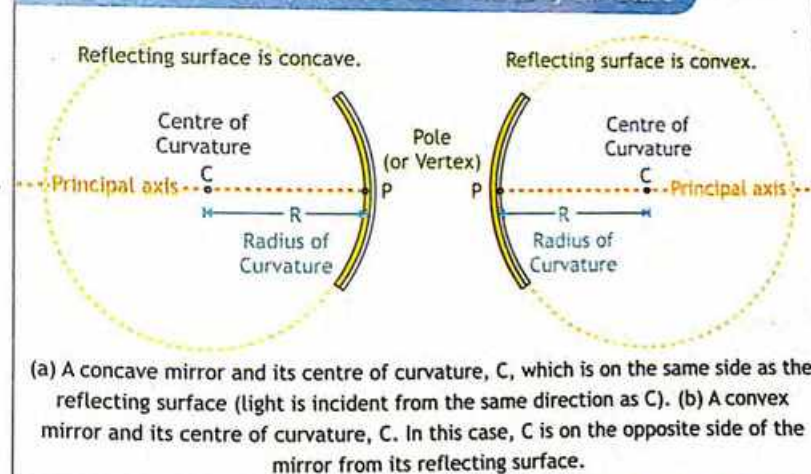


FIGURE 12.3 - Centre of curvature and radius of curvature



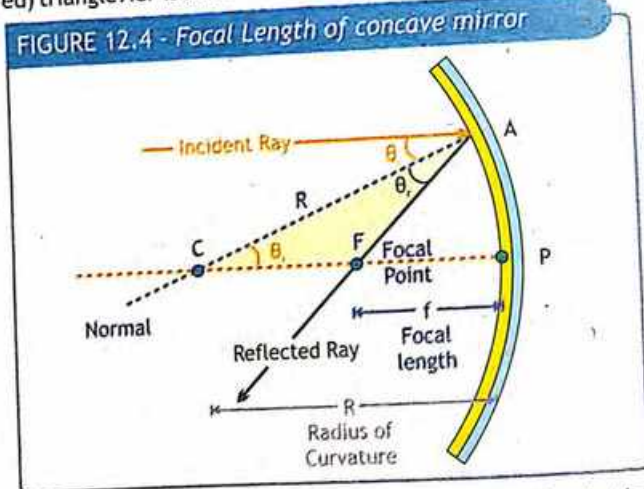
(a) A concave mirror and its centre of curvature, C, which is on the same side as the reflecting surface (light is incident from the same direction as C). (b) A convex mirror and its centre of curvature, C. In this case, C is on the opposite side of the mirror from its reflecting surface.

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Centre of curvature is the centre of the sphere with radius R (radius of curvature) of which the mirror is a section, and the principal axis is a straight line drawn through the centre of curvature and the midpoint of the mirror (pole or vertex) P . Note that the principal axis intersects the mirror at right angles. Figures 12.3 shows principal axis, centre of curvature and radius of curvature for both concave and convex mirrors.

12.2.1. Focal Point and focal length of concave mirror: Consider what happens when light rays that are parallel to the principal axis, strike a concave mirror. Follow the path of the incident ray in Figure 12.4. The light incident ray strikes the mirror at point A . The line segment CA is the radius of the mirror and, therefore, is normal to the spherical surface of the mirror at A . The Laws of Reflection tell us that the light ray reflects from the mirror such that the angle of reflection θ_r equals the angle of incidence θ_i . Furthermore, the angle ACF is also θ_i , because the radial line segment CP crosses two parallel lines. Since two of its angles are equal, the (coloured) triangle ACF is an isosceles triangle; thus, sides CF and FA are equal.

FIGURE 12.4 - Focal Length of concave mirror



When the incoming parallel light ray lies close to the principal axis, the angle of incidence, θ_i , is small, and the distance FA becomes similar in length to distance FP . Because θ_i is small, $CF = AF = FP$. Therefore, $FP = \frac{1}{2} CP$ and so point F lies halfway between the centre of curvature and the pole P (or vertex) of the mirror. Point F is called the focal point (or principal focus). The distance from the focal point to the pole P (or vertex) is called the focal length and is symbolized, f .

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The focal length, f , is one-half of the radius of curvature, R , for the spherical mirror. Point F is called the focal point because all the incident light rays that are parallel and close to the principal axis of the mirror, reflect from the mirror and pass through that one point, such that

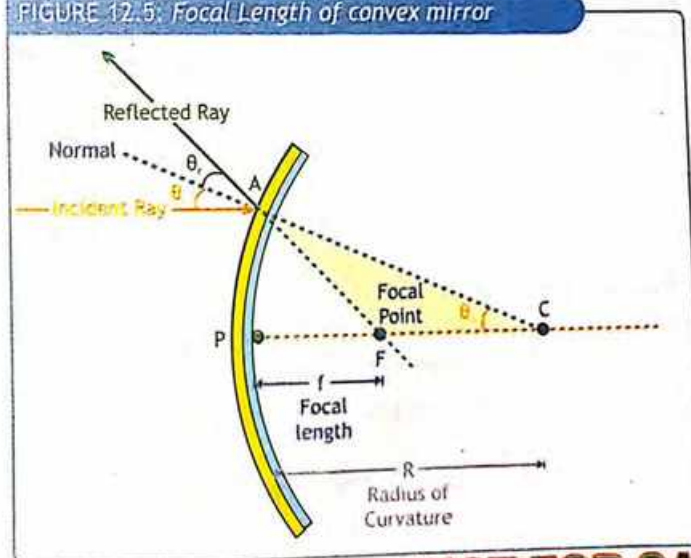
$$f = \frac{1}{2}R$$

12.2

SI unit: m

12.2.2. Focal Point and focal length of convex mirror: Figure 12.5, which shows a single ray reflecting from the convex mirror. The first thing to notice about this diagram is that a straight line drawn through the centre of curvature always intersects the mirror at right angles; hence, the line through C is the normal to the surface at the point of incidence, A . Since the incoming ray is parallel to the principal axis, it follows that the angle of incidence θ_i is equal to the angle CAF . Therefore from the laws of reflection the angle of reflection θ_r must also equal angle CAF . We see then, that $\triangle CAF$ is an isosceles triangle, with the sides CF and FA having equal length. Finally, for small angles the length CF is approximately equal to half the length $CA = R$; that is $CF = \frac{1}{2} R$. Therefore, to this same approximation, the distance FP (called focal length ' f ') is also $\frac{1}{2} R$.

FIGURE 12.5 - Focal Length of convex mirror



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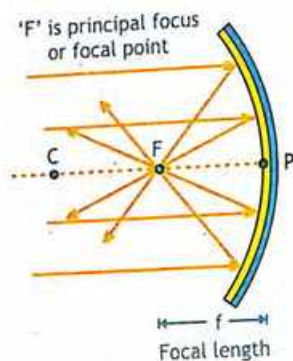
$$f = -\frac{1}{2}R$$

12.3

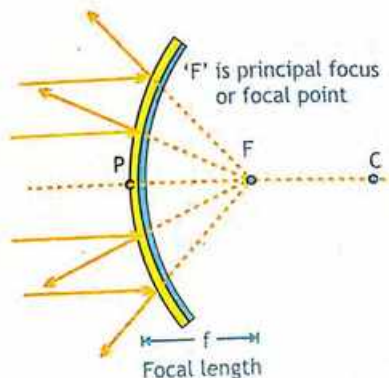
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The minus sign in this expression is used to indicate that the focal point lies behind the mirror.

FIGURE 12.6 - Principal focus



The concave mirror has the ability to converge the beam of light parallel to principal axis. Therefore it is also termed as converging mirror.



The convex mirror has the ability to diverge the beam of light parallel to principal axis. Therefore it is also termed as diverging mirror.

After reflecting from the mirror, the rays converge or diverge as if they originate from a single point called the focal point F (or principal focus) as shown in figure 12.6.

12.2.3. Image formation and ray tracing for spherical mirrors: Ray diagrams are used to show image formation in spherical mirrors.

1. Construct a figure showing the mirror and its principal axis. The figure should also show the focal point and the centre of curvature.
2. Draw the object at the appropriate point. One end of the object will often lie on the principal axis.
3. Draw four rays that arise from the tip of the object:

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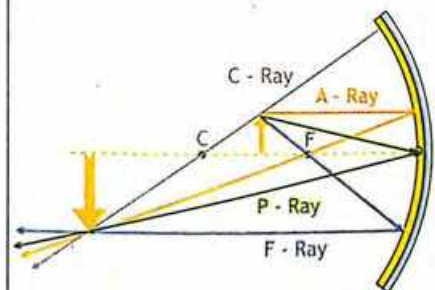
- (a) The **A-ray** (parallel ray hitting mirror at point A) is initially parallel to the axis. After reflection, this ray (or its extrapolation) passes through the focal point.
- (b) The **F-ray** (focal ray or its extrapolation) passes through the focal point. After reflection, this ray will be parallel to the axis.
- (c) The **P-ray** (pole ray or its extrapolation) is incident on the mirror at its pole or vertex, reflects making an equal angle with the axis (since the axis is normal to the mirror).
- (d) The **C-ray** (central ray or its extrapolation) passes through the centre of curvature of the mirror. After reflection, this ray passes back through the tip of the object.

These four rays are called the principal rays. Other rays can also be drawn but we prefer the principal rays because they are easier to draw. Most of the time we draw only two of the principal rays.

4. The point where the focal, parallel, pole and central rays (or their extrapolations) intersect is the image point. This point may be in front of the mirror giving a real image, or it may be necessary to extrapolate the rays back behind the mirror to locate a virtual image.

5. This ray-tracing procedure can be repeated for any desired point on the object. Thus, locating other points on the image, it is usually sufficient to consider just the tip of the image but additional points can be used if needed.

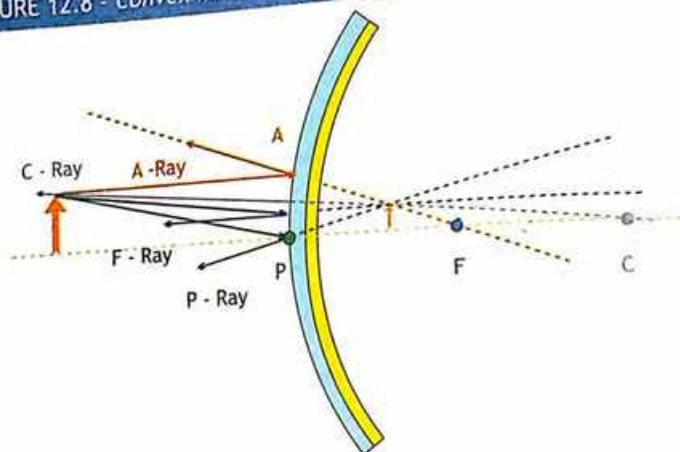
FIGURE 12.7 - Concave mirror ray tracing



1. A ray along the direction from the focal point to the mirror is reflected parallel to the principal axis.
2. F ray parallel to the principal axis is reflected through the focal point.
3. P ray incident on the vertex of the mirror reflects at an equal angle to the axis.
4. C ray along a radius is reflected back on itself.

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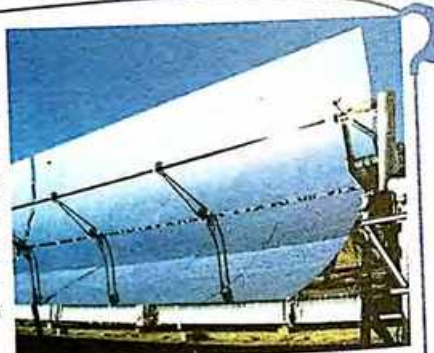
FIGURE 12.8 - Convex mirror ray tracing



1. A - ray parallel to the principal axis is reflected as if it came from a focal point.
2. F - ray directed toward the focal point is reflected parallel to the principal axis.
3. P - ray incident on the vertex of the mirror reflects at an equal angle to the axis.
4. C - ray along a radius is reflected back on itself.

TipBits

Capturing solar energy with mirrors. Parabolic mirrors are also used in one method of collecting solar energy for commercial purposes. Figure shows a long row of concave parabolic mirrors that reflect the sun's rays to the focal point. Located at the focal point and running the length of the row is an oil-filled pipe. The focused rays of the sun heat the oil.



In a solar-thermal electric plant, the heat from many such rows is used to generate steam. The steam, in turn, drives a turbine connected to an electric generator.

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How automobile headlights produce a beam of light? Another application of parabolic mirrors is in automobile headlights. Here, however, the situation is reversed from the operation of a solar collector. In a headlight, a high-intensity light source is placed at the focal point of the mirror, and light emerges parallel to the principal axis.



12.1 THE MIRROR EQUATION

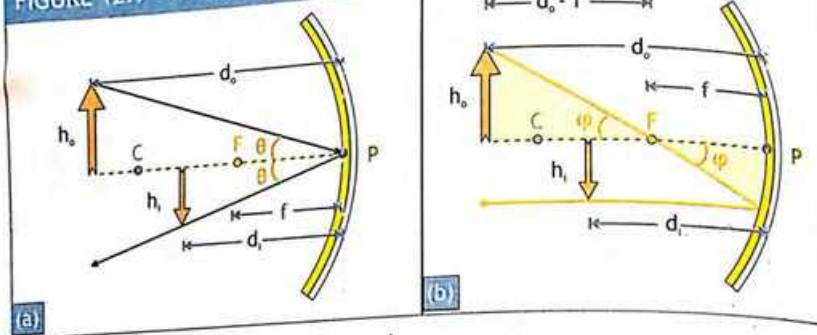
Drawing light ray diagrams are helpful in explaining and predicting the properties of images formed in plane, concave, and convex mirrors. To determine the magnification of the image, and its position with precision, it is necessary to draw very accurate scaled diagrams. The mirror equation on the other hand is a precise mathematical relationship between the object distance and the image distance for a given mirror. The following quantities are included in mirror formula:

- f = the focal length of the mirror
- h_o = the height of the object
- h_i = the height of the image
- d_o = the distance of the object from the mirror
- d_i = the distance of the image from the mirror
- M = the magnification of the image

To obtain this relation, we use the ray diagrams shown in Figure 12.9. The ray in Figure 12.9 (a) hits the mirror at its midpoint, where the principal axis is the normal to the mirror. As a result, the ray reflects at an angle θ below the principal axis that is equal to its incident angle θ above the axis. Therefore, the two green triangles in this diagram are similar, from which it follows that ratios of equivalent sides of similar triangles are equal.

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FIGURE 12.9 - Mirror Formula



$$\frac{h_o}{-h_i} = \frac{d_o}{d_i} \quad \text{--- (1)}$$

The image height is negative because it is inverted.

From the figure 12.9 (b) it is clear that the two yellow triangles in this diagram are also similar, since they are both right triangles and share the common angle ϕ . Thus, by using the same rule of similar triangles we get

$$\frac{h_o}{-h_i} = \frac{d_o - f}{f} \quad \text{--- (2)}$$

Comparing equation 1 and equation 2, we get

$$\frac{d_o}{d_i} = \frac{d_o - f}{f}$$

To isolate 'f', cross multiply

$$d_o f = d_i (d_o - f) \quad \text{or} \quad d_o f = d_i d_o - d_i f \quad \text{--- (3)}$$

Divide both sides of equation 3 by $d_o d_i f$, we get

$$\frac{d_o f}{d_i d_o f} = \frac{d_i d_o}{d_i d_o f} - \frac{d_i f}{d_i d_o f}$$

$$\text{or} \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} \quad \text{Hence}$$

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad \text{--- (12.4)}$$

The final equation 12.4 above is known as the mirror formula. It relates the distances of the object and the image to a concave mirror, in terms of the mirror's focal length. The ratio of the height of the image to the height of the object is defined as the magnification, M ;

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$$M = \frac{h_i}{h_o} = \frac{d_i}{d_o} \quad \text{--- (12.5)}$$

We derived the magnification and mirror equations for a concave mirror forming a real image, but the equations apply as well to convex mirrors and to virtual images if we use the sign conventions described below.)

12.3.1 Convention for mirror equation and magnification equation: To include all of the possible properties of both images and objects, the following sign convention has been established for both concave and convex spherical mirrors.

a. Object distance:

- d_o is positive for objects in front of the mirror (real objects)

b. Image distance:

- d_i is positive for objects in front of the mirror (real images)
- d_i is negative for objects behind the mirror (virtual images)

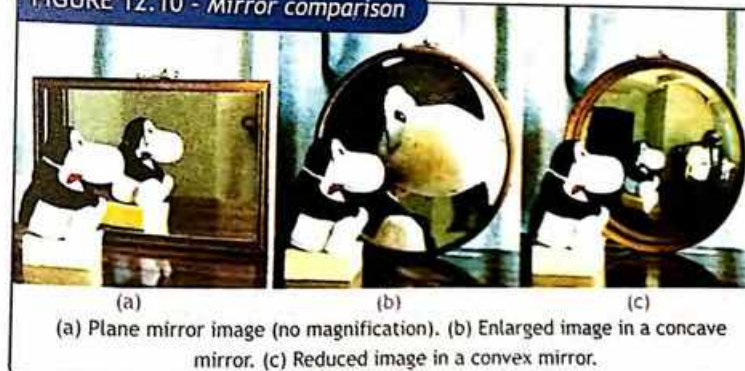
c. Image attitude:

- h_i is positive for images that are upright, compared to the object
- h_i is negative for images that are inverted, compared to the object

d. Focal length:

- f is positive for concave mirrors
- f is negative for convex mirrors

FIGURE 12.10 - Mirror comparison



(a) Plane mirror image (no magnification). (b) Enlarged image in a concave mirror. (c) Reduced image in a convex mirror.

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Table 12.1 Single-Mirror System Properties

Mirror Type	f	d_o	d_i	m
Plane	NA	$d_o > 0$	$d_i = d_o$ (negative)	Same size
Concave	+	$d_o > r$	$r > d_i > f$	Reduced, inverted
		$d_o = r$	$d_i = r$	Same size
		$r > d_o > f$	$d_i > r$	Enlarged, inverted
		$d_o = f$	$d_i = \infty$	No image
		$f > d_o > 0$	$d_i > d_o$ (negative)	Enlarged
Convex	-	$d_o > 0$	$f > d_i > 0$ (negative)	Reduced

EXAMPLE 12.1: IMAGE DISTANCE IN SPOON

The concave side of a spoon has a focal length of 5.00 cm. Find the image distance for this 'mirror' when the object distance is (a) 12.0 cm, (b) 10.0 cm, (c) 7.50 cm, (d) 5.00 cm and (e) 2.00 cm.



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Focal length ' f ' = 5.00 cm.

- (a) Object Distance ' d_o ' = 12.0 cm,
 (b) Object Distance ' d_o ' = 10.0 cm
 (c) Object Distance ' d_o ' = 7.50 cm
 (d) Object Distance ' d_o ' = 5.00 cm
 (e) Object Distance ' d_o ' = 2.00 cm

REQUIRED

- (a) Image Distance ' d_i ' = ?
 (b) Image Distance ' d_i ' = ?
 (c) Image Distance ' d_i ' = ?
 (d) Image Distance ' d_i ' = ?
 (e) Image Distance ' d_i ' = ?

SOLUTION: by mirror equation

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

Taking LCM

$$\frac{1}{d_i} = \frac{d_o - f}{fd_o} \quad \text{or} \quad d_i = \frac{fd_o}{d_o - f} \quad \text{--- (1)}$$

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(a) Putting values in equation 1

$$d_i = \frac{5.00 \text{ cm} \times 12.00 \text{ cm}}{12.00 \text{ cm} - 5.00 \text{ cm}}$$

$$\text{or } d_i = \frac{60.0 \text{ cm}^2}{7.00 \text{ cm}}$$

Therefore $d_i = 8.6 \text{ cm}$ — Answer

Hence, the image is closer to the mirror, as shown in figure 12.11 (a).

(b) Putting values in equation 1

$$d_i = \frac{5.00 \text{ cm} \times 10.00 \text{ cm}}{10.00 \text{ cm} - 5.00 \text{ cm}}$$

$$\text{or } d_i = \frac{50.0 \text{ cm}^2}{5.00 \text{ cm}}$$

Hence $d_i = 10.0 \text{ cm}$ — Answer

So the image distance is same as object distance, as shown in figure 12.11 (b).

(c) Putting values in equation 1

$$d_i = \frac{5.00 \text{ cm} \times 7.50 \text{ cm}}{7.50 \text{ cm} - 5.00 \text{ cm}}$$

$$\text{or } d_i = \frac{37.5 \text{ cm}^2}{2.50 \text{ cm}}$$

Therefore $d_i = 15.0 \text{ cm}$ — Answer

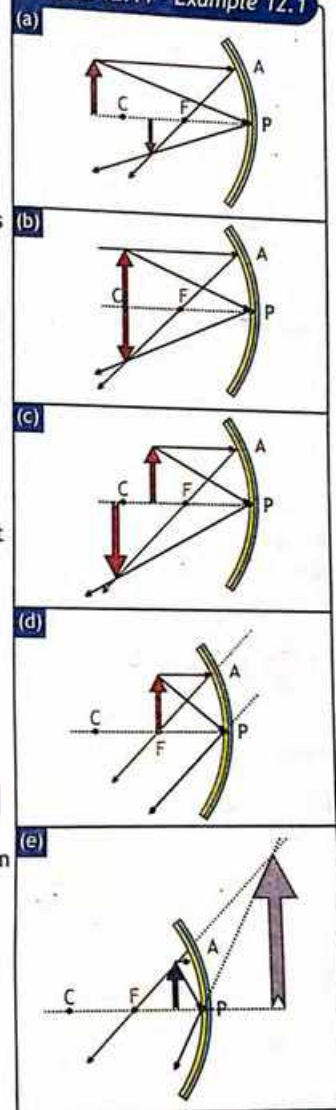
Hence, the image distance is greater than object distance, as shown in figure 12.11 (c).

(d) Putting values in equation 1

$$d_i = \frac{5.00 \text{ cm} \times 5.00 \text{ cm}}{5.00 \text{ cm} - 5.00 \text{ cm}}$$

$$\text{or } d_i = \frac{25.0 \text{ cm}^2}{0 \text{ cm}}$$

FIGURE 12.11 - Example 12.1



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Therefore $d_i = \infty$ — **Answer**

Hence, no image is formed, as shown in figure 12.11(d).

(e) Putting values in equation 1 $d_i = \frac{5.00 \text{ cm} \times 2.00 \text{ cm}}{2.00 \text{ cm} - 5.00 \text{ cm}}$ or $d_i = \frac{10.0 \text{ cm}^2}{-3.00 \text{ cm}}$

Therefore $d_i = -3.33 \text{ cm}$ — **Answer**

The negative sign of the image distance indicates that the image is formed on the other side of the mirror, as shown in figure 12.11(e).

ASSIGNMENT 12.1: DENTIST MIRROR

A dentist uses a concave mirror with focal length 2.0 cm to examine some teeth. If the tooth under examination is 1.1 cm high and mirror is placed at 0.9 cm. Calculate the distance of image formed, the height of the image and magnification.

EXTENSION EXERCISE 12.1

If the object height is 2 cm, what is the image height in each case? What is the magnification?



EXAMPLE 12.2: REAR VIEW MIRROR

An external rearview car mirror is convex with a radius of curvature of 16.0 m (Fig.). Determine (a) the focal length of the mirror, (b) location of the image and (c) its magnification for an object 10.0 m from the mirror.



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Object distance ' d_o ' = 10 m

Radius of curvature = R = 16.0 m

Solution: (a) radius of curvature and focal length for convex mirror are related by equation

$$f = -\frac{1}{2}R \quad \text{Putting values} \quad f = -\frac{1}{2}16 \text{ m}$$

Therefore $f = -8.0 \text{ m}$ — **Answer**

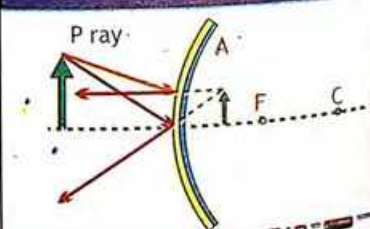
REQUIRED

(a) focal length ' f ' = ?

(b) image distance ' d_i ' = ?

(c) magnification ' M ' = ?

FIGURE 12.12 - Example 12.2



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The centre of curvature of a convex mirror is behind the mirror therefore its focal point is also negative.

(b) By mirror equation $\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$

Solving for image distance $\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$

Taking LCM $\frac{1}{d_i} = \frac{d_o - f}{fd_o}$ or $d_i = \frac{fd_o}{d_o - f}$ — ①

Putting values $d_i = \frac{-8.0 \text{ m} \times 10.0 \text{ m}}{10 \text{ m} - (-8.0 \text{ m})}$

or $d_i = -\frac{80.0 \text{ m}^2}{18.0 \text{ m}}$

Therefore $d_i = -4.4 \text{ m}$ — **Answer**

The negative sign shows that the image is behind the mirror.

(c) For convex mirrors the magnification is $M = -\frac{d_i}{d_o}$

Putting values $M = -\frac{-4.4 \text{ m}}{10 \text{ m}}$ Therefore $M = +0.44$ — **Answer**

The magnification is positive so the image is upright (same orientation as object, which is useful) and about half what it would be in a plane mirror. Objects in the convex mirror are closer than they appear because the convex mirror produces an image that is reduced in size, which makes the object look as if it is farther away. Therefore, the external rearview side mirrors on most cars carry this phrase "objects in the mirror are closer than they appear."

ASSIGNMENT 12.2: IMAGE IN A SECURITY MIRROR

A convex security mirror in a warehouse has a -0.50 m focal length. A 2.0 m tall forklift is 5.0 m from the mirror. What is the image position and image height?

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12.4 REFRACTION OF LIGHT

Although light travels in straight lines in a transparent material, such as air, if it passes into a different material, such as water, it changes direction at the boundary of the two material i.e. it is bent.

FIGURE 12.13 - Refraction

Refraction of light is the change of direction of light as it moves from one material (called medium) to another. Refraction of light is same as for other types of waves, such as refraction of water waves in ripple tank.

The following terms are used describing the refraction of light

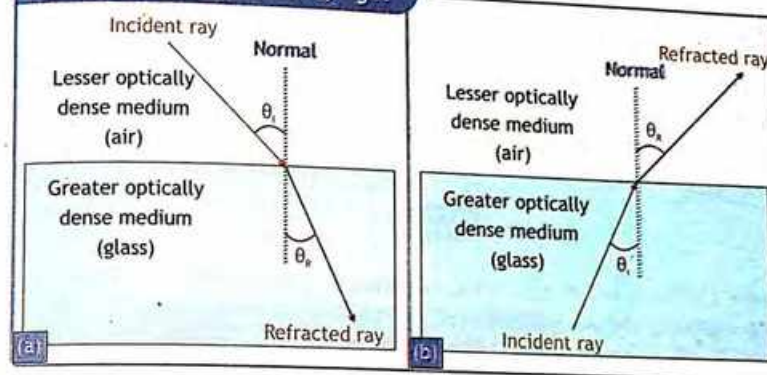
- i. **Incident ray:** The ray of light approaching the boundary (refracting surface) between two media.
- ii. **Refracted ray:** The ray of light moving away from the boundary between the two media.
- iii. **Point of incidence:** The point at which the incident ray strikes the refracting surface.
- iv. **Normal:** The line drawn at right angles to the refracting surface at the point of incidence.
- v. **Angle of incidence (θ_i):** The angle between the incident ray and the normal
- vi. **Angle of refraction (θ_r):** The angle of refraction ' θ_r ' is the angle between the refracted ray and the normal.
- vii. **Optically denser medium:** Medium having a greater refraction effect (slower speed of light); the actual density may or may not be greater.

A ray of light is bent towards the normal when it enters an optically denser medium at an angle, for example from air to glass as in Figure 12.14 (a). The angle of refraction θ_r is less than the angle of incidence θ_i ($\theta_i > \theta_r$). A ray of light is bent away from the normal when it enters an optically less dense medium, for example from glass to air as in Figure 12.14 (b). The angle of refraction θ_r is greater than the angle of incidence θ_i ($\theta_i < \theta_r$).

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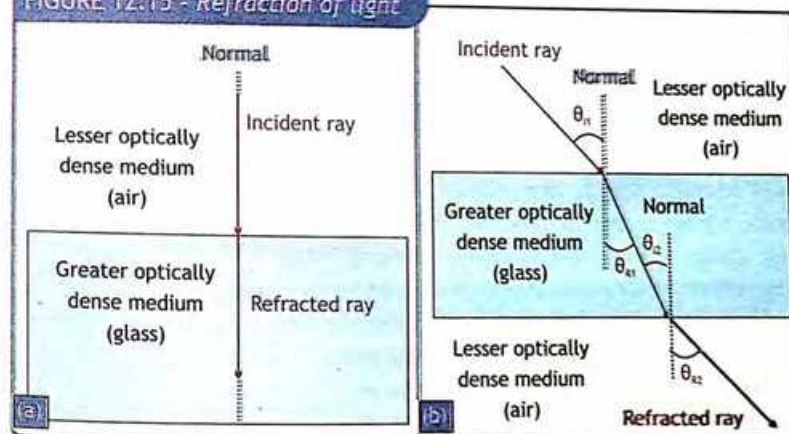


FIGURE 12.14 - Refraction of light



A ray travelling along the normal direction at a boundary is not refracted as shown in figure 12.15 (a). A ray emerging from a parallel-sided block is parallel to the ray entering, but is displaced sideways, like the ray in figure 12.15 (b).

FIGURE 12.15 - Refraction of light



LAB WORK

To verify the laws of refraction by using a glass slab.

NOT FOR SALE

a. Refractive Index (n):

Light travels at different speeds in different optically transparent materials. The ratio of the speed of light in vacuum divided by the speed of light in a material is called the refractive index (or index of refraction) of the material. The index of refraction n is given by

$$n = \frac{c}{v} \quad 12.6$$

where c is the speed of light in vacuum (about 3.00×10^8 m/s) and v is the speed of light in the medium. The speed of light in a physical medium such as glass is always less than that the speed of light in vacuum. Thus, the index of refraction of a material is always greater than or equal to 1, and by definition the index of refraction of vacuum is 1. Table 12.2 lists the indices of refraction for some common materials.

b. Snell's Law:

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the law of refraction, or 'Snell's Law,' which is stated as 'the product of the index of refraction of the first medium and the sine of the angle of incidence is equal to the product of the index of refraction of the second medium and the sine of the angle of refraction'. Mathematically

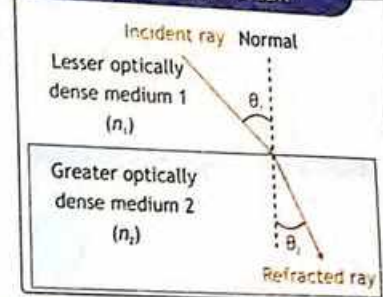
$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad 12.7$$

Table 12.2 Indices of Refraction for Some Common Materials

Material	Index of Refraction
Gases	
Air	1.000271
Helium	1.000036
Carbon dioxide	1.00045
Liquids	
Water	1.333
Methyl alcohol	1.329
Ethyl alcohol	1.362
Glycerine	1.473
Benzene	1.501
Typical oil	1.5
Solids	
Ice	1.310
Calcium fluoride	1.434
Fused quartz	1.46
Salt	1.544
Polystyrene	1.49
Typical glass	1.5
Crown glass	1.52
Quartz	1.544
Diamond	2.417

Here n_1 and n_2 are the indices of refraction for medium 1 and 2, and θ_1 and θ_2 are the angles between the rays and the perpendicular in medium 1 and 2, as shown in Figure 12.16.

FIGURE 12.16 - Snell's Law



EXAMPLE 12.3: SPEED OF LIGHT IN WATER

What is speed of light in water having index of refraction as 1.33?

GIVEN

Speed of light in vacuum ' c ' = 3.00×10^8 m/s

Index of refraction for water ' n ' = 1.33

REQUIRED

Speed of light in water ' v ' = ?

SOLUTION: The index of refraction is $n = \frac{c}{v}$ or $v = \frac{c}{n}$

Putting values $v = \frac{3.00 \times 10^8 \text{ m/s}}{1.33}$

Therefore $v = 2.26 \times 10^8 \text{ m/s}$ — Answer

Therefore the speed of light in water is 2.26×10^8 m/s

ASSIGNMENT 12.3: INDEX OF REFRACTION FOR KEROSENE OIL

If the speed of light in kerosene oil is 2.08×10^8 m/s, calculate the index of refraction.

EXAMPLE 12.4: CROWN GLASS AND WATER

Light travels from crown glass ($n_g = 1.52$) into water ($n_w = 1.33$). The angle of incidence in crown glass is 40.0° . What is the angle of refraction in water?

NOT FOR SALE

GIVEN

Index of refraction for crown glass ' n_g ' = 1.52

Index of refraction for water ' n_w ' = 1.33

Angle of incidence $\theta_i = 40.0^\circ$

REQUIRED

Angle of refraction $\theta_w = ?$

SOLUTION: By Snell's law $n_1 \sin \theta_1 = n_2 \sin \theta_2$ or $n_g \sin \theta_g = n_w \sin \theta_w$

$$\sin \theta_w = \frac{n_g}{n_w} \sin \theta_g \quad \text{Putting values} \quad \sin \theta_w = \frac{1.52}{1.33} \sin 40^\circ$$

$$\text{or } \sin \theta_w = 0.735 \quad \text{and} \quad \theta_w = \sin^{-1} 0.735$$

$$\text{Hence } \theta_w = 47.3^\circ \quad \text{--- Answer}$$

Note that $\theta_w > \theta_i$ since the light is moving to an optically lighter medium.

ASSIGNMENT 12.4: INDEX OF REFRACTION FOR UNKNOWN MEDIUM

Find the index of refraction for medium 2, if medium 1 is air with index of refraction $n_a = 1.00$, the incident angle is 30.0° and the angle of refraction is 22.0° . Compare the result with the table and identify the nature of medium 2.

Tip Bits

RED MOON: Refraction is responsible for the Moon appearing red during a lunar eclipse. A lunar eclipse occurs when Earth blocks sunlight towards the Moon. As a result, you might expect the Moon to be completely dark. Instead, the light from the sun refracts through the Earth's atmosphere and bends around Earth toward the Moon.



Since Earth's atmosphere scatters most of the blue and green light. Thus, mostly red light illuminates the Moon. Because the Moon reflects most colors of light equally well, it reflects the red light back to Earth, and therefore the Moon appears to be red.

NOT FOR SALE

12.5 TOTAL INTERNAL REFLECTION OF LIGHT

The angle of refraction is larger than the angle of incidence when light passes into a medium of a lower index of refraction, as shown in Figure 12.17 (a). This leads to an interesting phenomenon. As the angle of incidence increases, the angle of refraction also increases.

At a certain angle of incidence known as the critical angle, θ_c , the refracted light ray lies along the boundary of the two media, as shown in Figure 12.17 (b). When light strikes a transparent boundary, even though much of the light is transmitted, some is reflected. Total internal reflection occurs when light traveling from a region of a higher index of refraction to a region of a lower index of refraction strikes the boundary at an angle greater than the critical angle such that all light reflects back into the region of the higher index of refraction, as shown in Figure 12.17 (c).

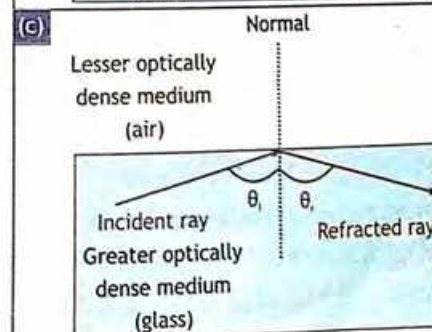
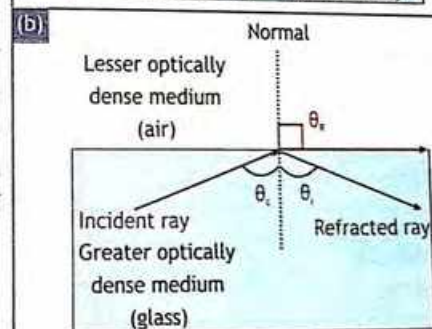
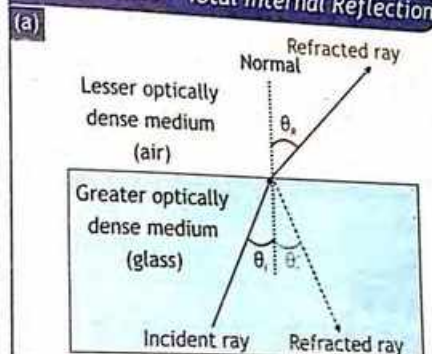
To construct an equation for the critical angle of any boundary, you can use Snell's Law and substitute $\theta_1 = \theta_c$ and $\theta_2 = 90^\circ$.

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$\text{since } \sin 90^\circ = 1$$

$$\text{therefore } n_1 \sin \theta_c = n_2$$

FIGURE 12.17 - Total Internal Reflection



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Hence

$$\sin \theta_c = \frac{n_2}{n_1}$$

13.8

Thus we can define critical angle as 'the sine of the critical angle is equal to the index of refraction of the refracting medium divided by the index of refraction of the incident medium.'

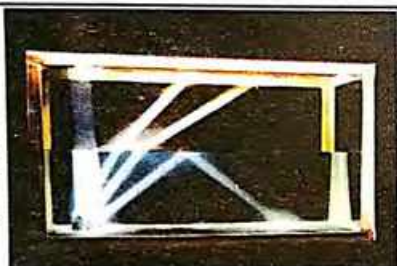
No transmitted ray for $\theta \geq \theta_c$

The two conditions required for total internal reflection to occur are as follows.

- The light must travel from an optically more dense medium into an optically less dense medium.
- The angle of incidence must exceed the critical angle, θ_c , associated with the material.

ACTIVITY

Make a beam of light to enter a tank of water from above and place mirrors oriented at different angles to reflect this beam of light. You will observe that the beam that makes an angle of incidence greater than the critical angle will totally be reflected back.

**LAB WORK**

To find the refractive index of water by using concave mirror.

EXAMPLE 12.5: CRITICAL ANGLE

Find the critical angle for light traveling from glass ($n = 1.502$) to (a) air ($n = 1.002$) and (b) water ($n = 1.332$)

GIVEN

index of refraction for ordinary glass ' n_g ' = 1.502

Index of refraction for air ' n_a ' = 1.002

Index of refraction for water ' n_w ' = 1.332

REQUIRED

Critical Angle $\theta_c = ?$

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SOLUTION: The critical angle is $\sin \theta_c = \frac{n_2}{n_1}$ or $\theta_c = \sin^{-1} \frac{n_2}{n_1}$

(a) When light goes from glass to air the critical angle is

$$\theta_c = \sin^{-1} \frac{n_a}{n_g} \quad \text{Putting values} \quad \theta_c = \sin^{-1} \frac{1.00}{1.50}$$

$$\theta_c = 41.8^\circ \quad \text{Answer}$$

(b) When light goes from glass to water the critical angle is

$$\theta_c = \sin^{-1} \frac{n_w}{n_g} \quad \text{Putting values} \quad \theta_c = \sin^{-1} \frac{1.33}{1.50}$$

$$\theta_c = 62.5^\circ \quad \text{Answer}$$

Note that the difference in index of refraction for glass - air is greater than glass - water. Hence light escapes from glass to air over less range of angles (00.0° to 41.8°) as compared to water (00.0° to 62.5°). In general, if the difference between indices of refraction values is large more light rays will undergo total internal reflection.

ASSIGNMENT 12.5: CRITICAL ANGLE FOR POLYSTYRENE

What is the critical angle for light traveling in a polystyrene (a type of plastic with index of refraction for polystyrene as 1.49) pipe surrounded by air (take index of refraction of air to be 1.00)?

Ans (6)
a) **Optical fibres:** Light can be trapped by total internal reflection inside a bent glass rod and 'piped' along a curved path called optical fibre.

Various glasses and plastics can be used to make optical fibres. Optical fibre transmits a beam of light by means of total internal reflection. Total internal reflection can occur in any transparent medium that has a higher index of refraction than the surrounding medium.

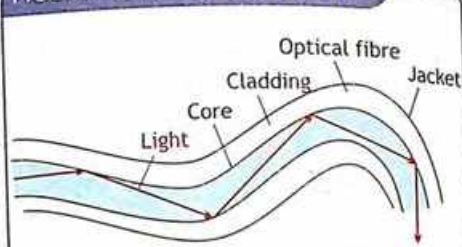
An optical fibre cable has a cylindrical shape and consists of three concentric sections: the core, the cladding and the jacket (Figure 12.18).

The core is the innermost section and consists of one or more very thin strands, made of glass or plastic. Each strand is surrounded by its own cladding, a glass or plastic coating that has optical properties different from the core.)

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The boundary between the core and cladding acts as a reflector to confine light that would otherwise escape the core. The outermost layer, surrounding one or a bundle of cladded fibres, is the jacket. The jacket is made of plastic and other material layered to protect against moisture, abrasion, crushing and other environmental dangers. As shown in figure 12.18 the light traveling through the transparent fibre always hits the internal boundary of the optical fibre at an angle greater than the critical angle, so all of the light is reflected and none of the light is transmitted through the boundary. Thus, the light maintains its intensity over the distance of the fibre.

FIGURE 12.18 - Optical fibre



Light sent into one end of an optical fibre like those shown here is transmitted to the opposite end with little loss of light through the sides of the fibre.

Point to Point
Surgical techniques have been revolutionized by the use of optical fibres. In arthroscopic surgery, a small surgical instrument (several millimeters in diameter) is mounted at the end of an optical fibre cable which allows light to be shone on the internal area and the reflected light to be viewed by the surgeon. The surgeon can insert the instrument and cable into a joint, such as the knee, with only a tiny incision and minimal damage to the surrounding tissue. Consequently, recovery from the procedure is relatively rapid compared to recovery from traditional surgical techniques.



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ENDOSCOPE

An endoscopy is a procedure where the inside of our body is examined using an instrument called an endoscope.

An endoscope is a long, thin, flexible tube that has a light source and camera at one end. Images of the inside of our body are displayed on computer monitor. Endoscopes can be inserted into the body through a natural opening, such as the mouth and down the throat, or through the bottom. An endoscope can also be inserted through a small cut (incision) made in the skin when keyhole surgery is being carried out.

Endoscopy can also be used by engineers to light up some awkward spot for inspection.

Picture shows a colonoscope revealing a benign (noncancerous) polyp attached to the wall of the colon (large intestine). Polyps that can turn cancerous or grow large enough to obstruct the colon are removed surgically.



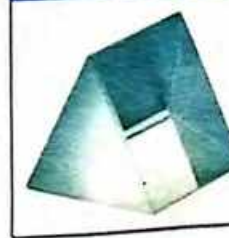
12.6 REFRACTION OF LIGHT THROUGH PRISM

Prism is a transparent optical element with flat, polished surfaces that refract light. At least two of the flat surfaces must have an angle between them. The exact angles between the surfaces depend on the application.

The traditional geometrical shape is that of a triangular prism with a triangular base and rectangular sides as shown in figure 12.19.¹

a. Deviation of light through prism: In the prism shown in Figure 12.20(a), the incident light ray travels through the air and enters the left side of the glass. The light bends toward the normal in the glass, because glass has a higher index of refraction (optical density) than the air.

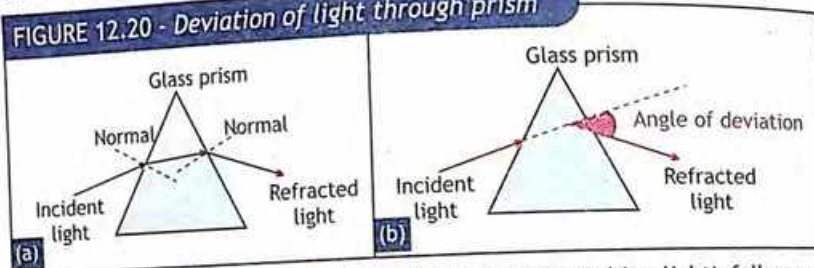
FIGURE 12.19 - Prism



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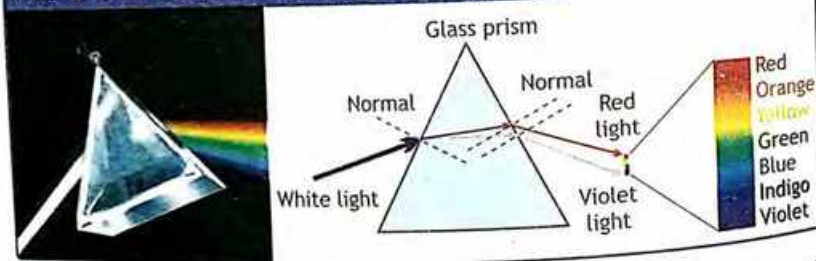
When the light leaves the glass and emerges into the air on the other side of the prism, the light is refracted away from the normal. Notice that the direction of the light leaving the glass is different from that of the light entering it. The change in direction of the light as it passes through the glass is known as its deviation. The amount of change is called the angle of deviation, θ_{dev} as shown in figure 12.20 (b). You can determine the angle of deviation for any shape of prism by applying Snell's law at each air-glass interface.

FIGURE 12.20 - Deviation of light through prism



b. **Dispersion of light through prism:** When sunlight (white light) falls on a triangular glass prism as in figure 12.21, a band of colours called a spectrum is obtained. The effect is termed dispersion. It arises because white light is a mixture of many colours; the prism separates the colours because the refractive index of glass is different for each colour (it is greatest for violet light).

FIGURE 12.21 - Dispersion of light through prism



LAB WORK

To trace the path of a ray of light through glass prism and measure the angle of deviation.

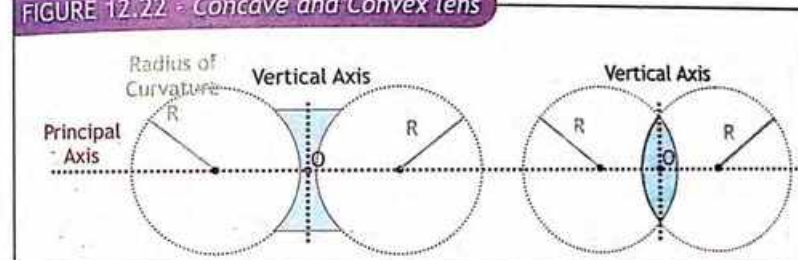
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12.7 LENSES

A lens is a piece of transparent optical material that focuses or disperses a beam of light by refraction. Lenses are made from materials such as glass or plastic, and are grounded and polished or molded to a desired shape. A lens can focus light to form an image.

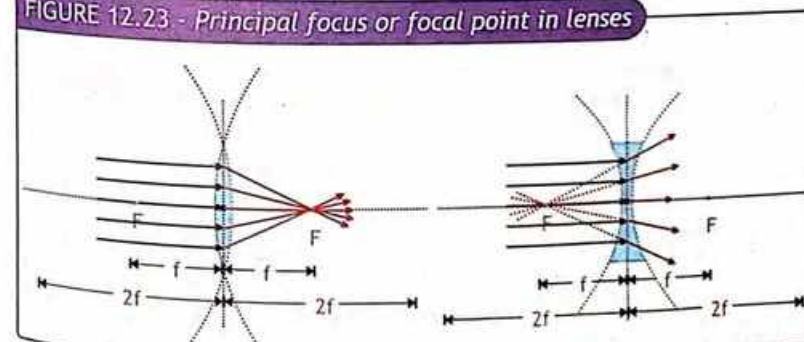
Each of a lens' two faces is part of a sphere and can be convex or concave (or one face may be flat). If a lens is thicker at the centre than the edges, it is a convex or converging lens since parallel rays will be converged to meet at the focus. A lens which is thinner in the centre than the edges is a concave or diverging lens since rays going through it will be spread out.

FIGURE 12.22 - Concave and Convex lens



Like mirrors, lenses have a principal axis perpendicular to their surface and passing through their midpoint as shown in figure 12.22. Lenses also have a vertical axis or principal plane through their middle. The centre point of the lens is called optical centre and denoted by O. There is no real centre of curvature, so $2F$ is used to denote twice the focal length.

FIGURE 12.23 - Principal focus or focal point in lenses



NOT FOR SALE

The point of convergence of the rays that enter parallel to the principal axis is called the principal focus or focal point. Since the rays of light can enter the lens from either side, there is a principal focus or focal point on the principal axis on each side of the lens. After refraction from lens, the rays converge or diverge as if they originate from a single point called the focal point F (or principal focus) as shown in figure 12.23. The distance from the vertical axis to either of the focal points is called the focal length of the lens.

12.7.1. Image formation and ray tracing for lenses: Ray diagrams are used to show image formation in lenses.

1. Construct a figure showing the lens and the principal axis of the lens. The figure should also show the focal points on both sides of the lens.
2. Draw the object at the appropriate point. One end of the object will generally lie on the axis.
3. Draw three rays that emanate from the tip of the object:

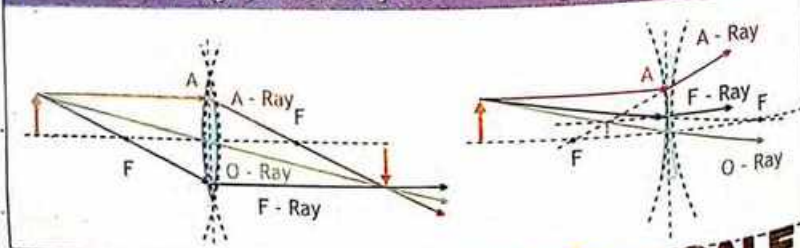
(a) The A - ray (parallel ray hitting lens at point A) is initially parallel to the axis. After passing through the lens, this ray or its extrapolation passes through one of the focal points.

(b) The F - ray (focal ray or its extrapolation) is directed at the focal point (same side for convex lens and the other side for concave lens). After passing through the lens, this ray is parallel to the axis.

(c) The O - ray (origin ray or its extrapolation) passes straight through the center of the lens called origin and is not deflected.

4. These three rays (figure 12.24) or their extrapolations intersect at the image. If the rays actually pass through the image, the image is real; if they do not, the image is virtual (just as in the case of images formed by a mirror).

FIGURE 12.24 - Image formation by concave and convex lenses



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When a lens forms a real image, the object and image are on opposite sides of the lens.

12.8 THE LENS EQUATION

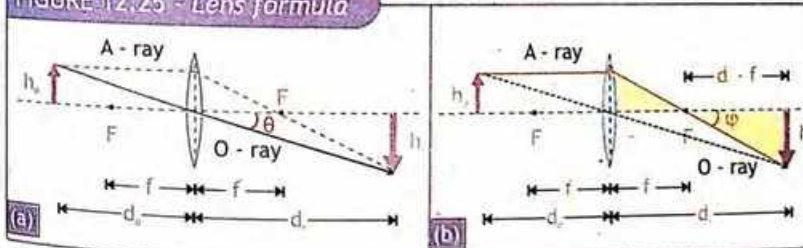
To calculate the precise location and size of the image formed by a lens, we use an equation that is analogous to the mirror equation. The lens equation is a precise mathematical relationship between the object distance and the image distance for a given lens. This formula is accurate only if the thickness of the lens is small compared to its diameter.

The following quantities are included in lens formula:

- f = the focal length of the lens
- h_o = the height of the object
- h_i = the height of the image
- d_o = the distance of the object from the lens
- d_i = the distance of the image from the lens
- M = the magnification of the image

This equation can be derived by referring to Figure 12.25, which shows the image produced by a convex lens, along with the O and A - rays that locate the image.

FIGURE 12.25 - Lens formula



The ray in Figure 12.25 (a) hits the lens at its midpoint, where the principal axis is the normal to the lens. As a result, the ray refracts at an angle θ below the principal axis that is equal to its incident angle θ above the axis. Therefore, the two green triangles in this diagram are similar, from which it follows that ratios of equivalent sides of similar triangles are equal.

$$\frac{h_o}{-h_i} = \frac{d_o}{d_i} \quad \text{--- ①}$$

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The image height is taken negative because it is inverted.

From the figure 12.25 (b) it is clear that the two yellow triangles in this diagram are also similar, since they are both right triangles and share the common angle ϕ . Thus, by using the same rule of similar triangles, we get

$$\frac{h_o}{h_i} = \frac{f}{d_i - f} \quad \text{--- (2)}$$

Comparing equation 1 and equation 2, we get

$$\frac{d_o}{d_i} = \frac{f}{d_i - f}$$

To isolate 'f', Cross Multiply

$$\begin{aligned} d_i f &= d_o (d_i - f) \\ \text{or} \\ d_i f &= d_o d_i - d_o f \quad \text{--- (3)} \end{aligned}$$

Divide both sides of the equation 3 by $d_o d_i f$

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}$$

$$\frac{d_i f}{d_o d_i f} = \frac{d_o d_i}{d_o d_i f} - \frac{d_o f}{d_o d_i f}$$

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad \text{--- 12.9}$$

The final equation 12.9 is known as the lens formula. You will probably recognize this equation as the mirror formula. Because the lens and mirror equations are the same, the equation is often called the mirror/lens formula.

The ratio of the height of the image to the height of the object is defined as the magnification, M . From Equation 1, we see that

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad \text{--- 12.10}$$

We derived the magnification and thin lens equations for a convex lens forming a real image, it can be used to calculate the image properties of both converging lenses and diverging lenses, provided we use the sign conventions described below.

12.8.1 Convention for the lens equation: To include all of the possible properties of both images and objects, the following sign convention has been established

a. **Object distance:**

- d_o is positive for real objects (from which light diverges).

b. **Image distance:**

- d_i is positive for real images (images on the opposite side of the lens from the object).
- d_i is negative for virtual images (images on the same side of the lens as the object).

c. **Image attitude:**

- h_i is positive for images that are upright, compared to the object
- h_i is negative for images that are inverted, compared to the object

d. **Focal length:**

- f is positive for converging (convex) lenses.
- f is negative for diverging (concave) lenses.

12.8.2 Power of lens: The degree of convergence or divergence of light rays falling on lens is called power of lens. Instrument makers often quote the power of a lens rather than its focal length. The power of a lens in diopters D , is given by the equation

$$D = \frac{1}{f} \quad \text{--- 12.11}$$

where f is the focal length of the lens expressed in metres. Eyeglass lenses are typically characterized in terms of **diopters**. The power of a lens in diopters should not be confused with the familiar concept of power in watts. It is an unfortunate fact that the word "power" is used for two completely different concepts. If you examine a prescription for eyeglasses, you will note lens powers given in diopters. If you examine the label on a motor, you will note energy consumption rate given as a power in watts.

LAB WORK

To determine the critical angle of glass using a semi circular slab and a light ray box/or by prism.

To find the focal length of a convex lens by parallax method.

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EXAMPLE 12.6: IMAGE DISTANCE IN CONVEX LENS

A converging lens of focal length 10.0 cm forms images of an object situated at various distances. (a) If the object is placed 30.0 cm from the lens, locate the image, state whether it's real or virtual, and find its magnification. Repeat the problem (b) when the object is at 20.0 cm and (c) when the object is 15.0 cm from the lens (d) when the object is 10.0 cm from the lens and (e) when the object is 5.00 cm from the lens.

GIVEN

Focal length ' f ' = 10.00 cm.

- (a) Object Distance ' d_o ' = 30.0 cm,
 (b) Object Distance ' d_o ' = 20.0 cm
 (c) Object Distance ' d_o ' = 15.0 cm
 (d) Object Distance ' d_o ' = 10.00 cm
 (e) Object Distance ' d_o ' = 5.00 cm

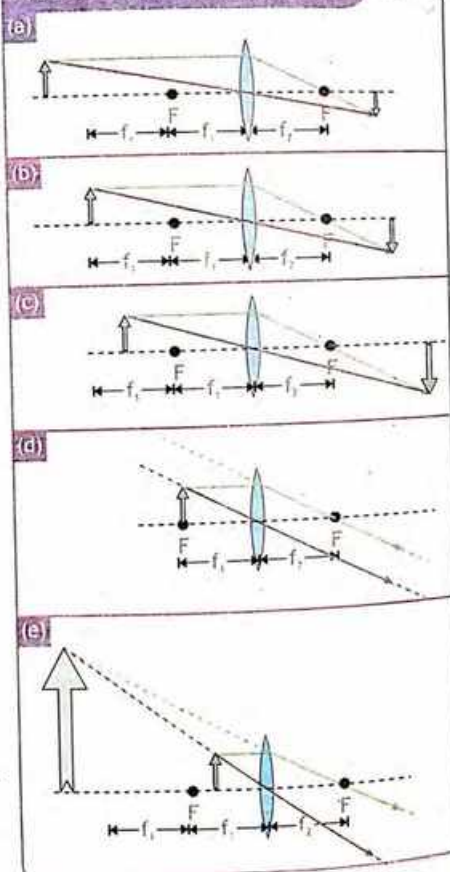
REQUIRED

- (a) Image Distance ' d_i ' = ?
 Magnification M = ?
 (b) Image Distance ' d_i ' = ?
 Magnification M = ?
 (c) Image Distance ' d_i ' = ?
 Magnification M = ?
 (d) Image Distance ' d_i ' = ?
 Magnification M = ?
 (e) Image Distance ' d_i ' = ?
 Magnification M = ?

SOLUTION: by mirror equation

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$$

$$\frac{1}{d_i} = \frac{d_o - f}{fd_o}$$

NOT FOR SALE**FIGURE 12.26 - Example 12.6**

$$\text{or} \quad d_i = \frac{fd_o}{d_o - f} \quad \text{--- (1)}$$

(a) to get image distance putting values in equation 1

$$d_i = \frac{10.0 \text{ cm} \times 30.0 \text{ cm}}{30.0 \text{ cm} - 10.0 \text{ cm}} \quad \text{or} \quad d_i = \frac{300 \text{ cm}^2}{20 \text{ cm}}$$

$$\text{Therefore} \quad d_i = +15.0 \text{ cm} \quad \text{--- Answer}$$

The magnification formula is $M = -\frac{d_i}{d_o} = -\frac{15 \text{ cm}}{30 \text{ cm}}$

$$\text{Therefore} \quad M = -0.500 \quad \text{--- Answer}$$

As shown in figure 12.26 (a) and the answers in this example it is confirmed that when the object distance is greater than twice the focal length ($d_o > 2f$), for convex lens

- the image distance is less than twice the focal length, (i.e. $15 \text{ cm} < 20 \text{ cm}$)
- the image is real, (distance positive)
- the image is inverted, (magnification negative) and
- the image is smaller than the object. (magnification less than 1)

(b) to get image distance putting values in equation 1

$$d_i = \frac{10.0 \text{ cm} \times 20.0 \text{ cm}}{20.0 \text{ cm} - 10.0 \text{ cm}} \quad \text{or} \quad d_i = \frac{200 \text{ cm}^2}{10.0 \text{ cm}}$$

$$\text{Hence} \quad d_i = +20.0 \text{ cm} \quad \text{--- Answer}$$

The magnification formula is $M = -\frac{d_i}{d_o} = -\frac{20 \text{ cm}}{20 \text{ cm}}$

$$\text{Therefore} \quad M = -1.00 \quad \text{--- Answer}$$

Therefore, when the object distance is equal to twice the focal length ($d_o = 2f$) as shown in figure 12.26 (b) it is confirmed that for convex lens:

- the image distance is equal to twice the focal length (20 cm),
- the image is real (distance positive),
- the image is inverted (magnification negative), and
- the image is the same size as the object (magnification is equal to 1).

NOT FOR SALE

(c) to get image distance putting values in equation 1

$$d_i = \frac{10.0\text{ cm} \times 15.0\text{ cm}}{15.0\text{ cm} - 10.0\text{ cm}} \quad \text{or} \quad d_i = \frac{150\text{ cm}^2}{5.00\text{ cm}}$$

Hence $d_i = 30\text{ cm}$ — Answer

The magnification formula is $M = -\frac{d_i}{d_o} = -\frac{30\text{ cm}}{15\text{ cm}}$

Therefore $M = -2.00$ — Answer

Therefore, when the object distance is less than twice the focal length but greater than the focal length ($2f < d_o < f$) as shown in figure 12.26 (c), it is confirmed that for convex lens:

- the image distance is greater than twice the focal length ($30\text{ cm} > 20\text{ cm}$),
- the image is real (distance positive),
- the image is inverted (magnification negative), and
- the image is larger than the object (magnification is greater than 1).

(d) to get image distance putting values in equation 1

$$d_i = \frac{10.0\text{ cm} \times 10.0\text{ cm}}{10.0\text{ cm} - 10.0\text{ cm}} \quad \text{or} \quad d_i = \frac{100\text{ cm}^2}{0\text{ cm}}$$

Hence $d_i = \infty$ — Answer

The magnification formula is $M = -\frac{d_i}{d_o} = -\frac{\infty}{20\text{ cm}}$

Therefore $M = \infty$ — Answer

Hence, the result confirms that when the object distance is equal to the focal length ($d_o = f$) as shown in figure 12.26 (d) for convex lens:

- no image exists because the refracted rays are parallel. You could say that the image lies at infinity.

NOT FOR SALE

(e) to get image distance putting values in equation 1

$$d_i = \frac{10.0\text{ cm} \times 5.00\text{ cm}}{5.00\text{ cm} - 10.0\text{ cm}} \quad \text{or} \quad d_i = \frac{50\text{ cm}^2}{-5\text{ cm}}$$

Hence $d_i = -10\text{ cm}$ — Answer

The magnification formula is $M = -\frac{d_i}{d_o} = -\frac{-10\text{ cm}}{5\text{ cm}}$

Therefore $M = +2$ — Answer

Hence, the results confirms that when the object distance is less than the focal length but greater than zero ($f > d_o > 0$) as shown in figure 12.6 (e), for convex lens:

- the image distance is greater than the object distance, ($10\text{ cm} > 5\text{ cm}$)
- the image is virtual (distance negative),
- the image is upright (magnification positive), and
- the image is larger than the object (magnification is greater than 1).

EXTENSION EXERCISE 12.2

Repeat the problem of Example 12.6 for a diverging lens of focal length 10.0 cm .

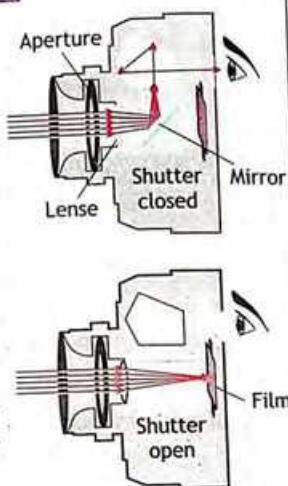
ASSIGNMENT 12.6: LENS COMPARISON

An object is placed 30.0 cm in front of a converging lens and then 12.5 cm in front of a diverging lens. Both lenses have a focal length of 10.0 cm . For both cases, find the image distance and the magnification. Describe the images.

NOT FOR SALE

HOW A CAMERA WORKS

One of the simplest optical instruments is the camera, which often has only one lens to produce an image, or even—in a pinhole camera—no lens. Figure shows a simple 35-mm camera. The camera uses a converging lens to form a real image on the film. The image must be real in order to expose the film (i.e., cause a chemical reaction). Light rays from a point on an object being photographed must converge to a corresponding point on the film. A digital camera has replaced film with a CCD (charge-coupled device) array.

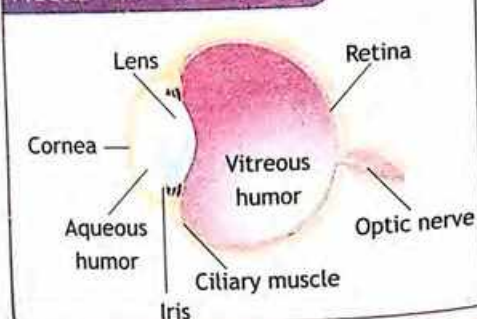


12.9 HUMAN EYE

The human eye is like a camera in its basic structure (figure 12.27), but is more complex. The interior of the eye is filled with a transparent gel-like substance called the vitreous humor with index of refraction $n = 1.337$. Light enters in this region through the cornea and lens. Between the cornea and lens is a watery fluid, the aqueous humor with $n = 1.336$. The iris adjusts automatically to control the amount of light entering the eye, similar to a camera.

The retina, which plays the role of the film or sensor in a camera, is on the curved back surface of the eye. The retina is composed of a many nerves which act to change light energy into electrical signals that travel along the nerves to brain for interpretation.)

FIGURE 12.27 - Human Eye

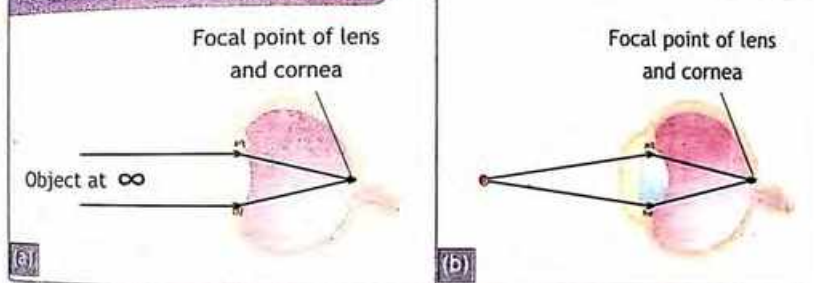


NOT FOR SALE

The lens of the eye ($n = 1.386$ to 1.406) does little of the bending of the light rays. Most of the refraction is done at the front surface of the cornea at its interface with air. The lens acts as a fine adjustment for focusing at different distances. This is accomplished by the ciliary muscles, which change the curvature of the lens so that its focal length is changed.

To focus on a distant object, the ciliary muscles of the eye are relaxed and the lens is thin, as shown in figure 12.28 (a), and parallel rays focus at the focal point (on the retina). To focus on a nearby object, the muscles contract, causing the centre of the lens to thicken, figure 12.28(b), thus shortening the focal length so that images of nearby objects can be focused on the retina, behind the new focal point. This focusing adjustment is called accommodation.

FIGURE 12.28 - Accommodation



The closest distance at which the eye can focus clearly is called the near point of the eye. For young adults it is typically 25 cm, as people grow older, the ability to accommodate is reduced and the near point increases.

A given person's far point is the farthest distance at which an object can be seen clearly. Since we can focus on the Moon and stars, it is clear that the normal far point is essentially infinity.

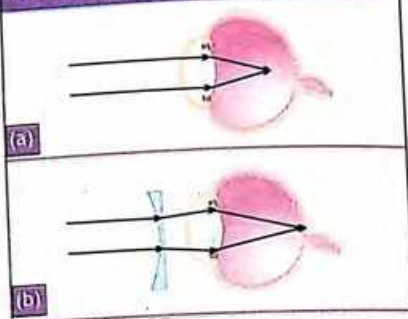
Many people have eyes that do not accommodate within the normal range of 25 cm to infinity or have some other defect. Two common defects are short-sightedness and long-sightedness. Both can be corrected to a large extent with lenses—either eyeglasses or contact lenses.

12.9.1 Short-sightedness: In short-sightedness (nearsightedness or myopia), the human eye can focus only on nearby objects. The far point is not infinity but some shorter distance, so distant objects are not seen clearly.

NOT FOR SALE

Short-sightedness is usually caused by an eyeball that is too long, although sometimes it is the curvature of the cornea that is too great. In either case, images of distant objects are focused in front of the retina as shown in figure 12.29 (a). A diverging lens, because it causes parallel rays to diverge, allows the rays to be focused at the retina as shown in figure 12.29 (b) and thus can correct nearsightedness.

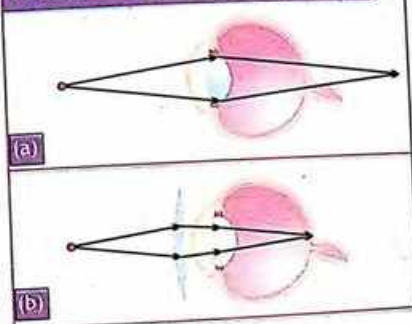
FIGURE 12.29 - Short-sightedness



12.9.2 Long-sightedness:

In long-sightedness (farsightedness or hyperopia), the eye cannot focus on nearby objects. Although distant objects are usually seen clearly, the near point is somewhat greater than the "normal" 25 cm, shown in figure 12.30 (a). This defect is caused by an eyeball that is too short or (less often) by a cornea that is not sufficiently curved. It is corrected by a converging lens, shown in figure 12.30 (b).

FIGURE 12.30 - Long-sightedness



12.9.3 Resolving power: Resolving power is the capacity of an instrument to distinctively separate two points which are close together. The sharpness of vision—in particular, the ability to visually separate closely spaced objects—is referred to as resolution.

For example, in the first photo we see a bright light in the distance that may be the single headlight of an approaching motorcycle or the unresolved image of two headlights on a car. As the car approaches closer, the separation between the lights will increase. As the car continues to approach, its individual headlights become increasingly distinct, as shown in the third photo. So the two points are distinctively separated.

NOT FOR SALE

FIGURE 12.31 - Resolving the headlights of an approaching car



12.10 ANGULAR MAGNIFICATION AND SIMPLE MICROSCOPE

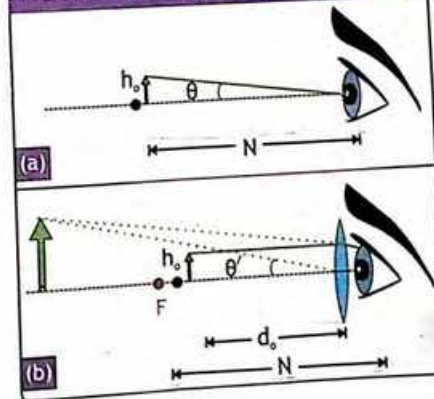
An optical instrument, such as a magnifying glass, allows us to view small or distant objects because it produces a larger image on the retina than would be possible otherwise. In other words, an optical instrument magnifies the angular size of the object. The angular magnification (or magnifying power) m , is the angular size θ' of the final image produced by the instrument divided by a reference angular size θ . The reference angular size is the angular size of the object when seen without the instrument.

$$\text{Angular Magnification} = \frac{\text{Angular size of final image produced by optical instrument}}{\text{Reference angular size of object seen without optical instrument}}$$

$$\text{or } m_0 = \frac{\theta'}{\theta} \quad \text{--- ①}$$

Assume an object with height h_o , without a magnifier, the largest angle θ that we can attain and see the object clearly is when we place the object at the near point N as in Figure 12.32 (a). We can get a magnified image of the object by placing the object just inside the focal length of a converging lens as shown in Figure 12.32 (b). If we look through the lens at the image, we can see the enlarged, upright, virtual image.

FIGURE 12.32: SIMPLE MICROSCOPE



NOT FOR SALE

Figure 12.32 (a) shows that the angle subtended by the object without the magnifier is given by

$$\tan(\theta) = \frac{\text{Perpendicular}}{\text{base}} = \frac{h_o}{N} \quad (2)$$

Assuming that the eye is next to the magnifying glass, the angular size as seen by the eye as in figure 12.32 (b) is

$$\tan(\theta') = \frac{\text{Perpendicular}}{\text{base}} = \frac{h_o}{d_o} \quad (3)$$

where f is the focal length of the lens. We assume that the object is placed at the focal length of the lens, so the image is at minus infinity. For small-angles $\tan \theta = \theta$ and $\tan \theta' = \theta'$. Thus, by putting values from equation 2 and equation 3 in equation 1, the angular magnification of a magnifier can be written as

$$m_o = \frac{h_o'/d_o}{h_o/N} \quad \text{or} \quad m_o = \frac{N}{d_o} \quad (4)$$

According to the thin-lens equation, d_o is related to the image distance d_i and the focal length f of the lens by

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \text{or} \quad \frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} \quad (5)$$

putting values from equation 5 in equation 4, we get

$$m_o = \left[\frac{1}{f} - \frac{1}{d_i} \right] N \quad (12.12)$$

Two special cases of this result are of interest, depending on whether the image is located as close to the eye as possible or as far away as possible.

To be seen clearly, the closest the image can be relative to the eye is at the near point, or $d_i = -N$. The minus sign indicates that the image lies to the left of the lens and is virtual. In this event, Equation 12.12 becomes

$$m_o = \left[\frac{1}{f} - \frac{1}{-N} \right] N \quad \text{or} \quad m_o = \left[\frac{1}{f} + \frac{1}{N} \right] N$$

NOT FOR SALE

$$\text{and} \quad m_o = \frac{N}{f} + \frac{N}{N} \quad \text{Therefore} \quad m_o = \frac{N}{f} + 1 \quad (12.12 \text{ (a)})$$

The farthest the image can be from the eye is at infinity ($d_i = \infty$); this occurs when the object is placed at the focal point of the lens. When the image is at infinity, equation 12.12 can be written as

$$m_o = \left[\frac{1}{f} - \frac{1}{\infty} \right] N \quad \text{as} \quad \frac{1}{\infty} = 0$$

$$\text{or} \quad m_o = \left[\frac{1}{f} - 0 \right] N \quad \text{Hence} \quad m_o = \frac{N}{f} \quad (12.12 \text{ (b)})$$

EXAMPLE 12.7: INSECT EXAMINATION

A biologist with a near-point distance of $N = 26$ cm, examines an insect wing through a magnifying glass whose focal length is 4.3 cm. Find the angular magnification when the image produced by the magnifier is (a) at the near point and (b) at infinity.



GIVEN

Focal length of magnifying glass ' f ' = 4.3 cm.

Near-point distance of $N = 26$ cm

Object Distance ' d_o ' = 1.1 cm

REQUIRED

Angular Magnification ' m_o ' = ?

SOLUTION: The magnification when the image is at near point for magnifying glass is

$$m_o = \frac{N}{f} + 1 \quad \text{Putting values} \quad m_o = \frac{26 \text{ cm}}{4.3 \text{ cm}} + 1$$

$$\text{Hence} \quad m_o = 7 \quad \text{Answer}$$

The magnification when the image is at infinity for magnifying glass is

$$m_o = \frac{N}{f} \quad \text{Putting values} \quad m_o = \frac{26 \text{ cm}}{4.3 \text{ cm}}$$

$$\text{Hence} \quad m_o = 6 \quad \text{Answer}$$

The relaxed eye results in decrease in magnification of 1.0—from 7.0 to 6.0.

NOT FOR SALE

ASSIGNMENT 12.7: JEWELER'S LOUPE

An 8-cm-focal-length converging lens is used as a 'jeweler's loupe,' which is a magnifying glass. Estimate the magnification (a) when the eye is relaxed, and (b) if the eye is focused at its near point

FIGURE 12.33 - Magnifying glass

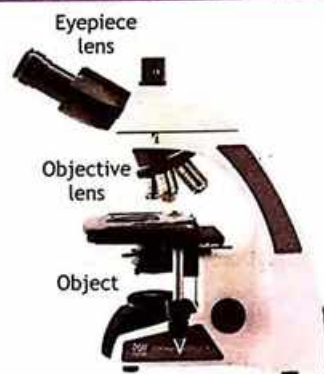
**12.11 COMPOUND MICROSCOPE**

To increase the angular magnification beyond that possible with a magnifying glass, an additional converging lens can be included to "pre-magnify" the object before the magnifying glass comes into play.

The result is an optical instrument known as the compound microscope. The magnifying glass is called the eyepiece and the additional lens is called the objective as shown in figure 12.34.

The overall magnification of a microscope is the product of the magnifications produced by the two lenses.

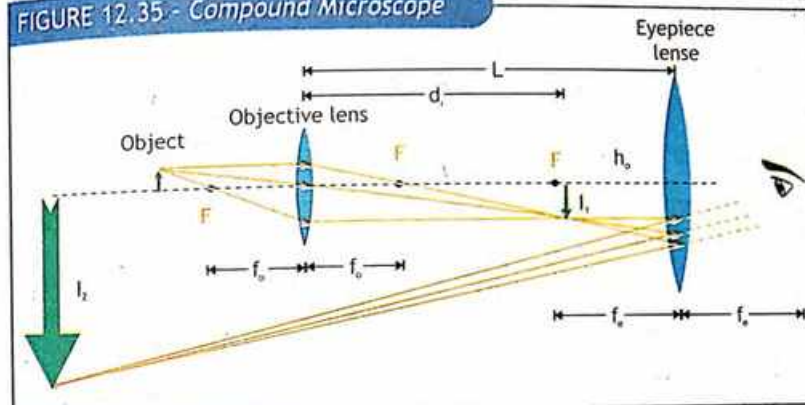
FIGURE 12.34 - Compound Microscope

**NOT FOR SALE**

In a typical situation shown in figure 12.35, the object to be examined is placed only a small distance beyond the focal point of the objective ' f_o ', which means that $d_o \approx f_o$. The magnification ' M_o ' produced by the objective is given

$$M_o = -\frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{d_i}{f_o} \quad \text{--- (1)}$$

FIGURE 12.35 - Compound Microscope



The eyepiece acts like a simple magnifier. If we assume that the eye is relaxed, the eyepiece angular magnification m_e by equation 12.12 (b) can be written as

$$m_e = \frac{N}{f_e} \quad \text{--- (2)}$$

Since the eyepiece enlarges the image formed by the objective, the overall magnification ' m ' is the product of the magnification of the objective lens, ' M_o ', times the angular magnification ' m_e ' of the eyepiece lens

$$m = M_o \times m_e \quad \text{--- (3)}$$

putting equation 1 and equation 2 in equation 3, we get

$$m = -\frac{d_i}{f_o} \times \frac{N}{f_e} \quad \text{--- 12.13}$$

The minus sign indicates that the image is inverted.

NOT FOR SALE

EXAMPLE 12.8: BIOLOGY LABORATORY

In biology class, a student with a near-point distance of $N = 25$ cm uses a microscope to view an amoeba. If the objective has a focal length of 1.0 cm, the eyepiece has a focal length of 2.5 cm, and the amoeba is 1.1 cm from the objective, what is the magnification produced by the microscope?

GIVEN

Focal length of eye piece ' f_e ' = 2.5 cm.

Focal length of objective ' f_o ' = 1.0 cm.

Near-point distance of $N = 25$ cm

Object distance ' d_o ' = 1.1 cm

REQUIRED

Total magnification ' m ' = ?

SOLUTION: The image distance d_i for the equation 12.13 is not given, therefore we will first find the image distance by using the thin-lens formula

$$\frac{1}{f_o} = \frac{1}{d_i} + \frac{1}{d_o} \quad \text{or} \quad \frac{1}{d_i} = \frac{1}{f_o} - \frac{1}{d_o} \quad \text{taking LCM} \quad \frac{1}{d_i} = \frac{d_o - f_o}{f_o d_o}$$

$$\text{or} \quad d_i = \frac{f_o d_o}{d_o - f_o} \quad \text{--- (1)}$$

$$\text{Putting values} \quad d_i = \frac{1.1 \text{ cm} \times 1.0 \text{ cm}}{1.0 \text{ cm} - 1.1 \text{ cm}} \quad \text{and} \quad d_i = \frac{1.1 \text{ cm}^2}{-0.1 \text{ cm}}$$

$$d_i = -11 \text{ cm}$$

The magnification formula for compound microscope is $m = -\frac{d_i}{f_o} \times \frac{N}{f_e}$

$$\text{Putting values} \quad m = -\frac{11 \text{ cm}}{1.0 \text{ cm}} \times \frac{25 \text{ cm}}{2.5 \text{ cm}}$$

$$\text{Therefore} \quad m = -110 \quad \text{--- Answer}$$

Thus, the amoeba appears 110 times larger and is inverted. If the amoeba is to be viewed with a relaxed eye, the image formed by the objective should be at the focal point of the eyepiece, which will then form an image at infinity. Therefore, the length of the tube containing the objective and eyepiece is $L = 11 \text{ cm} + 2.5 \text{ cm} = 13.5 \text{ cm}$ in this case.

NOT FOR SALE

ASSIGNMENT 12.8: EYE PIECE FOCAL LENGTH

If the focal length of the eyepiece is increased, does the magnitude of the magnification increase or decrease? Check your response by calculating the magnification when the focal length of the eyepiece is 3.5 cm.

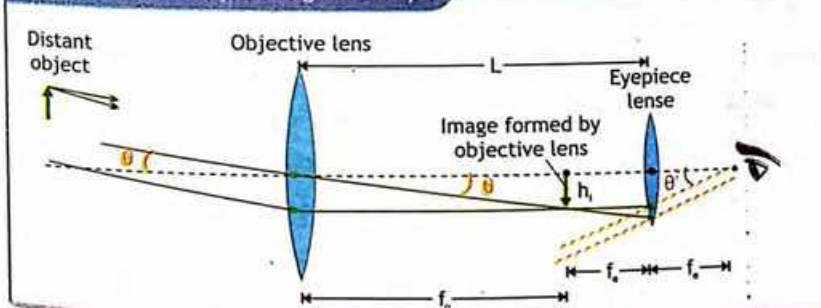
12.12 REFRACTING TELESCOPE

A telescope is an instrument for magnifying distant objects, such as stars and planets. Like a microscope, a telescope consists of an objective lens that forms a real image of the object; and an eyepiece (also called the ocular) is used to view this real image.

Since the object is far away, the angular size seen by the unaided eye is nearly the same as the angle θ subtended at the objective of the telescope as shown in Figure 12.37. Moreover, θ is also the angle subtended by the first image, therefore

$$\theta = -\frac{h_i}{f_o} \quad \text{--- (1)}$$

Here h_i is the height of the first image and f_o is the focal length of the objective. A minus sign has been inserted into this equation because the first image is inverted relative to the object and the image height h_i is a negative number.

FIGURE 12.37 - Refracting telescope

NOT FOR SALE

FIGURE 12.36 - Telescope



As in the microscope, the image of the objective is the object for the eyepiece, which is basically a magnifier. Thus, if the image of the objective is placed at the focal point of the eyepiece, it will form an image that is at infinity, as indicated in Figure 12.37. In this configuration, the observer can view the final image of the telescope with a completely relaxed eye.

The angular size of the image formed by the eyepiece is approximately

$$\theta' = \frac{h_i}{f_e} \quad \text{--- (2)}$$

To find the total angular magnification of the telescope, we consider the definition of angular magnification as

$$m_\theta = \frac{\theta'}{\theta} \quad \text{--- (3)}$$

putting equation 1 and equation 2 in equation 3, we get

$$m_\theta = \frac{h_i/f_e}{-h_o/f_o} \quad \text{Therefore} \quad m_\theta = -\frac{f_o}{f_e} \quad \text{--- 12.14}$$

LAB WORK

To set up a microscope and telescope.

EXAMPLE 12.9: WORLD'S LARGEST TELESCOPE

The largest optical refracting telescope in the world is located at the Yerkes Observatory in Wisconsin. The objective lens has a focal length of 19 m, and the eyepiece has a focal length of 10 cm. (a) Calculate the total magnifying power of this telescope. (b) Estimate the length of the telescope.



GIVEN

Focal length of objective ' f_o ' = 19 m.

Focal length of eye piece ' f_e ' = 10 cm.

REQUIRED

(a) Angular magnification ' m_θ ' = ?

(b) Approximate length ' L ' = ?

SOLUTION: (a) The magnification of telescope is $m_\theta = -\frac{f_o}{f_e}$

Putting values $m_\theta = -\frac{19m}{0.10m}$ Therefore $m_\theta = -190$ --- **Answer**

NOT FOR SALE

(b) For a relaxed eye, the image is at the focal point of both the eyepiece and the objective lenses. The distance between the two lenses is thus

$$L \approx f_o + f_e \approx 19m + 0.10m$$

$$\text{Therefore } L \approx 19m \quad \text{--- Answer}$$

which is essentially the length of the telescope.

ASSIGNMENT 12.9: ASTRONOMICAL TELESCOPE

An astronomical telescope has the following specifications: $f_o = 985$ mm and $f_e = 5.00$ mm. From these data points, find (a) the angular magnification and (b) the approximate length of this telescope.

The Reflection of Light: When light reflects from a smooth surface, the reflected light obeys the law of reflection.

Spherical Mirrors: A spherical mirror has the shape of a section from the surface of a hollow sphere.

The Mirror Formula: The formula specifying the relation between the object distance d_o , the image distance d_i , and the focal length f of the mirror.

Refraction: The changing of a light ray's direction when it passes through variations in matter.

Snell's Law: The ratio of sine of angle of incidence and sine of angle of refraction is constant for a given pair of media.

Total Internal Reflection: When the angle of incidence exceeds the critical angle, all the incident light is reflected back into the material from which it came.

Lens: A transmissive optical device that focuses or disperses a light beam by means of refraction.

Thin Lens Formula: The formula specifying the relation between the object distance d_o , the image distance d_i , and the focal length f of thin lens.

Power of Lens: The degree of convergence or divergence of light rays falling on lens.

Short Sightedness: Defect of an eye so that distant objects are not seen clearly.

Long Sightedness: Defect of an eye so that nearby objects are not seen clearly.

Simple Microscope: A converging lens, which works by allowing an object to be viewed at a distance less than the near-point distance.

Compound Microscope: Instruments for enlarging the detail that we cannot see with the unaided eye.

Telescope: Device meant for viewing distant objects, producing an image that is larger than the image that can be seen with the unaided eye.

NOT FOR SALE

GROUP A 'OPTICS': Interview an optometrist, optician, or ophthalmologist. Find out what equipment and tools each uses. What kinds of eye problems are curable? What training is necessary for each career? Publish the interview(s) in school magazine.

GROUP B 'MUSLIM SCIENTIST': The Egyptian scholar Alhazen (Ibn-al-Haytham) studied lenses, mirrors, rainbows, and other light phenomena early in the Middle Ages. Research his scholarly work, his life, and his relationship with the Caliph al-Hakim. How advanced were Alhazen's inventions and theories? Summarize your findings and report them to the class.

GROUP C 'OPTICAL FIBRES': Prepare a chart presentation on the use of optical fibres in telecommunications and medical field and present your chart to classroom and display it in classroom.

GROUP D 'MICROSCOPES AND TELESCOPES': Research the internet for advance microscopes and telescopes. Prepare a presentation to be presented in class.

GROUP E 'TELESCOPE OR MICROSCOPE': Buy few lenses and make your own telescope or compound microscope of simple objects and donate it to the school laboratory.

EXERCISE

- 1 If the angle of incidence is 30° the angle of reflection will be
A. 30° B. 45° C. 90° D. 210°
- 2 When r is the radius of curvature of concave mirror. Real diminished image will be formed when the object is at
A. $d_o > r$ B. $d_o = r$ C. $r > d_o > f$ D. $d_o = f$
- 3 The focal length of convex mirror with radius of curvature 10 cm is
A. +10 cm B. +5 cm C. -10 cm D. -5 cm
- 4 An object is placed 7 cm from a concave mirror whose radius of curvature is 10 cm, the image formed will be
A. real and upright B. virtual and upright
C. real and inverted D. virtual and inverted
- 5 Which one of the following materials will refract light more
A. water B. glass C. air D. diamond

NOT FOR SALE

- Geometrical Method*
- 1 A convex lens with focal length 8.00 cm has the power of lens
A. 2.05 D B. 4.00 D C. 12.5 D D. 16.0 D
 - 2 If the distance from your eye's lens to the retina is shorter than for a normal eye, you will struggle to see objects that are
A. nearby B. colorful C. far away D. moving fast.
 - 3 Who benefits more from using a magnifying glass, a person whose near point is located at a distance away from the eyes of
A. 75 cm B. 50 cm C. 35 cm D. 25 cm
 - 4 The human eye forms the image of an object at its
A. iris B. retina C. pupil D. cornea

CONCEPTUAL QUESTIONS

Give a brief response to the following questions

- 1 Which type of lens would you use to start fire from light from sun concave or convex, would work best? At what distance from the lens should the paper be held for best results?
- 2 If a concave mirror produces a real image, is the image necessarily inverted? Explain.
- 3 Are rearview mirrors used in cars concave or convex?
- 4 A magician during a show makes a glass lens with $n = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?
- 5 Suppose that you were handed a lens and a ruler and told to determine the focal length of the lens. How would you proceed?
- 6 Can we achieve total internal reflection from optically rare medium to optically dense medium?
- 7 Will a nearsighted person who wears corrective lenses in her glasses be able to see clearly underwater when wearing those glasses?
- 8 When you use a simple magnifying glass, does it matter whether you hold the object to be examined closer to the lens than its focal length or farther away? Explain.
- 9 In blind turns on hilly roads, mirrors are used to help drivers. Are these mirrors plane mirrors, concave mirrors or convex mirrors? Explain.

NOT FOR SALE

COMPREHENSIVE QUESTIONS

Give an extended response to the following questions

- 1 What is meant by reflection of light? State and explain laws of reflection with diagrams.
- 2 Derive spherical mirror formula.
- 3 What is meant by refraction of light? What is the index of refraction?
- 4 State and explain laws of refraction with diagrams.
- 5 What is total internal reflection? How we can calculate the critical angle for total internal reflection? What are the conditions for total internal reflection?
- 6 What are optical fibres? Give some applications of optical fibers.
- 7 Describe the behavior for a ray of light after passing through a prism.
- 8 Derive thin lens equation.
- 9 Describe power of lens and its resolving power. What are its units?
- 10 How the human eye works? How the defects in the eye like short sightedness and long sightedness be corrected by using lenses?
- 11 What is a simple microscope? Using a ray diagram explain its working, angular magnification and magnifying power.
- 12 What is a compound microscope? Using ray diagram for a compound microscope, mention its magnifying power.
- 13 What is a telescope? Using a ray diagram explain its working, angular magnification and magnifying power.

NUMERICAL QUESTIONS

- 1 A 1.50-cm-high object is placed 20.0 cm from a concave mirror with radius of curvature 30.0 cm. Determine (a) the position of the image, and (b) its size, also draw the ray diagrams.
- 2 A candle of height 8.0 cm is located at a distance of 300 mm from a convex mirror, its virtual image is formed behind the mirror at a distance of 3.0 cm from the pole (or vertex). Find the focal length of the mirror and height of the image formed.
- 3 Calculate the speed of light in zircon with index of refraction $n = 1.923$, a material used in jewelry to replicate diamond.

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- 4 A light ray strikes an air/water surface at an angle of 46° with respect to the normal. The refractive index for water is 1.33. Find the angle of refraction when the direction of the ray is (a) from air to water and (b) from water to air.
- 5 An optical fiber is made from flint glass with index of refraction 1.66 and is surrounded by a cladding made of crown glass with index of refraction 1.52. What is the critical angle?
- 6 Suppose the book page is held 7.50 cm from a Concave lens of focal length 10.0 cm and concave lens of focal length -10 cm. What magnification is produced in each case?
- 7 Gulalai is viewing a flea using a magnifier with $f = 3.0$ cm. If her near point is at $N = 25$ cm then calculate the maximum magnification she can get.
- 8 A telescope has a magnification of 40.0 and a length of 1230 mm. What are the focal lengths of the objective and eyepiece?



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