

TEST EDITION



THE TEXTBOOK OF

MATHEMATICS

For Class

XII

SINDH TEXTBOOK BOARD, JAMSHORO

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PREFACE

It is with great pleasure and immense gratitude that we present to you the mathematics textbook for Class XII, developed under the guidance of the Sindh Textbook Board, Jamshoro. As an institution committed to shaping the educational landscape of the province, we take immense pride in offering a comprehensive and meticulously crafted resource that aligns with the dynamic curriculum set forth by the Directorate of Curriculum, Assessment, and Research (DCAR) in 2019.

This textbook is a collaborative effort that owes its existence to the dedication and expertise of countless individuals. We extend our sincere thanks to the authors, editors, and reviewers who tirelessly contributed their knowledge, insights, and time to create a textbook that not only imparts mathematical concepts but also fosters a love for learning and discovery.

The mathematics textbook for Class XII delves into the fascinating worlds of Calculus and Analytic Geometry. These subjects form the cornerstone of modern mathematics and have far-reaching applications in various fields. The content has been meticulously structured to provide students with a solid foundation in these areas, preparing them to tackle real-world challenges and pursue higher studies with confidence.

This textbook is designed with the utmost care to enhance students' understanding of mathematical concepts through clear explanations, illustrative examples, and thought-provoking exercises. It is our hope that this resource will serve as a valuable companion to both students and teachers, facilitating a deeper grasp of the subject matter and encouraging analytical thinking.

We believe that education is a continuous journey, and as such, we welcome your feedback and suggestions. Our commitment to improvement and innovation remains unwavering, and we encourage students and educators alike to share their thoughts and ideas. Together, we can create an educational ecosystem that empowers learners to excel and educators to inspire.

In closing, we extend our heartfelt wishes for your academic journey with this textbook. May it instill in you a passion for learning, a thirst for knowledge, and a profound appreciation for the beauty of mathematics.

Chairman



Contents

Sr. No	Unit Name	Page #
Unit 1	Introduction to Symbolic Package: MAPLE	1
Unit 2	Functions and Limits	20
Unit 3	Differentiation	58
Unit 4	Higher Order Derivatives and Applications	103
Unit 5	Differentiation of Vector Functions	133
Unit 6	Integration	148
Unit 7	Plane Analytic Geometry: Straight Line	192
Unit 8	Circle	241
Unit 9	Parabola, Ellipse and Hyperbola	274
Unit 10	Differential Equations	351
Unit 11	Partial Differentiation	375
Unit 12	Introduction To Numerical Methods	388
	Answers	409



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Unit

1

Introduction to Symbolic Package: MAPLE

1.1 Introduction

In the discipline of mathematics, the powerful software package Maple is widely utilized. Symbolic and numeric operations, data visualization, and the development of interactive applications are all possible inside this framework. Due to its extensive feature set, Maple, a program by Maplesoft, is excellent for learning and exploring many areas of mathematics.

Maple's capacity for symbolic computation is one of its main advantages. It enables the symbolic manipulation of mathematical expressions and equations, allowing the user to perform algebraic operations on variables and functions rather than simply entering numbers. Since it can simplify expressions, solve equations, and execute operations of calculus, Maple is a useful tool for many mathematical tasks.

Maple also allows for numerical computations, so you can use it to get close to an answer by plugging in numbers. Matrix calculations, numerical integration, and differential equations are only a few of the advanced mathematical procedures that it can handle. Mathematical problems can be investigated from several angles when symbolic and numerical computations are combined.

Maple also has many visualization tools that can be used to generate charts, graphs, and interactive representations of mathematical topics. This graphical depiction can help you better grasp mathematical concepts and better communicate your findings to others.

Maple provides a nice interface and a versatile programming language that facilitates the development of unique algorithms and functions. Maple is a useful material in some situations. It can be used for mathematical research, problem-solving, and conceptualization. Maple can be used to gain a deeper understanding of mathematics and develop problem-solving skills.

1.1.1 Reconfigure MAPLE Environment

The main interface of Maple software is shown in Fig. 1.1. Maple session is started by double clicking on the Maple icon which will start MAPLE package.



Fig. 1.1 Main interface of Maple software

you can see the starting page containing points of important resources for new users as follows:

User workspace:

User workspace forms the largest part of the Maple interface and serves as an area where the user can insert objects, evaluate expressions or write input commands and code. There are two basic modes for Maple workspace:

- Document mode and
- Worksheet mode

These two basic modes have their advantages and disadvantages.

Document Mode:

The document mode in Maple offers quick problem-solving rich content composition and allows the user to enter text and expressions on the same line. Expressions can be evaluated without the need of typing a specific command which guarantees better readability. Details about functions and key commands are available in Maple Help under Create New Document → Create and Open Document → Work in Document Mode.

Worksheet Mode:

Visually, worksheet mode can be distinguished from document mode by a symbol ">" located at the start of each line signifying a prompt. To evaluate expressions the user needs to



enter specific commands, which, while hindering coherence, offers advanced functionality and customized control. In worksheet mode, further assistance may be located in Create Maple Worksheets → Create and Open Worksheets → Work in Worksheet Mode.

Document Menu Bar:

The document menu bar is located at the uppermost part of the interface and allows the user to access almost all functions of the system. It contains both functions essential for working with the document itself as well as features and tools for creating and manipulating objects, expressions etc., within the document. The full description of all the menu items can be found in the Maple help system under Getting Started → Menus → Document Menu Bar. The document menu bar is shown in Fig. 1.2.



File Edit View Insert Format Evaluate Tools Window Help

Fig. 1.2 The Document Menu Bar of Maple software

Worksheet Toolbar:

The worksheet toolbar is a part of the Maple interface that serves as a location for commonly performed tasks. The settings allow it to be either hidden or shown. A search box is located at the end allowing the user to browse Maple resources to a great extent. Features such as tasks, tutors and help for specific topics can be accessed in this way. The worksheet Toolbar is shown in Fig. 1.3.

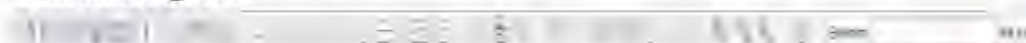


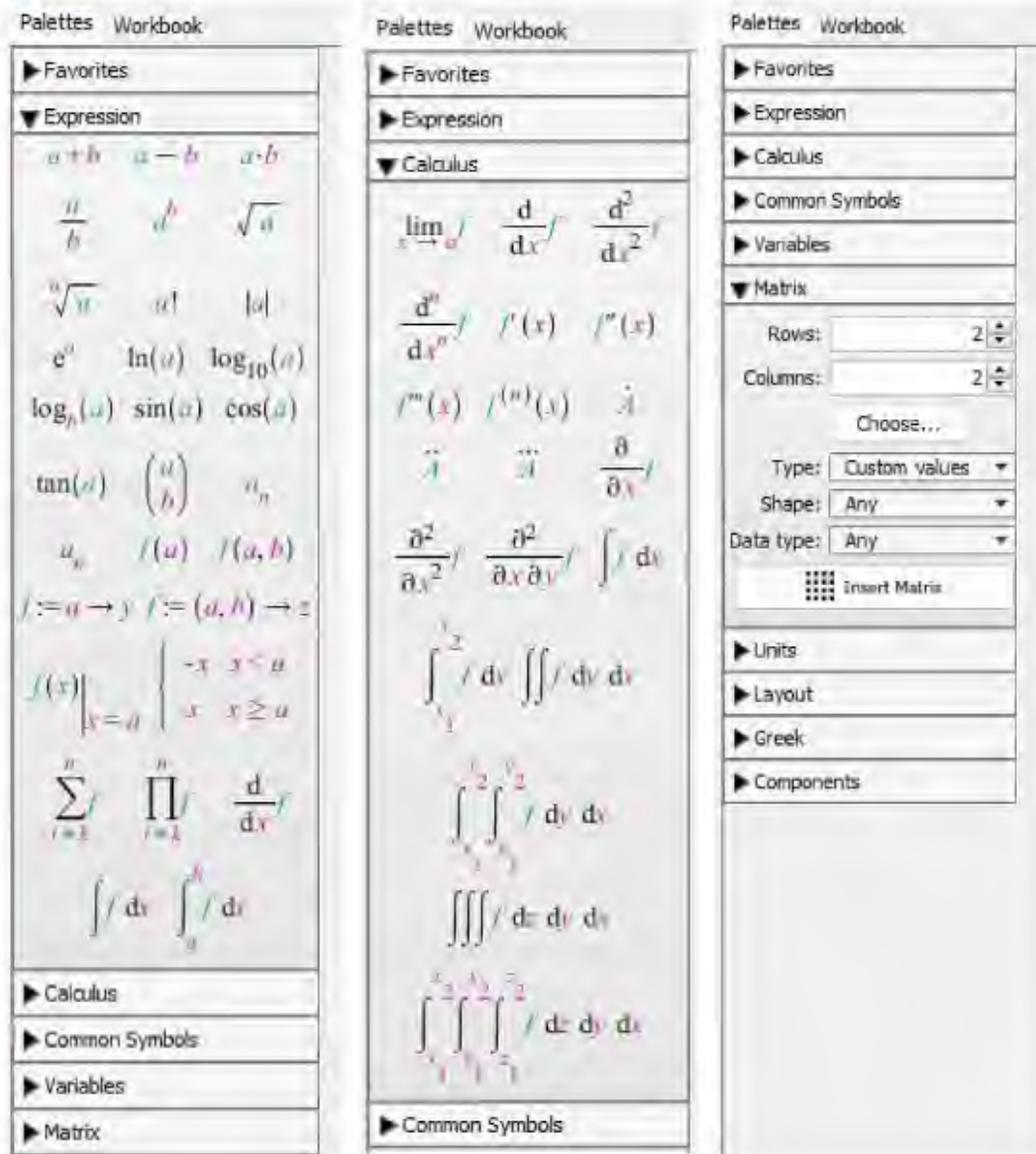
Fig. 1.3 The Worksheet Toolbar of Maple software

Toolbar Menus:

The toolbar menus can be found under the worksheet toolbar and are a set of menus whose function changes depending on the activity performed or required by the user. A total of five modes can be used, namely text, math, drawing, plot and animation. Each one serves a function that its name suggests, for example, in text mode the user is able to adjust font style, size or text alignment while in plot mode it is possible to modify plots. As with the document menu bar, further description of this part of the interface can be found in the Maple Help system, this time under Getting Started → Menus → Toolbar Menus (Fig. 1.4) shows the Toolbar Menus of Maple software.



Fig. 1.4 The Toolbar Menus of Maple software

Palettes:**Fig. 1.5 Palettes in Maple**

Palettes are a part of the (Graphical User Interface) that is located on its left side and can be described as collections of related items. These items can be inserted into the document by either clicking or dragging-and-dropping. The items include expressions, matrices, symbols and others. The Palettes of Maple software are shown in Fig. 1.5.



Context menus:

Maple context menus improve the user experience and provide quick access to a wide range of capabilities. Context menus change depending on what the user interacts with. Right-clicking on a graph, equation, or variable opens a context menu with related commands and operations. These menus provide shortcuts to basic functions, making mathematical calculations and data analysis easy. Maple's context menus make it easy to adjust plot parameters, calculate, and use specialist tools. Fig. 1.6. shows the Maple's context Menus.

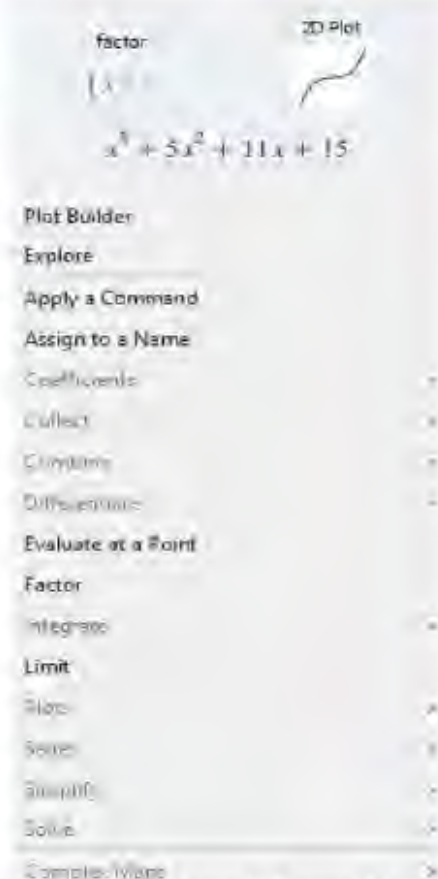


Fig. 1.6 The Maple's Context Menu

Note:

- (i) Every command in Maple must end with a semicolon. The semicolon assures that the computer will execute your instruction. To execute your instructions, you must press the Enter key.
- (ii) Maple is case sensitive. For example, x is not the same as $\times$ because Maple distinguishes between upper case and lower-case letters, and it will take them as being two different variables.
- (iii) Always type Maple commands in lower case, unless your instructor tells you otherwise. In such cases be very careful to type everything exactly as your instructor tells you.

Finally, if you make a mistake, do not continue typing on the next line. Go back, fix it! Maple behaves like a text editor and you can always go back to fix mistakes or to make changes by moving the mouse and clicking over the place where you would like to make the change.



You can also use the arrow keys to move the cursor around. Once you make the change, press the Enter key to re-execute the command (if needed). It is not necessary to move the cursor to the end of the line.

1.1.2 Recognize basic MAPLE command

Basic Maple Commands:

The prompt is the symbol $>$. Maple commands are entered to the right of the prompt. Each command ends with either ":" or ";". If the colon is used, the command is executed but the output is not printed. When the semicolon is used, the output is printed. List of some of the most useful commands in Maple are as follows.

Command Execution	Result
Using [ENTER] Type "1 + 2 [ENTER]".	3
Using [Alt]+[ENTER] Type "x + 5 - 2" then [Alt]+ [ENTER].	$x + 3$

Command Prompt $>$	Description
$> \text{restart}$	Resets all variables and unload all Packages
$> \text{eval}$	Evaluates an Expression
$> \text{evalf}(\text{Pi}, 5)$	Evaluates π to 5 digits
$\text{subs}(x = 2 * x + y, x^2)$	Substitutes $x = 2 * x + y$ into x^2 to give $(2 + x * y)^2 = 4x^2y^2$
$\text{diff}(5 * x * y, x)$	Differentiates with respect to x
$\text{diff}(5 * x * y, x\$2)$	Differentiates with respect to x twice
$\int \sin(x) dx$	Integrates with respect to x
$\left[> \int_0^{\infty} \frac{\sin(x)}{x} dx \right]$	Integrates from 0 to infinity to give $\frac{\pi}{2}$
$\text{simplify}(x * y + 2 * x * y - 3 * x)$	Simplifies the expression (this is really useful)
$\text{expand}((x + 1)(x + 2))$	Expands a factored expression
$\text{factor}(x^2 + 2x + 1)$	Factors an expression
$\text{exp}(\text{Pi} * I)$	The exponential function $e^{\pi i}$
$\text{solve}(5 * x + x * y^2 = 3, x)$	Solves the equation for x
$> \text{solve}(\{\text{eq1}, \text{eq2}, \text{eq3}\}, [x, y, z])$	Solves a system of 3 equations for x, y , and z variables



1.1.3 Use MAPLE as a Calculator



Fig. 1.7 Using Maple software as a Calculator

Maple Calculator App can be started with double click on the icon , get that calculator interface through which you can easily solve many mathematical problems regarding arithmetic, geometry, graphics and calculus etc. The Maple's calculator interface is shown in Fig. 1.7.

Maple Calculator uses the world-renowned Maple mathematics engine, so it can solve many kinds of problems, including:

- ❖ **Basic Math:** Arithmetic, fractions, decimals, integers, factors, square roots, powers and many more.
- ❖ **Algebra:** Solving and graphing linear equations, solving and graphing systems of equations, working with polynomials, quadratic equations and functions, logarithmic and exponential functions, trigonometric functions, trigonometric identities and many more.
- ❖ **Precalculus:** Graphing, piecewise functions, absolute value, inequalities, implicit functions and many more.
- ❖ **Calculus:** Derivatives, limits, definite and indefinite integrals.
- ❖ **Linear Algebra:** Matrix multiplication, inverse, determinant, transpose, Gaussian elimination and many more.
- ❖ **Differential Equations:** Ordinary differential equations and a lot more.

Command Calculator	Result
$> 2 * \left(\frac{1}{2} + \frac{3}{4}\right);$	$\frac{5}{2}$
$> \sin\left(30 * \frac{\pi}{180}\right);$	$\frac{1}{2}$
$> \ln(2.718);$	0.9998963157
$> \int \frac{1}{x} dx;$	$\ln(x)$



1.1.4 Use online MAPLE help

Maple Help Resources

Resources	Description
Maple Tour	An interactive overview of Maple is given by Maple Tour. From the Help menu, select Take a Tour of Maple.
Online Manual	The Maple Online Manual is an extensive and comprehensive documentation resource that provides detailed information about how to use Maple effectively. You can execute examples, copy content into other documents, and search the contents using the Maple Help System. The Maple Getting Started Guide provides extensive information for new users on using Maple and the resources available on the Maplesoft Website (http://www.maplesoft.com). Each procedure and concept is accompanied by visual depictions to help you identify Maple resources and tools. From the Help menu, select Manuals, Dictionary, and more>Manuals.
Quick Help	A list of key commands and concepts are given in Quick Help. From the Help menu, select Quick Help. Alternatively, press F1. For additional information, click an item in the Quick Help.
Quick Reference	This is a table of commands and information for new users that opens in a new window. It contains hyperlinks to help pages for more information. From the Help menu, select Quick Reference. Alternatively, press Ctrl + F2.
Help Page	Help for Maple features, commands, packages, and more can be found on Help Page. Help pages include examples and screenshots to help you quickly learn. From the Help menu, select Maple Help. You can search for a help topic, perform a text search, or browse the Table of Contents. You can also open a help page by entering? at the input prompt (in Worksheet mode) or in Math mode (in Document mode).
Task Templates	This is a set of commands with placeholders that you can use to quickly perform a task. From the Tools menu, select Tasks, and then Browse.
Applications and Example Worksheets	These are executable documents that demonstrate the power and flexibility of the Maple interactive document or provide an overview of computations in a particular field. From the Help menu, select Manuals, Dictionary, and more, and then Applications and Examples.
Mathematics and Engineering Dictionary	Over 5000 definitions, including 300 figures and plots can be found. From the Help menu, select Manuals, Dictionary, and more, and then Dictionary.



1.2 Polynomials

A Maple polynomial is an expression in powers of an unknown. Univariate polynomials are polynomials in one unknown. For example, $x^3 - 2x + 13$.

Multivariate polynomials are polynomials in multiple unknowns, such as,

$$x^2y - \frac{3}{2}xy^2 + 7x$$

The coefficients can be integers, rational numbers, irrational numbers, floating-point numbers, complex numbers, variables, or a combination of these types. One such example is given below:

$$> a^2 + 7 * x - \frac{b}{2};$$

$$a^2 + 7x - \frac{1}{2}b$$

1.2.1 Factoring a polynomial

To factor a polynomial using Maple software, you can use the '**factor**' command. Given below are some examples for how to factor an algebraic expression in Maple:

Example 1:

$$\left[\begin{array}{l} > \text{factor}(6x^2 + 18x - 24); \\ 6(x + 4)(x - 1) \end{array} \right] \quad \dots(\text{i})$$

Example 2:

$$\left[\begin{array}{l} > \text{factor}(x^3 - 8x^2 + 17x - 10); \\ (x - 5)(x - 1)(x - 2) \end{array} \right] \quad \dots(\text{ii})$$

Example 3:

$$\left[\begin{array}{l} > \text{factor}\left(\frac{1}{x^2-1} + \frac{1}{x^2+3x+2}\right); \\ \frac{2x+1}{(x+2)(x+1)(x-1)} \end{array} \right] \quad \dots(\text{iii})$$

Example 4:

$$\left[\begin{array}{l} > \text{factor}(x^3 - y^3); \\ (x - y)(x^2 + xy + y^2) \end{array} \right] \quad \dots(\text{iv})$$

Example 5:

$$\left[\begin{array}{l} > \text{factor}\left(\frac{x^3-y^3}{x^4-y^4}\right); \\ \frac{x^2+xy+y^2}{(x+y)(x^2+y^2)} \end{array} \right] \quad \dots(\text{v})$$

Note:

- Never put space in between command and expression.
- *ifactor* command means integer factorization.

1.2.2 Expanding an expression

The **'expand'** command distributes products over sums. This is done for all polynomials. For quotients of polynomials, only sums in the numerator are expanded; products and powers are left alone. See the **normal** command for dealing with quotients of polynomials.

Example 1:

$$\left[\begin{array}{l} > \text{expand}((x+1) * (x+2)); \\ x^2 + 3x + 2 \end{array} \right]$$

Example 2:

$$\left[\begin{array}{l} > \text{expand}\left(\frac{x+1}{x+2}\right); \\ \frac{x}{x+2} + \frac{1}{x+2} \end{array} \right]$$

1.2.3 Simplifying an expression

To simplify an expression: Use the **'simplify'** command. The **'simplify'** command applies simplification rules to an expression. Maple has simplification rules for various types of expressions and forms, including trigonometric functions, radicals, logarithmic functions, exponential functions, powers, and various special functions. You can also specify custom simplification rules using a set of side relations. Look at the following structures:

```
simplify (expr, n1, n2, ..., opt);
simplify (expr, side1, side2, ..., opt);
simplify (expr, assume=prop, opt);
simplify (expr, size, evaluate_known_functions = eval_boolean);
simplify (expr, symbolic, opt);
```

Example 1:

$$\left[\begin{array}{l} > \text{simplify}\left(4\frac{1}{2} + 3\right); \\ 5 \end{array} \right]$$

Example 2:

$$\left[\begin{array}{l} > \text{simplify}(e^{a+\ln(be^c)}); \\ be^{a+c} \end{array} \right]$$

Note: In Example 2, the symbol 'e' must be taken from palettes of Maple software.

1.2.4 Simplifying a rational expression

To simplify a rational expression using Maple software, you can use the **simplify** command. Here's an example of how to simplify a rational expression in Maple:

Open Maple software and create a new document or worksheet.

Define your rational expression. For example, let's say we have the expression $\text{expr} = (2x^2 + 4x + 2) / (4x)$. You can define it in Maple using the following syntax:



```
expr := (2*x^2 + 4*x + 2) / (4*x);
```

Use the **simplify** command to simplify the expression. Enter `simplify(expr)` and execute the command. Maple will return the simplified form of the rational expression.

Here is the complete code snippet:

```
expr := (2*x^2 + 4*x + 2) / (4*x);
simplify(expr);
```

When you run this code in Maple, it will output the simplified form of the rational expression as shown below:

$$\frac{(x+1)^2}{2x}$$

Maple will apply various simplification techniques, such as factoring, canceling common factors, expanding and combining like terms, to simplify the expression as much as possible. If the expression cannot be simplified further, Maple will return the input expression as the output.

Remember to substitute your own rational expression for `expr` in the code above to simplify a different expression.

$$\left[\begin{array}{l} > \text{simplify} \left(\frac{x^4 - y^4}{x^2 - y^2} \right); \\ x^2 + y^2 \end{array} \right] \quad \dots(i)$$

1.2.5 Substituting into an expression

In Maple one would use the '**subs**' command to substitute a variable by a value in an expression. In Maple, '**subs**' takes two arguments: an equality `>` (variable=substitution value, the expression).

$$\left[\begin{array}{l} > \text{subs}(x = 2, \frac{x^2 + x + 1}{7}); \end{array} \right] \quad \dots(i)$$

$$\left[\begin{array}{l} > \text{subs}(x = y, \frac{4 * x + 3 * y}{7y}); \end{array} \right] \quad \dots(ii)$$

$$\left[\begin{array}{l} > \text{subs}(z = x + y, \frac{2 * z + x + y}{3x + 3y}); \end{array} \right] \quad \dots(iii)$$

1.3 Graphics

We can use context panel to create and modify plots such as graph of curves, and surfaces.

• Create a Plot:

To create a plot of a curve or surface, Maple uses defaults for initial range, style and orientation. From the displayed context panel, select Plots and then a plot type of your choice. The context panel shows context-sensitive operations you can apply to the expression.

- **Modify a Plot:**

To modify the attributes of a plot, an individual curve, or a surface; use context panel and select the attribute and value you want. You can change the plot range, style and other attributes. Alternatively, use the plot menu from the main menu at the top of the worksheet.

- **Plot and Curve Settings:**

Curves and other elements in a plot object can have attributes specified for them such as color, symbol, and font. The plot itself also has attributes which are used when the attribute has not been set for the individual curve. When you set an attribute from the plot menu or the context panel it applies to the individual curve selected, if there is one. If none is selected, it applies to the plot, setting the plot's default attribute.

1.3.1 Plot a two-dimensional graph

Two-Dimensional Plots

There are several types of two-dimensional plots in Maple. They provide for the plotting of the graph of a function of one variable, of a data set, of a parametrically defined curve, and of an implicitly defined curve. For example,

- To plot the graph of a function of one variable in the plane:

Maple Command: $> \text{plot}(f(x), x = a..b)$

For example:

$> \text{plot}(\sin(x), x = 0..Pi);$ [Press Enter]

The graph of $\sin(x)$ on the interval $[0, \pi]$ in Maple software is shown in Fig. 1.8.

- To plot the graphs of several functions of one variable on the same coordinate axes:

Maple Command: $> \text{plot}(\{f(x), g(x), \dots, h(x)\}, x = a..b)$

For example:

$> \text{plot}(\{x^2, x, x^2 + 7\}, x = -8..8);$ [Press Enter]

Figure 1.9. shows the graphs of

$f(x) = x^2, g(x) = x, h(x) = x^2 + 7$ in Maple software on the same coordinate axes.

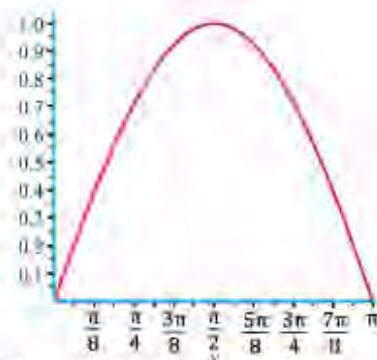


Fig. 1.8 The graph of $\sin(x)$ on the interval $[0, \pi]$ in Maple software

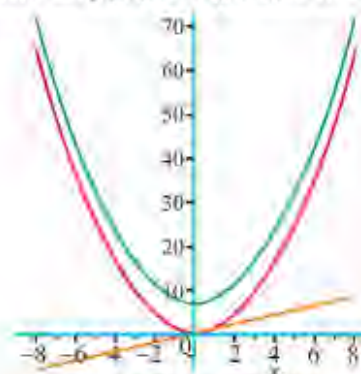


Fig. 1.9 The graphs of $f(x) = x^2, g(x) = x, h(x) = x^2 + 7$ in Maple software on the same coordinate axes



- Notice that the functions are separated by commas and the list is enclosed in braces.

For example:

`> plot({sin(x), cos(x)}, x = 0..Pi);` [Press Enter]

Look at Fig. 1.10 for the above example.

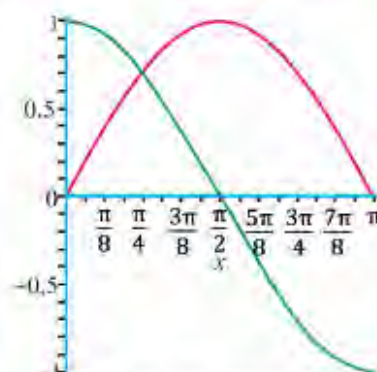


Fig. 1.10 The graphs of $f(x) = \sin(x)$, $g(x) = \cos(x)$ in Maple software on the same coordinate axes

1.3.2 Demonstrate domain and range of a plot

If no range is specified, Maple tries to determine a reasonable domain to plot the expression. For trigonometric plots, Maple will often use the range

$$-2\pi \dots 2\pi$$

For low degree polynomials, Maple ensures that the axis of symmetry of the graph is in the center of the plot.

Plot Multiple Functions on the Same Graph.

`> plot([ln(x), sqrt(x), exp(x)], x = -2..2, color = ["Red", "Green", "Blue"]);`

Note: The functions can also be enclosed in square brackets. Look at Fig. 1.11. for the above three graphs.

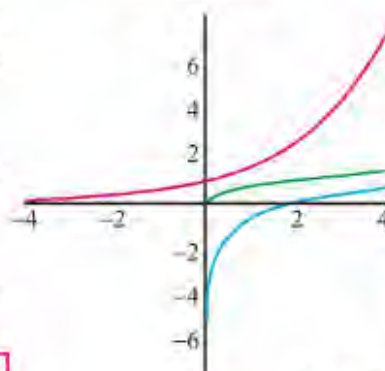


Fig. 1.11

- Plots Using Specified Range

`> plot(cos(2x), x = 0 .. 4*Pi);`

Look at Fig. 1.12 for the graph of $\cos(2x)$ from 0 to 4π .

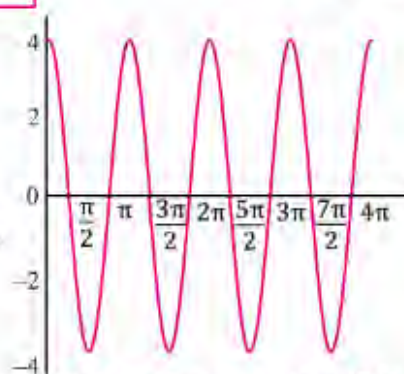


Fig. 1.12 The graph of $\cos(2x)$



```
>plot(cos(2x), x = 0 .. 2*Pi);
```

Look at Fig. 1.13 for the graph of $\cos(2x)$ from 0 to 2π .

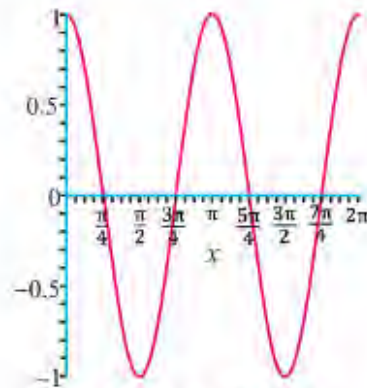


Fig. 1.13 The graph of $\cos(2x)$ from 0 to 2π

1.3.3 Sketch parametric equations

- To plot a curve defined parametrically by
`plot([f(t),g(t),t=a..b])`

For example:

```
> plot([sin(t), cos(t), t=0..2*Pi])
```

Look at Fig. 1.14 for the parametric curve of the above example.

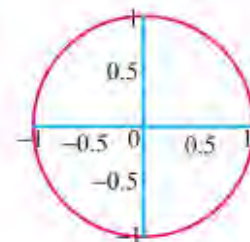


Fig. 1.14

- To plot **several** parametric curves on the same plot, pack several "square brackets" together inside of "curly braces":

```
>plot({[t,t^2,t=-2..2],[1+t,1+2*t,t=-1..1]})
```

Look at Fig. 1.15 for the above example.

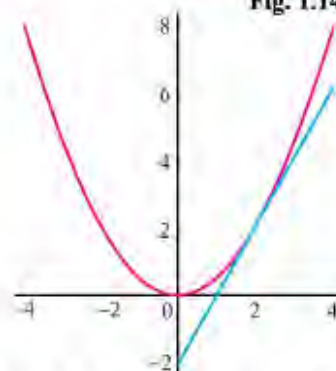


Fig. 1.15 Several parametric curves on the same plot

1.3.4 Know plotting options

There are many options that can be applied to the plot command, and other plots accept most of those options. They include grid display options (gridlines, axes, captions, and more), plot generation options (adaptive point sampling and discontinuity detection), and plot display



options (colors, line and fill styles, transparency, etc.)

Most of the options that can be applied to a plot command can also be applied interactively to an existing plot, by using the plot context menus.

The plot tools package provides commands to generate basic graphical objects and alter existing plot structures. Objects include geometric shapes, arrows, and points, and can be displayed either in existing plots or on separate axes. Transformations involve rotation, translation, and scaling of any type of plot. Also available is text plot, for adding text or 2-D math to an existing plot (or `textplot3d` for 3-D plots). You can also add text, 2-D math, various shapes, or free form drawing to a 2-D plot using a set of drawing tools.

1.4 Matrices

There are four ways to construct a Matrix in Maple. Which method you use depends on your data and needs. The following are some guidelines on when to use which method:

1. Use the Matrix construction shortcuts to quickly construct a Matrix with a small number of elements.
 2. Use the Matrix palette to specify the initial type and shape for the Matrix as well as its data type.
 3. Use the Matrix function to access more initialization options (for example, using a procedure or lists as initializers) and options that maximize the efficiency of reading from the Matrix, storing the Matrix, or both.
 4. Use the Import Matrix function to import data stored in a file into a Matrix.
- Use a pair of matching angle brackets ($\langle \rangle$) to enclose the comma-separated values of the Matrix elements.
 - To have sequences of comma-separated values, define the rows of your Matrix, separate the rows with a semicolon (;).

`Matrix1 := < a , b , c ; d , e , f >`

1.4.1 Regular matrix and vector entry arrangement

Creating Matrices and Vectors:

Creating Matrices:

You can create a Matrix using:

- ❖ The Matrix command
- ❖ The angle bracket shortcut notation
- ❖ The Matrix palette (see Figure 1.16).

When creating a Matrix using the Matrix command, there are several input formats available. For example

A 2×2 Matrix with all elements to be zero:



$$\begin{bmatrix} > \text{Matrix}(2); \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \quad \dots(i)$$

A 2×3 Matrix with all zero elements:

$$\begin{bmatrix} > \text{Matrix}(2,3); \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \quad \dots(ii)$$

A 3×3 Matrix with shape of identity:

$$\begin{bmatrix} > \text{Matrix}(3, \text{shape} = \text{identity}); \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{bmatrix} \quad \dots(iii)$$

A 3×3 Matrix with all entries of 5:

$$\begin{bmatrix} > \text{Matrix}(1..3, 1..3, 5); \\ \begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{bmatrix} \end{bmatrix} \quad \dots(iv)$$

Specify all element as a list of lists that define the element values row-wise. Lists are enclosed in square brackets and commas are used to separate elements of the list. For example, to create a 2×3 matrix with 1 2 3 in the first row and 4 5 6 in the second row, we will need a list of two lists — the two lists are the first row [1,2,3] and the second row [4,5,6] which are then put in a list, i.e., inside square brackets.

Arrangement of Matrix with set of entries:

$$\begin{bmatrix} > \text{Matrix}([[1,2,3], [4,5,6]]); \\ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \end{bmatrix} \quad \dots(v)$$

Alternatively, use the angle bracket shortcut, $< >$. Separate items in a column with commas, and separate columns with vertical bars, |.

$$\begin{bmatrix} > < 1,2,3 | 4,5,6 | 7,8,9 >; \\ \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \end{bmatrix} \quad \dots(vi)$$

Use the Matrix palette (Fig. 1.16.) to interactively create a matrix without commands:

Palette

In the Matrix palette, you can specify the matrix size (see Figure 1.17.) and properties. To insert a matrix, click the Insert Matrix button.

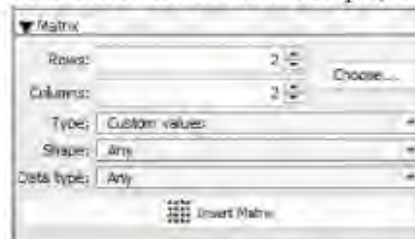


Fig 1.16. Matrix Palettes

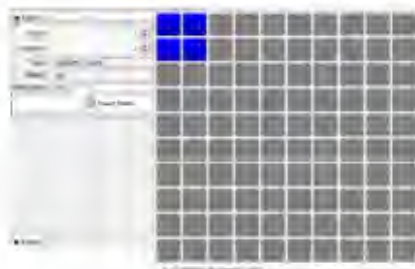


Fig 1.17

Matrix Palettes: Choosing the Size



Creating Vectors

You can create a Vector using angle brackets ($\langle \rangle$).

To create a column vector, specify a comma-delimited sequence, $\langle a, b, c \rangle$. The number of elements is inferred from the number of expressions.

$$\begin{aligned} & \text{[> } \langle 1, 2, 3 \rangle \text{ ;} \\ & \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{aligned} \quad \dots(i)$$

To create a row vector, specify a vertical-bar-delimited ($|$) sequence, $\langle a|b|c \rangle$. The number of elements is inferred from the number of expressions.

$$\begin{aligned} & \text{[> } \langle 1|2|3 \rangle \text{ ;} \\ & \quad [1 \ 2 \ 3] \end{aligned} \quad \dots(ii)$$

For information on the Vector command options, refer to the Vector help page.

You can also create vectors using the Matrix palette. If either the number of rows or number of columns specified is 1, then you have the option of inserting a matrix, or inserting a vector of the appropriate type. See Figure 1.18. to insert a vector using the Palettes.



Fig 1.18 Insert a Vector

To define a column vector using the Vector constructor, specify:

- ❖ The number of elements. If you explicitly specify all element values, this argument is not required.
- ❖ A list of expressions that define the element values.
- ❖ Parameters such as shape, datatype, and fill that set properties of the vector.

The following two calling sequences are equivalent:

$$\begin{aligned} & \text{[> } \text{Vector}([0, 0, 0]) \\ & \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \quad \dots(i)$$

$$\begin{aligned} & \text{[> } \text{Vector}(3, \text{shape} = \text{'zero'}) \text{ ;} \\ & \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned} \quad \dots(ii)$$

To create a row vector using the Vector constructor, include row as an index.

$$\begin{aligned} & \text{[> } \text{Vector}[\text{row}](3, \text{'fill'} = 1) \text{ ;} \\ & \quad [1 \ 1 \ 1] \end{aligned} \quad \dots(iii)$$



1.4.2 Apply matrix operations

Command	Description
Matrix Addition: $A := \text{Matrix}([[1, 2, 3], [4, 5, 6]]);$ $A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $B := \text{Matrix}([[1, 3, 6], [2, 1, 5]]);$ $B := \begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 5 \end{bmatrix}$ $A + B;$ $\begin{bmatrix} 2 & 5 & 9 \\ 6 & 6 & 11 \end{bmatrix}$	Take two matrices A and B of order 2×3 and apply maple command for their addition and subtraction and get the result as $A + B$ $A - B$ Similarly multiply Matrices A and B, which are multipliable as per their orders, that is, number of columns of A should be same as that of number of rows in B.
Matrix Subtraction: $\begin{bmatrix} > A := \text{Matrix}([[1, 2, 3], [4, 5, 6]]); \\ A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \\ > B := \text{Matrix}([[1, 3, 6], [2, 1, 5]]); \\ B := \begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 5 \end{bmatrix} \\ > A - B; \\ \begin{bmatrix} 0 & -1 & -3 \\ 2 & 4 & 1 \end{bmatrix} \end{bmatrix}$	

1.4.3 Compute inverse and transpose of a matrix

In Context Menu, by selecting “standard operation” option, we have both **Inverse** and **Transpose** commands of Matrix A that can be used as follows:

$A := \text{Matrix}(3, [1, 2, 3, 3, 4, 0, 3, 2, 1]);$

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix} \quad \dots (i)$$

Inverse
→

$$A := \begin{bmatrix} -\frac{1}{5} & -\frac{1}{5} & \frac{3}{5} \\ \frac{3}{20} & \frac{2}{5} & -\frac{9}{20} \\ \frac{3}{10} & -\frac{1}{5} & \frac{1}{10} \end{bmatrix} \quad \dots (ii)$$

$A := \text{Matrix}(3, [1, 2, 3, 3, 4, 0, 3, 2, 1]);$

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 0 \\ 3 & 2 & 1 \end{bmatrix} \quad \dots (i)$$

Transpose

→

$$\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 2 \\ 3 & 0 & 1 \end{bmatrix}$$

... (ii)

Exercise 1

- Expand the following expressions using the "**expand**" command in Maple.
 - $(x + y)^3$
 - $\cos(x + y) + \sin(x - y)$
 - $(2a - b)^2 + (a + 3b)^2$
 - $e^x(x + y) - e^x(x - y)$
 - $(x + 2)(x - 3)$
 - $\text{sqrt}(x + y)$
- Use the "**factor**" command in Maple to factorize the following expressions and write the factored form of the expressions:
 - $x^3 - 8$
 - $x^4 - y^4$
 - $x^2 - 4$
 - $3x^3 + 12x^2 - 15x$
 - $2x^2 + 6xy + 4y^2$
 - $x^2 + 4x + 4$
- Simplify the following expressions using the "**simplify**" command in Maple and write the simplified form of the expressions:
 - $\frac{(x^2 + 2x + 1)}{(x + 1)}$
 - $\frac{(x^3 - x^2 - x + 1)}{(x^2 + 1)}$
 - $\sin(x)^2 + \cos(x)^2$
 - $\frac{(3x^2 + 2x - 1)}{(x^2 + 1)}$
- Evaluate the numerical value of $\sin \frac{\pi}{6}$ using the **evalf** command in Maple.
- Find the inverse of the matrix $A := \text{Matrix}(2, [1, 2, 3, 4])$; using the **Matrix Inverse** command in Maple.
- Find the transpose of the matrix $B := \text{Matrix}(2, 3, [1, 2, 3, 4, 5, 6])$; using the **Transpose** command in Maple.
- Perform matrix multiplication for matrices $A := \text{Matrix}(2, [1, 2, 3, 4])$; $B := \text{Matrix}(2, [5, 6, 7, 8])$ using the command in Maple.
- Plot the functions $f(x) = x^2$ and $g(x) = 2x - 1$ on the same graph in the range $[-5, 5]$.
- Generate a graph that includes the functions $h(x) = \sin(x)$ and $k(x) = \cos(x)$ in the interval $[0, 2\pi]$.
- Plot the parametric curves $x(t) = \cos(2t)$ and $y(t) = \sin(3t)$ for t in the range $[-2\pi, 2\pi]$.

Note: Do not forget to type **with(LinearAlgebra):** in Maple software before executing the commands in Q. No. 05, Q. No. 06, and Q. No. 07.