

EXERCISE NO. 5

-:5.1:-

- (i) $11! = 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 39916800$
- (ii) $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3628800$
- (iii) $\frac{12!}{8!0!} = \frac{12 \times 11 \times 10 \times 9 \times 8!}{8! \times 0!} = 12 \times 11 \times 10 \times 9 = 11880$
- (iv) $\frac{12!8!}{9!6!} = \frac{12 \times 11 \times 10 \times 9 \times 8! \times 6!}{9! \times 6!} = 12 \times 11 \times 10 \times 8 \times 7 = 73920$
- (v) $\frac{10!8!}{9!5!} = \frac{(10 \times 9!) \times (8 \times 7 \times 6 \times 5!)}{9! \times 5!} = 10 \times 8 \times 7 \times 6 = 3360$
- (vi) $\frac{13!4!}{5!2!} = \frac{13! \times 4 \times 3 \times 2 \times 1!}{15 \times 14 \times 13! \times 2 \times 1!} = \frac{4 \times 3}{15 \times 14} = \frac{2}{35}$
- (vii) $\frac{10!3!}{13!} = \frac{10! \times 3 \times 2 \times 1}{13 \times 12 \times 11 \times 10!} = \frac{3 \times 2 \times 1}{13 \times 12 \times 11} = \frac{6}{1716} = \frac{1}{286}$
- (viii) $\frac{13!}{10!3!} = \frac{13 \times 12 \times 11 \times 10!}{10! \times 3 \times 2 \times 1} = 286$

-:5.2:-

- (i) ${}^{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7!}{7!} = 10 \times 9 \times 8 = 720$
- (ii) ${}^9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{5!} = 9 \times 8 \times 7 \times 6 = 3024$
- (iii) ${}^{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{4!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!}$
 $= 10 \times 9 \times 8 \times 7 \times 6 \times 5 = 151200$
- (iv) ${}^8P_7 = \frac{8!}{(8-7)!} = \frac{8!}{1!} = 8 \times 7 \times 6 \times 5 \times 4 \times 2 \times 1! = 40320$
- (v) ${}^{25}P_5 = \frac{25!}{(25-5)!} = \frac{25!}{20!} = \frac{25 \times 24 \times 23 \times 22 \times 21 \times 20!}{20!}$
 $= 25 \times 24 \times 23 \times 22 \times 21 = 6375600$

-:5.3:-

The number of digits are 6. The numbers of 3 digits can be formed in the following ways:

$${}^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 6 \times 5 \times 4 = 120$$

-:5.4:-

The number of travelers are three and number of hotels are four. They can take their quarters each in the following ways:

$${}^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4 \times 3 \times 2 \times 1 = 24$$

-:5.5:-

The number of students = 4, The number of doors = 6, 4 students can enter in 6 doors in following different ways.

$${}^n P_r = {}^6 P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 6 \times 5 \times 4 \times 3 = 360$$

-:5.6:-

Here $n = 8, r = 4$

$${}^n P_r = {}^8 P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$$

-:5.7:-

The word Statistics contains 10 letters in all, of these the 3S's are alike of one kind, the 3T's are alike of one kind, 2I's are alike. There is one C and One A. Thus we have

Total No. $N = 10$

No. of S's = 3, No. of T's = 3, No. of C's = 1, No. of A = 1, No. of I = 2

Hence required number of arrangements are given below:

$$P = \frac{10!}{3!3!2!1!1!} = 75600$$

-:5.8:-

The word FAISALABAD contains 10 letters, we have

Total No. $n = 10$

No. of A's = 4, No. of B's = 1, No. of D's = 1, No. of S's = 1, No. of F's = 1,
No. of L's = 1, No. of I's = 1

Hence required number of arrangements are given below:

$$P = \frac{10!}{3!1!1!1!1!1!} = 151200$$

-:5.9:-

Here $n = 6$ Flags. Red Flags = 3, Green Flags = 2, Yellow Flags = 1
 Required number of arrangements are given

$$P = \frac{6!}{3!2!1!} = 60$$

-:5.10:-

$$nCr = \frac{n!}{r!(n-r)!}$$

(i) ${}^8C_5$ Here $n = 8, r = 5$

$${}^nC_r = {}^8C_5 = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \frac{8 \times 7 \times 6 \times 5!}{5 \times 3 \times 2 \times 1} = 8 \times 7 = 56$$

(ii) ${}^7C_3$ Here $n = 7, r = 3$

$${}^nC_r = {}^7C_3 = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$

(iii) ${}^{24}C_4$ Here $n = 24, r = 4$

$$\begin{aligned} {}^nC_r = {}^{24}C_4 &= \frac{24!}{4!(24-4)!} = \frac{24!}{4!20!} = \frac{24 \times 23 \times 22 \times 21 \times 20!}{4 \times 3 \times 2 \times 1 \times 20!} \\ &= 23 \times 22 \times 21 = 10626 \end{aligned}$$

(iv) ${}^{19}C_4$ Here $n = 19, r = 4$

$$\begin{aligned} {}^nC_r = {}^{19}C_4 &= \frac{19!}{4!(19-4)!} = \frac{19!}{4!15!} = \frac{19 \times 18 \times 17 \times 16 \times 15!}{4 \times 3 \times 2 \times 1 \times 15!} \\ &= 19 \times 6 \times 7 = 3876 \end{aligned}$$

(v) ${}^{52}C_{50}$ Here $n = 52, r = 50$

$$\begin{aligned} {}^nC_r = {}^{52}C_{50} &= \frac{52!}{50!(52-50)!} = \frac{52!}{50!2!} = \frac{52 \times 51 \times 50!}{50! \times 2 \times 1} \\ &= 1326 \end{aligned}$$

(vi) ${}^{2n}C_{r+1}$ Here $2n - r, r = r + 1$

$${}^nC_r = {}^{2n}C_{r+1} = \frac{2n!}{(r+1)!(2n-r-1)!}$$

-:5.11:-

Here $n = 8, r = 6$

$${}^nC_r = {}^8C_6 = \frac{8!}{6!(8-6)!} = \frac{8!}{6!2!} = \frac{8 \times 7 \times 6!}{6! \times 2 \times 1} = 28$$

-:5.12:-

Here $n = 12$, $r = 5$, 5 books including one specified book can be selected in the following ways:

$${}^{11}C_4 \times {}^1C_1 = \frac{11!}{4!(11-4)!} = \frac{11!}{4!7!} = \frac{11 \times 10 \times 9 \times 8 \times 7!}{4 \times 3 \times 2 \times 1 \times 7!} = 330 \text{ ways}$$

-:5.13:-

2 balls out of 4 white and 6 black balls can drawn in the following ways:

Here $n = 4 + 6 = 10$, $r = 2$

$${}^{10}C_2 = {}^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!} = 5 \times 9 = 45 \text{ ways}$$

-:5.14:-

The committee of 3 men and 2 women from 6 men and 4 women can be formed in the following ways:

$$\begin{aligned} {}^6C_3 \times {}^4C_2 &= \frac{6!}{3!(6-3)!} \times \frac{4!}{2!(4-2)!} = \frac{6!}{3!3!} \times \frac{4!}{2!2!} \\ &= \frac{6 \times 5 \times 4 \times 3!}{3 \times 2 \times 1 \times 3!} \times \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = (5 \times 4)(2 \times 3) \\ &= 20 \times 6 = 120 \text{ ways} \end{aligned}$$

-:5.15:-

There are 9 balls of which 5 are yellow and 4 black. A group of 3 yellow and 3 black balls from 9 balls can be drawn in the following ways:

$$\begin{aligned} {}^5C_3 \times {}^4C_3 &= \frac{5!}{3!(5-3)!} \times \frac{4!}{2!(4-2)!} = \frac{5!}{3!2!} \times \frac{4!}{2!2!} \\ &= \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{4 \times 3 \times 2!}{2 \times 1 \times 2!} = 10 \times 6 = 60 \text{ ways} \end{aligned}$$

-:5.16:-

Expand $(x + y)^7$

$$\begin{aligned} (x + y)^7 &= x^7 + {}^7C_1 x^6 y + {}^7C_2 x^5 y^2 + {}^7C_3 x^4 y^3 + {}^7C_4 x^3 y^4 + {}^7C_5 x^2 y^5 \\ &\quad + {}^7C_6 x y^6 + y^7 \\ &= x^7 + 7x^6 y + 21x^5 y^2 + 35x^4 y^3 + 35x^3 y^4 + 21x^2 y^5 + 7x y^6 + y^7 \end{aligned}$$

-:5.17:-

Expand $(x - 2y)^4$

$$(x-2y)^4 = x^4 + {}^4C_1 x^3(-2y) + {}^4C_2 x^2(-2y)^2 + {}^4C_3 x(-2y)^3 + (-2y)^4$$

$$= x^4 + 8x^3y + 24x^2y^2 - 32xy^3 + 16y^4$$

-:5.18:-

Expand $(x-3y)^5$

$$(x-3y)^5 = x^5 + {}^5C_1 x^4(-3y) + {}^5C_2 x^3(-3y)^2 + {}^5C_3 x^2(-3y)^3 + {}^5C_4 x(-3y)^4 + (-3y)^5$$

$$= x^5 - 15x^4y + 90x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5$$

-:5.19:-

Expand $(2x-y)^5$

$$(2x-y)^5 = (2x)^5 + {}^5C_1 (2x)^4(-y) + {}^5C_2 (2x)^3(-y)^2 + {}^5C_3 (2x)^2(-y)^3$$

$$+ {}^5C_4 (2x)(-y)^4 + (-y)^5$$

$$= 32x^5 + 5(16x^4)(-y) + 10(8x^3)(-y)^2 + 10(4x^2)(-y)^3 + 5(2x)(-y)^4 + (-y)^5$$

$$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$$

-:5.20:-

Expand $(1-3a^2)^6$

$$(1-3a^2)^6 = 1^6 + {}^6C_1 1^5(-3a^2) + {}^6C_2 1^4(-3a^2)^2 + {}^6C_3 1^3(-3a^2)^3 + {}^6C_4 1^2$$

$$(-3a^2)^4 + {}^6C_5 1(-3a^2)^5 + (-3a^2)^6$$

$$= 1 + 6(1)(-3a^2) + 15(1)(-3a^2)^2 + 20(1)(-3a^2)^3 + 15(1)(-3a^2)^4$$

$$+ 6(1)(-3a^2)^5 + (-3a^2)^6$$

$$= 1 - 18a^2 + 135a^4 - 540a^6 + 1215a^8 - 145a^{10} + 729a^{12}$$

-:5.21:-

Expand $\left(a - \frac{1}{a}\right)^6$

$$\left(a - \frac{1}{a}\right)^6 = a^6 + {}^6C_1 a^5\left(-\frac{1}{a}\right) + {}^6C_2 a^4\left(-\frac{1}{a}\right)^2 + {}^6C_3 a^3\left(-\frac{1}{a}\right)^3 + {}^6C_4 a^2\left(-\frac{1}{a}\right)^4$$

$$+ {}^6C_5 a\left(-\frac{1}{a}\right)^5 + \left(-\frac{1}{a}\right)^6$$

$$= a^6 + 6a^5\left(-\frac{1}{a}\right) + 15a^4\left(-\frac{1}{a}\right)^2 + 20a^3\left(-\frac{1}{a}\right)^3 + 15a^2\left(-\frac{1}{a}\right)^4$$

$$+ 6a\left(-\frac{1}{a}\right)^5 + \left(-\frac{1}{a}\right)^6$$

$$= a^4 - 6a^4 + 15a^2 - 20 + \frac{15}{a^2} - \frac{6}{a^4} + \frac{1}{a^6}$$

-:5.22:-

Find the 4th term of $(x - 5)^{13}$ Here $n = 13$, $r = 3$, $a = x$, $b = -5$

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

$$T_4 = T_{3+1} = {}^{13}C_3 x^{13-3} (-5)^3 = 286x^{10}(-125) = -35750x^{10}$$

-:5.23:-

Find the 5th term of $(a + 2x^3)^{17}$ Here $n = 17$, $r = 4$, $a = a$, $b = 2x^3$

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

$$T_5 = T_{4+1} = {}^{17}C_4 a^{17-4} (2x^3)^4 = 2380a^{13} (16x^{12}) = 38080a^{13} x^{12}$$

-:5.24:-

Find the middle term of $\left(\frac{a}{x} + \frac{x}{a}\right)^{10}$ Here $n = 10$

The total number of terms will be 11 and the middle term is 6th. Therefore,

$$T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{a}{x}\right)^{10-5} \left(\frac{x}{a}\right)^5 = 252 \left(\frac{a}{x}\right)^5 \left(\frac{x}{a}\right)^5 = 252 = \frac{a^5}{x^5} + \frac{x^5}{a^5} = 252$$

-:5.25:-

Find the 7th term in the expansion of $(x - 2)^{15}$ Here $n = 15$, $r = 6$, $a = x$, $b = -2$

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

$$T_7 = T_{6+1} = {}^{15}C_6 x^{15-6} (-2)^6 = 5005x^9(64) = 5005(64)x^9 = 320320x^9$$

-:5.26:-

$$\sqrt{1.07} = \sqrt{1 + 0.07} = (1 + 0.07)^{\frac{1}{2}}$$

$$= (1)^{\frac{1}{2}} + \left(\frac{1}{2}\right)(1)^{-\frac{1}{2}}(0.07) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(1)^{-\frac{3}{2}}(0.07)^2$$

$$+ \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(1)^{-\frac{5}{2}}(0.07)^3$$

$$= 1 + 0.035 - 0.0006125 + 0.000214 + \dots = 1.0356339$$

-:5.27:-

Find 23 correct to 3 places of decimals

$$\sqrt{23} = (25 - 2)^{\frac{1}{2}} = 5\left(1 - \frac{2}{25}\right)^{\frac{1}{2}} = 5(1 - 0.08)^{\frac{1}{2}}$$

$$5(1 - 0.08)^{\frac{1}{2}} = 5\left[1 + \frac{1}{2}(-0.08) + \frac{\frac{1}{2}(\frac{1}{2} - 1)}{2!}(-0.08)^2 + \right.$$

$$\left. \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{3!}(-0.08)^3 + \dots\right]$$

$$= 5[1 - 0.047864] = 5[0.952136] = 4.76068 = 4.76$$

∴5.28:-

Find 0.97 to 6 places of decimals

$$\sqrt{0.97} = \sqrt{1 - 0.03} = (1 - 0.03)^{\frac{1}{2}}$$

$$(1 - 0.03)^{\frac{1}{2}} = 1 + \frac{1}{2}(-0.03) + \frac{\frac{1}{2}(\frac{1}{2} - 1)}{2!}(-0.03)^2$$

$$+ \frac{\frac{1}{2}(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{3!}(-0.03)^3 + \dots$$

$$= 1 + \frac{1}{2}(-0.03) + \frac{\frac{1}{2}(-\frac{1}{2})}{2!}(-0.03)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{3!}(-0.03)^3 + \dots$$

$$= 1 - 0.015 - 0.0001125 - 0.0000016 + \dots = 1 - 0.151141$$

$$= 0.9848859 = 0.984886$$

∴5.29:-

Find $(1.2)^7$ correct to four decimals places.

$$(1.2)^7 = (1 + 0.2)^7$$

$$(1 + 0.2)^7 = (1)^7 + {}^7C_1(1)^6(0.2) + {}^7C_2(1)^5(0.2)^2 + {}^7C_3(1)^4(0.2)^3 + {}^7C_4(1)^3(0.2)^4$$

$$+ {}^7C_5(1)^2(0.2)^5 + {}^7C_6(1)(0.2)^6 + (0.2)^7$$

$$= 1 + 7(1)(0.2) + 21(1)(0.2)^2 + 35(1)(0.2)^3 + 35(1)(0.2)^4 + 21(1)(0.2)^5$$

$$+ 7(1)(0.2)^6 + (0.2)^7$$

$$= 1 + 7(0.2) + 21(0.04) + 35(0.008) + 35(0.0016) + 21(0.00032)$$

$$+ 7(0.00064) + 0.0000128$$

$$= 1 + 1.4 + 0.84 + 0.28 + 0.056 + 0.00672 + 0.000448 + 0.0000128$$

$$= 3.5831808 = 3.5832$$

∴5.45:-

(i) Sample Space = $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

Let A represent the event, an odd number appears in a single throw. $n(A) = 3$

$$P(\text{Having Odd No.}) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) $n(S) = 6$

Let A represent the event of an ace appears. $n(A)=1$

$$P(\text{Having an ace}) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

(iii) $n(S) = 6$

Let A represent the event, a five appears. $n(A)=1$

$$P(\text{Having a 5}) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

(iv) The two dice can be thrown in $6 \times 6 = 36$ ways. The number of sample points in sample space are $n(S) = 36$

Let A represent the event that sum of faces is 8.

$A = (6,2), (5,3), (4,4), (3,5), (2,6); n(A) = 5$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

(v) There are two possible outcomes, head and tail. $n(S)=2$

Let A represent the event that head occur. $n(A)=1$

$$P(\text{Head}) = P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$$

(vi) The sample space in three tosses of a fair coin is

$S = \{HHH, HHH, HTH, THH, HTT, THT, TTH, TTT\}$

$n(S) = 8$

Let A represent the event that at least one head appears.

$n(A)=7$

$$P(A) = \frac{n(A)}{n(S)} = \frac{7}{8}$$

(vii) Once card from 52 cards can be drawn 52 ways. $n(S)=52$

Let A represent the event of drawing a king, ace, jack of clubs or queen of diamonds. $n(A)=10$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{52} = \frac{5}{26}$$

(viii) $n(S)=6$

Let A represent the event, an even number appears. $n(A)=3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

-:5.46:-

(i) Let A represent the event that the card drawn is an ace. The number of ace cards is 4. $n(A)=4$

Hence

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

- (ii) Let B represent the event that the card drawn is heart. The number of outcomes favourable to the event B is 13, $n(B)=13$
Hence

$$P(B) = \frac{n(B)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

- (iii) Let C represent the event that card drawn is a red. The number of outcomes favourable to event C is 26. $n(C)=26$

$$P(C) = \frac{n(C)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

- (iv) Let D represent the event that the card drawn is a pictured card. The number of outcomes favourable to the event D is 16. $n(D)=16$

$$P(D) = \frac{n(D)}{n(S)} = \frac{16}{52} = \frac{3}{13}$$

- (v) Let E represent the event that the card drawn is a king of heart. The favourable case to the event E is one, $n(E)=1$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{52}$$

- (vi) Let F represent the event that card drawn is a black card. The number of outcomes favourable to the event F is 26, $n(F)=26$

$$P(F) = \frac{n(F)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

- (vii) Let H represent the event that the card drawn is a card of diamond. The number of outcomes favourable to the event H is 13, $n(H)=13$

$$P(H) = \frac{n(H)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

- (viii) Let G represent the event that card drawn is a face card. The number of outcomes favourable to event G is 16. $n(G)=16$

$$P(G) = \frac{n(G)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

:-5.47:-

- (a) When 2 dice are thrown, the number of various total number of dots are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
(b) The two dice can be thrown $6 \times 6 = 36$ ways. There are 36 sample points in sample space.
Let A represent the event that the sum of faces is 10 with 2 dice.
 $A = (6,4), (5,5), (4,6)$; $n(A)=3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$$

- (c) The two dice can be thrown $6 \times 6 = 36$ ways, $n(S) = 36$
 Let B represent the event that the sum of faces is 11 with 2 dice
 $B = (6, 5), (5, 6)$ $n(B) = 2$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$$

- (d) There are 36 sample points in sample space i.e. $n(S) = 36$
 Let A represent the event that a total of more than 7 occurs. Then the event A has 15 following outcomes.

$A = (2, 6), (3, 6), (4, 6), (5, 6), (6, 6), (3, 5), (4, 5), (5, 5), (6, 5), (4, 4), (5, 4), (6, 4), (5, 3), (6, 3), (6, 2)$

$n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{36}$$

Let B represent the event that total of less than 7 occurs. Then event B has 15 following outcomes.

$B = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)$

$n(B) = 15$

$$P(B) = \frac{n(B)}{n(S)} = \frac{15}{36}$$

Hence $P(A) = P(B)$

- (e) Two dice can be thrown in $6 \times 6 = 36$ $n(S) = 36$
 Let A represent the event that is sum shown is 8, and B represent the event that two show the same number then

$A = (6, 2), (5, 3), (4, 4), (3, 5), (2, 6)$

$n(A) = 5$ and

$B = (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)$

$n(B) = 6$

Hence

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

-:5.48:-

- (a) There are 35 students of which 15 are boys and 20 are girls. One student is chosen. The number of sample points in the sample space are $n(S) = {}^{35}C_1 = 35$

Let A represent the event of choosing a girls, $n(A) = {}^{20}C_1 = 20$

$$P(A) = \frac{n(A)}{n(S)} = \frac{20}{35} = \frac{4}{7}$$

- (b) The number of sample points in the sample space are

$$n(S) = {}^{52}C_1 = 52$$

Let A represent the event of drawing a spade card.

$$n(A) = {}^{13}C_1 = 13$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

- (c) There are 9 chips of which 5 are black and 4 are pink. One chip is drawn. The number of sample points in the sample space are

$$n(S) = {}^9C_1 = 9$$

- (i) Let A represent the event of drawing black chip.

$$n(A) = {}^5C_1 = 5$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{9}$$

- (ii) Let B represent the event of drawing pink chip

$$n(B) = {}^4C_1 = 4$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{9}$$

:-5.49:-The three dice can be thrown in $6 \times 6 \times 6 = 216$ ways.

- (a) Possible outcomes to get 7 with 3 dice

Outcomes	No of Outcomes
(1,1,5)	3
(1,2,4)	6
(1,3,3)	3
(2,2,3)	3
	15

Let A represent the event of getting 7 with 3 dice, $n(A) = 15$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{216} = 0.069$$

- (b) Let B represent event of 9 with 3 dice.

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
1	8	5
2	7	6
3	6	4
4	5	3
5	4	2
6	3	1
		25

$$n(B) = 25, \quad n(S) = 216$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{25}{216}$$

(c) Let C represent the event of getting 15 with 3 dice.

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
3	12	1
4	11	2
5	10	3
6	9	4
		10

$$P(C) = \frac{n(C)}{n(S)} = \frac{10}{216}$$

(d) Let D represent the event of getting 13 with 3 dice.

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
1	12	1
2	11	2
3	10	3
4	9	4
5	8	5
6	7	6
		21

$$n(D) = 21$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{21}{216}$$

(e) We know from (b) that

$$P(A) = \frac{25}{216}$$

Let B represent 11 with 3 dice

3rd Die	Sum of 1st & 2nd Die	No of Outcomes
1	10	3
2	9	4
3	8	5
4	7	6
5	6	5
6	5	4
		27

$$n(B) = 27$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{27}{216}$$

By using addition law of probability

$$P(9 \text{ or } 11) = P(A \text{ or } B) = P(A) + P(B)$$

$$= \frac{25}{216} + \frac{27}{216} = \frac{13}{54}$$

∴5.50:-

The two dice can be thrown $6 \times 6 = 36$ ways. $n(S) = 36$

- (i) Let A represent the event that the product of the number on dice is between 8 and 16.

$A = (4, 2), (3, 3), (2, 4), (5, 2), (4, 3), (3, 4), (2, 5), (6, 2), (5, 3), (4, 4), (3, 5), (2, 6)$

$$n(A) = 12$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

- (ii) Let B represent the event that the product of the numbers on the dice is divisible by 4.

$B = (4, 1), (1, 4), (4, 2), (2, 4), (2, 6), (6, 2), (4, 4), (2, 6), (4, 5), (4, 3), (2, 2), (3, 4), (5, 4), (6, 4), (4, 6), (6, 6)$

$$n(B) = 15$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{15}{36}$$

∴5.51:-

The number of sample points in the sample space are

$$n(S) = {}^{22}C_2 = 1326$$

Let A represent the event that one card is king and the other card is queen.

$$n(A) = {}^4C_1 \times {}^4C_1 = 4 \times 4 = 16 \text{ ways}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{16}{1326} = \frac{8}{663}$$

∴5.52:-

There are 9 balls of which 4 are white and 5 are black. Three are drawn. The number of sample points in the sample space are

$$n(S) = {}^9C_3 = 84 \text{ ways}$$

Let A represent the event of drawing 3 black balls.

$$n(A) = {}^5C_3 = 10 \text{ ways}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{84}$$

-:5.53:-

There are 13 balls of which 5 are white and 8 are black. Four balls are drawn at random. The number of sample points in the sample space are.

$$n(S) = {}^{13}C_4 = 715$$

Let A represent the event of drawing 4 black balls.

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{715} = \frac{1}{143}$$

-:5.54:-

There are 8 cards of which one joker and seven other cards. Five cards are chosen at random. The number of sample points in sample space are

$$n(S) = {}^8C_5 = 56$$

A can choose one joker and 4 other cards.

$$n(S) = {}^1C_1 \times {}^7C_4 = 35$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{35}{56} = \frac{5}{8}$$

-:5.55:-

There are 9 persons of which 3 men, 2 women and 4 children. Four persons are chosen at random. The number of sample points in sample space are

$${}^9C_4 = 126$$

Let A represent the event to choose 4 persons of which exactly 2 are children. Four persons can be chosen by the following ways.

(A₁) 2 men 0 women 2 children

(A₂) 0 men 2 women 2 children

(A₃) 1 man 1 woman 2 children

$$n(A_1) = {}^3C_2 \times {}^2C_0 \times {}^4C_2 = 3 \times 1 \times 6 = 18$$

$$n(A_2) = {}^3C_0 \times {}^2C_2 \times {}^4C_2 = 1 \times 1 \times 6 = 6$$

$$n(A_3) = {}^3C_1 \times {}^2C_1 \times {}^4C_2 = 3 \times 2 \times 6 = 36$$

$$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{18}{126}$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{6}{126}$$

$$P(A_3) = \frac{n(A_3)}{n(S)} = \frac{36}{126}$$

Hence

$$\begin{aligned} P(A) &= P(A_1) + P(A_2) + P(A_3) \\ &= \frac{18}{126} + \frac{6}{126} + \frac{36}{126} = \frac{60}{126} = \frac{10}{21} \end{aligned}$$

-:5.56:-

There are 15 balls of which 5 white, 5 red and 4 black. Three balls are drawn at random. The number of sample points in sample space are

$$n(S) = {}^{15}C_3 = 455$$

- (i) Let A represent the event of drawing all the three white balls.

$$n(A) = {}^5C_3 \times {}^6C_0 \times {}^4C_0 = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{455} = \frac{2}{91}$$

- (ii) let A represent the white balls and B represent the red balls. Then

$$n(A \text{ or } B) = {}^5C_3 \times {}^6C_0 \times {}^4C_0 + {}^5C_0 \times {}^6C_3 \times {}^4C_0 \\ = 10 + 20 = 30$$

$$P(A \text{ or } B) = P(\text{White or black})$$

$$P(A \text{ or } B) = \frac{n(A \text{ or } B)}{n(S)} = \frac{30}{455} = \frac{6}{91}$$

- (iii) Let C represent the event of drawing one red and 2 black ball.

$$n(C) = {}^6C_1 \times {}^4C_2 = 6 \times 6 = 36$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{36}{455}$$

- (iv) Let D represent the event of drawing one ball of each colour.

$$n(D) = {}^5C_1 \times {}^6C_1 \times {}^4C_1 = 5 \times 6 \times 4 = 120$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{120}{455} = \frac{24}{91}$$

-:5.57:-

There are 15 balls of which 4 white, 5 red and 6 black balls. Three balls are drawn at random. The number of sample points in sample space are

$$n(S) = {}^{15}C_3 = 455$$

- (i) Let A represent the event of drawing no black ball.

$$n(A) = {}^9C_3 = 84$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{84}{455} = \frac{12}{65}$$

- (ii) Let B represent the event of drawing exactly 2 black and one other ball.

$$n(B) = {}^6C_2 \times {}^9C_1 = 135$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{135}{455} = \frac{27}{91}$$

- (iii) Let C represent the event of drawing all the same colour

$$n(C) = {}^4C_3 + {}^5C_3 + {}^6C_3 \\ = 4 + 10 + 20 = 34 \text{ ways.}$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{34}{455}$$

-:5.58:-

There are 50 lamps in which 3 are defective and 47 are non-defective. Five lamps are chosen at random. The number of sample points in sample space are

$$n(S) = {}^{50}C_5 = 2118760$$

- (a) Let A represent the event that non of lamp will be defective.

$$n(A) = {}^{47}C_5 = 1533939$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{1533939}{2118760} = 0.734$$

- (b) Let B be the event of choosing exactly 2 defective and three other non defective lamps.

$$\begin{aligned} n(B) &= {}^3C_2 \times {}^{47}C_3 \\ &= 3 \times 16215 = 48645 \end{aligned}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{48645}{2118760} = 0.023$$

-:5.59:-

There are 15 balls in which 6 white 5 red and 4 black. Four balls are drawn at random. The number of sample points in sample space are

$$n(A) = {}^{15}C_4 = 1365$$

Let A represent the event of getting at least 2 red balls.

$$\begin{aligned} n(A) &= {}^5C_2 \times {}^{10}C_2 + {}^5C_3 \times {}^{10}C_1 + {}^5C_4 \times {}^{10}C_0 \\ &= 450 + 100 + 5 = 555 \end{aligned}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{555}{1365} = \frac{57}{91}$$

-:5.60:-

There are 12 balls of which 5 white and 7 black. 3 balls are drawn at random. The number of sample points in sample space are

$$n(S) = {}^{12}C_3 = 220$$

- (a) Let A represent the event of drawing all white balls.

$$n(A) = {}^5C_3 \times {}^7C_0 = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{220} = \frac{1}{22}$$

- (b) Let B represent the event of drawing 2 white and one black ball

$$n(B) = {}^5C_2 \times {}^7C_1 = 70$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{70}{220} = \frac{7}{22}$$

- (c) Let C represent the event of drawing all 3 of the same colour

$$n(C) = {}^5C_3 \times {}^7C_0 + {}^5C_0 \times {}^7C_3 \\ = 10 + 35 = 45$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{45}{220} = \frac{9}{44}$$

-:5.61:-

There are 14 balls of which 4 are red, 5 black and 5 white. Six balls are drawn at random. The number of sample points in sample are

$$n(S) = {}^{14}C_6 = 3003$$

- (i) Let A represent the event of drawing 3 red and 3 other balls

$$n(A) = {}^4C_3 \times {}^{10}C_3 = 4 \times 120 = 480$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{480}{3003}$$

- (ii) Let B represent the event of drawing at least 2 white balls

$$n(B) = {}^5C_2 \times {}^9C_4 + {}^5C_3 \times {}^9C_3 + {}^5C_4 \times {}^9C_2 + {}^5C_5 \times {}^9C_1 \\ = 1260 + 840 + 180 + 9 = 2289$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2289}{3003}$$

-:5.62:-

There are 14 balls of which 3 white 2 red, 5 green and 4 yellow. Four balls are selected at random. The number of sample points in sample space are

$$n(S) = {}^{14}C_4 = 1001$$

Let A represent the event of drawing 4 balls of all colour. As there are 4 colours, one ball of each colour is drawn.

$$n(A) = {}^3C_1 \times {}^2C_1 \times {}^5C_1 \times {}^4C_1 \\ = 3 \times 2 \times 5 \times 4 = 120$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{120}{1001}$$

-:5.63:-

There are two bags, one bag contains 6 balls of which 4 white and 2 black and the other bag contains 8 balls of which 3 white and 5 black. One ball is drawn from each bag.

- (i) Let A represent the event drawing both white ball

Probability of drawing one white ball from first, one white ball from second bag is

$$P(A) = \frac{{}^4C_1 \times {}^2C_0}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_0}{{}^8C_1} = \frac{4}{6} \times \frac{3}{8} = \frac{12}{48} = \frac{1}{4}$$

- (ii) Let B represent the event of drawing both black ball.

The probability of drawing one black ball from first, one black ball from second bag is

$$P(B) = \frac{{}^4C_0 \times {}^2C_1}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_1}{{}^8C_1} = \frac{2}{6} \times \frac{5}{8} = \frac{10}{48} = \frac{5}{24}$$

- (iii) Let C represent the event of drawing one white, one black ball.
One white and one black ball can be drawn by the following ways.

	Bag I		Bag II	
	White 4	Black 2	White 2	Black 5
Case I	White ball		Black ball	
Case II	Black ball		White ball	

Probability of drawing one white from first, one black from the second bag is

$$P_1 = \frac{{}^4C_1 \times {}^2C_1}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_1}{{}^8C_1} = \frac{4}{6} \times \frac{5}{8} = \frac{5}{12}$$

Probability of drawing one black from first and one black from second bag is

$$P_2 = \frac{{}^4C_0 \times {}^2C_1}{{}^6C_1} \times \frac{{}^3C_1 \times {}^5C_0}{{}^8C_1} = \frac{2}{6} \times \frac{3}{8} = \frac{1}{8}$$

Hence the required probability is

$$P(C) = P_1 + P_2 = \frac{5}{12} + \frac{1}{8} = \frac{13}{24}$$

-:5.64:-

There are three bags A, B, C. Bag A contains 12 balls of which 5 are white, 7 are black. Bag B contains 10 balls of which 4 are white, 6 black. Bag C contains 6 balls of which 2 are white and 4 black. One ball is drawn from a bag selected at random white ball is selected.

There are three bags and each these has an equal chance of being selected. The probability of each bag is equal to $1/3$.

The following three possible cases in the selection of bags are

- First bag may be selected.
- Second bag may be selected.
- Third bag may be selected.

The probability of a white ball from first bag

$$= \frac{{}^5C_1 \times {}^7C_0}{{}^{12}C_1} = \frac{5}{12}$$

The probability of a white ball with the probability of selection of first bag.

$$P_1 = \frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$$

The probability of a white ball from second bag

$$= \frac{{}^4C_1 \times {}^6C_0}{{}^{10}C_1} = \frac{4}{10}$$

The probability of a white ball with the probability of selection of second bag

$$P_2 = \frac{1}{3} \times \frac{4}{10} = \frac{4}{30}$$

Similarly for the probability of a white ball from third bag with the probability of the selection of third bag is

$$= \frac{1}{3} \times \frac{2}{6} = \frac{2}{18}$$

$$P_1 = \frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$$

Hence the required probability is

$$P = P_1 + P_2 + P_3 = \frac{5}{36} + \frac{4}{30} + \frac{2}{18} = \frac{69}{180}$$

∴5.65:-

There are 13 balls of which 5 white and 8 black. Two drawings each of 3 balls are made.

Let A represent the event that first drawing contains 3 white balls. B represent the event that second drawing contains 3 black balls. The number of sample points in sample space are

$$n(S) = {}^{13}C_3 = 286$$

$$n(A) = {}^5C_3 \times {}^8C_0 = 10$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{10}{286}$$

$$n(B) = {}^5C_0 \times {}^8C_3 = 56$$

To find the probability of B; when the white balls drawn in the first drawing are not being replaced i.e. $P(B/A)$, the sample space S is reduced.

$$n(S) = {}^{10}C_3 = 120$$

$$P(B/A) = \frac{n(B)}{n(S)} = \frac{56}{120}$$

We have to find

$$P(A \cap B) = P(A) \cdot P(B/A) = \frac{10}{286} \times \frac{56}{120} = \frac{7}{249}$$

∴5.66:-

(i) With Replacement:

$$n(S) = {}^{52}C_1 = 52$$

Let A_1 represent the event of drawing first ace card

$$n(A_1) = 4$$

$$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{4}{52}$$

Let A_2 represent the event of drawing second ace

$$n(A_2) = 4$$

$$P(A_2) = \frac{n(A_2)}{n(S)} = \frac{4}{52}$$

Hence

$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

(ii) Without replacement

$$n(S) = 51; n(A_2) = 3$$

$$P(A_2 / A_1) = \frac{n(A_2)}{n(S)} = \frac{3}{51}$$

Hence

$$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2 / A_1) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{121}$$

-:5.67:-

Let A represent the event of football players and B represent the event of basketball players. It is given that

$$P(A) = \frac{60}{100}; P(B) = \frac{50}{100}; P(A \cap B) = \frac{30}{100}$$

(i)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{60}{100} + \frac{50}{100} - \frac{30}{100} = \frac{80}{100} = 0.8$$

(ii)

$$P(\bar{A}) = 1 - \frac{60}{100} = \frac{40}{100}, P(\bar{B}) = 1 - \frac{50}{100} = \frac{50}{100}$$

$$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B}) = \frac{40}{100} \times \frac{50}{100} = 0.2$$

-:5.68:-

There are 5 balls in a bag of which 2 red and 3 white. The number of sampling points in sample space are

$$n(S) = {}^5C_1 = 5$$

Let A_1 represent the first drawing of a red ball

$$n(A_1) = {}^2C_1 = 2$$

$$P(A_1) = \frac{n(A_1)}{n(S)} = \frac{2}{5}$$

let A_2 represent the second drawing of red ball. First ball is being replaced.
Hence

$$\begin{aligned}n(S) &= 5; \quad n(A_2) = {}^2C_1 = 2 \\P(A_2) &= \frac{n(A_2)}{n(S)} = \frac{2}{5} \\P(A_1 \cap A_2) &= P(A_1) \cdot P(A_2) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}\end{aligned}$$

-:5.69:-

When the first red ball is not replaced, then the sample space reduced.

$$n(S) = 4$$

The remaining red ball is one, hence

$$\begin{aligned}P(A_2 / A_1) &= \frac{1}{4} \\P(A_1 \cap A_2) &= (P(A_1) \cdot P(A_2 / A_1)) = \frac{2}{5} \times \frac{1}{4} = \frac{1}{10}\end{aligned}$$

-:5.70:-

Let A represent the event that sum of dots is odd and B represent the event that sum of dots is 7. Then we have to find $P(B/A)$. The number of sample points in sample space are.

$$n(S) = 36; \quad n(A) = 18$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

There are 6 sample points in AB

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Hence

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

-:5.71:-

Let A_1 represent the event that the employee has accounting back ground and A_2 be the event that employee is an executive.

$$P(A_1) = 0.20; \quad P(A_1/A_2) = 0.05$$

Hence required probability is

$$P(A_2 / A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.05}{0.20} = 0.25$$

-:5.72:-

The sample space S for this experiment consists of the following 36 equally likely outcomes, that is

$$n(S) = 36$$

Let A represent the sum is 7, B represent sum is odd, C represent sum is greater than 6 and D represent the two dice had the same outcome. Then

$$n(A) = 6, \quad P(A) = \frac{6}{36} = \frac{1}{6}$$

- (i) B represent the event that sum is odd

$$n(B) = 18$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{18}{36} = \frac{1}{2}$$

$$n(A \cap B) = 6$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

- (ii) Let C represent the event that sum is greater than 6

$$C = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (2,6), (3,5), (4,4), (5,3), (6,2), (3,6), (4,5), (5,4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6)\}$$

$$n(C) = 21$$

$$P(C) = \frac{n(C)}{n(S)} = \frac{21}{36}$$

$$n(A \cap C) = 6$$

$$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{6}}{\frac{21}{36}} = \frac{2}{7}$$

- (iii) Let D represent the event having same outcomes.

$$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

$$n(D) = 6$$

$$P(D) = \frac{n(D)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$A \cap D = \phi$$

$$P(A \cap D) = 0$$

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0}{\frac{1}{6}} = 0$$

5

PROBABILITY

QUESTION NO. 1

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	An experiment of three coins are tossed has sample points	4	✓ 8	9	None of these
(Faisalabad Board, 2008)					
2)	The probability of an event always lies between	-1 and +1	-1 and 0	✓ 0 and +1	0 and ∞
(Lahore Board, 2008 & 2010)					
3)	The probability is the measure of	✓ Un-certainty	Certainty	Chances	None of these
(Lahore Board, 2008)					
4)	When two coins are tossed, the possible outcomes are	6	✓ 4	12	3
(Gujranwala Board, 2009) (Faisalabad Board, 2009 & 2010) (Lahore Board, 2009)					
5)	The probability of sure even is	0	100	✓ 1	1/2
(Lahore Board, 2009)					
6)	If $P(A) = 0.70$, then $P(\bar{A}) =$ is	0.70	✓ 0.30	0	1
(Faisalabad Board, 2010)					
7)	For two mutually exclusive events A and B: $P(A) = 0.70$; $P(B) = 0.3$ then $P(A \cup B)$ is	0	0.40	0.30	✓ 1
(Lahore Board, 2010)					

S#	Questions	Answers			
8)	Four books can be arranged in a shelf is	8 ways	2 ways	16 ways	✓ 24 ways
(Lahore Board, 2010)					
9)	The probability of a black card from pack of 52 cards is	$\frac{1}{52}$	✓ $\frac{1}{2}$	$\frac{13}{52}$	$\frac{3}{4}$
(Lahore Board, 2010)					
10)	The value of probability can not be greater than	Zero	✓ Unity	Sample	Two
(Gujranwala Board, 2010)					
11)	When two coins are tossed simultaneously, the probability of one head is	✓ $\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
(Rawalpindi Board, 2010)					
12)	If n coins are tossed, the possible outcomes are	n	✓ 2n	2^n	n^2
(Rawalpindi Board, 2010)					
13)	The complete sample space is called	Empty	Impossible event	Equally likely event	✓ Sure event
(Sargodha Board, 2010)					

QUESTION NO. 2

Possible answers are given to each statement. Tick (✓) the correct one.

S#	Questions	Answers			
1)	The idea of probability was first applied in gambling games in:	Pakistan	America	✓ France	India
2)	Today the application of the theory of probability is the most important in making decisions in:	Business	Insurance	Management	✓ All above

S#	Questions	Answers			
3)	n-factorial is denoted by:	✓ $n!$	$m!$	$r!$	$p!$
4)	$(n+2)! =$ _____	$n(n-1)!$	$(n-1)(n-2)!$	✓ $(n-2)(n+1)n!$	$(n \times 2)!$
5)	Three objects can be arranged in:	3 ways	4 ways	5 ways	✓ 6 ways
6)	The total number of possible combinations of a set of a different objects, taken r at a time is denoted by:	${}^n P_r$	✓ ${}^n C_r$	${}^n D_r$	None of these
7)	${}^n C_r =$ _____	$\frac{n!}{r!}$	$\frac{n!}{(n-r)!}$	✓ $\frac{n!}{r!(n-r)!}$	$\frac{r!}{n!}$
8)	A set having zero as its element is called:	Empty set	Infinite set	Singleton set	Null set
9)	Two books are to be selected at random without replacement out of four books. The number of possible selections are:	✓ 6	2	4	3
10)	Three books of different colours are to be arranged in book-shelf. The possible arrangements are:	3	1	✓ 6	2
11)	A set consisting of all the elements under consideration is called:	Empty set	Disjoint set	✓ Universal set	Sub set
12)	A set consisting of all the elements under consideration is called:	U	λ	✓ ϕ	μ

S#	Questions	Answers			
13)	Sets having elements in common are called:	✓ Overlapping sets	Disjoint sets	Infinite sets	None of these
14)	The set A is a subset of the set B if every element of A is an element of B. We use the symbol as:	$A \supseteq B$	✓ $A \subseteq B$	$A = B$	$A + B$
15)	The union of two sets is denoted by:	✓ $(A \cup B)$	$(A \cap B)$	$A = B$	$A + B$
16)	The intersection of two sets is denoted by:	$A \cup B$	✓ $A \cap B$	$A \subseteq B$	$A - B$
17)	An act or the process of obtaining an observation is called:	✓ Experiment	Sample space	Event	Set
18)	If an experiment provides different results under essentially identical conditions, it is called:	Sample space	Simple event	✓ Random experiment	Sub set
19)	A set of all possible outcomes of the random experiment is called:	✓ Sample space	Impossible event	Union of sets	Sub set
20)	Any subset of a sample space is called:	Finite set	Nul set	✓ Event	All of these
21)	An event that contains more than one sample point is called:	Simple event	✓ Compound event	Impossible event	None of these
22)	When a die is rolled, all possible out comes are:	2	4	✓ 6	8
23)	When 2 dice are rolled, all possible out comes are:	4	10	22	✓ 36

S#	Questions	Answers			
24)	When n dice are rolled, the possible sample points in the sample space are:	6^n	n^n	6^n	None of these
25)	When two coins are tossed, the possible outcomes are:	6	4	36	12
26)	If three coins are tossed, the possible outcomes are:	8	3	1	10
27)	If n coins are tossed, the possible outcomes are:	n	$2n$	2^n	n^2
28)	If two events A and B can not occur together, then they are:	Mutually exclusive	Not mutually exclusive	Independent	All of these
29)	If two events A and B have same chance to occur, then they are:	Mutually exclusive	Equally likely	Exhaustive	None of these
30)	If A and B are mutually exclusive events, $P(A)=0.25$, $P(B)=0.50$, then $P(A \cup B) =$	0.75	0.25	0.50	1
31)	The probability of an event always lies between:	-1 and +1	-1 and 0	0 and 1	None of these
32)	$P(A \cup B)$ $P(A)+P(B)$ if A and B are:	Mutually exclusive events	Independent events	Non mutually exclusive events	None of these
33)	The probability of appearing a tail when a fair coin is tossed is:	0	$\frac{1}{2}$	$\frac{1}{4}$	1

S#	Questions	Answers			
34)	The probability of drawing a queen from well-shuffled pack of 52 playing cards is:	✓ $\frac{4}{52}$	$\frac{2}{52}$	$\frac{13}{52}$	$\frac{26}{52}$
35)	If two cards are drawn at random from a standard deck of cards, the number of possible sample points is:	52	52!	✓ 1326	2000
36)	The probability of drawing a jack from a deck of cards is:	✓ $\frac{1}{13}$	$\frac{1}{4}$	$\frac{1}{52}$	$\frac{5}{52}$
37)	The probability of selecting a jack of club from a deck of cards is:	$\frac{13}{52}$	$\frac{4}{52}$	✓ $\frac{1}{52}$	$\frac{2}{52}$
38)	The probability of getting exactly one head when two coins are tossed is:	$\frac{1}{4}$	✓ $\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$
39)	Five cards are selected at random from a pack of 52 cards. The possible outcomes are:	52	✓ ${}^{52}C_5$	52^5	5^{52}
40)	A fair coin is tossed 200 times, the expected number of heads is:	✓ 100	7	50	20
41)	When two dice are rolled, the maximum total on the two faces of the dice will be:	6	36	✓ 12	10
42)	If A and B are independent events then:	$P(A \cup B) = P(A) + P(B)$	$P(A \cap B) = P(A) + P(B)$	✓ $P(A \cap B) = P(A) \cdot P(B)$	None of these

S#	Questions	Answers			
43)	If $P(A/B) = P(A)$ and $P(B/A) = P(B)$, then the events A and B are said to be:	✓ Independent	Dependent	Mutually exclusive	Simple
44)	If $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$, then events A and B are said to be:	Equally likely	Mutually exclusive	✓ Independent	Exhaustive
45)	Probability of an event can never be:	One	✓ Negative	Positive	Zero
46)	The conditional probability $P(A/B)$ is:	$\frac{P(A \cap B)}{P(B)}$	$\frac{P(A \cap B)}{P(A)}$	✓ $\frac{P(A \cap B)}{P(B)}$	$\frac{P(A \cap B)}{P(A)}$

QUESTION NO. 3

Write the short answer to the following questions in the space provided

Q. 1: What is difference between permutation and combination?

Ans: A permutation of a number of objects is an arrangement of these objects in some order. A selection of distinct objects without some order is called a combination, symbolically

$${}^n P_r = \frac{n!}{(n-r)!} \text{ is a formula for permutation } n$$

$${}^n C_r = \frac{n!}{r!(n-r)!} \text{ is a formula for combination } n$$

Q. 2: What do you mean by n!?

Ans: The product of first "n" natural numbers is called n factorial denoted by $n! = n(n-1)(n-2) \dots 1$

Q. 3: Define the term experiment.

Ans: An act or the process of obtaining an observation is called experiment.

Q. 4: Define the term trail.

Ans: Performing of an experiment is called a trail.

Q. 5: Define the term random experiment.

(Lahore Board, 2009 & 2010

(Rawalpindi Boar, 2010)

(Sargodha Board, 2010)

Ans: An experiment is said to be random, if it provides different results, if it is repeated a large number of times under identical conditions.

Q. 6: What is a sample space?

Ans: A set of all possible outcomes of a random experiment is called a sample space. It is denoted by S .

Q. 7: Give two examples of a random experiment.

Ans: The tossing of a fair coin and throwing of a balanced die are examples of random experiment.

Q. 8: What is an event?

(Gujranwala Board, 2010)

Ans: Any subset of the sample space is called an event. It is denoted by A , B , C , D etc.

Q. 9: Make a sample space, when a cubical die is rolled.

(Gujranwala Board, 2010)

(Sargodha Board, 2010)

Ans: $S = \{1, 2, 3, 4, 5, 6\}$

Q. 10: Define sample points.

Ans: The elements of the sample space are called sample points.

Q. 11: What are the properties of a random experiment?

Ans: A random experiment has three properties.

- (i) The experiment can be repeated
- (ii) The experiment always has at least two outcomes
- (iii) The outcome of each trial is unpredictable

Q. 12: Define simple event.

(Lahore Board, 2010)

(Rawalpindi Board, 2010)

Ans: A simple event is one which contains only one sample point of the sample space.

Q. 13: Define compound event.

Ans: Compound event contains two or more sample points of the sample space.

Q. 14: What is impossible event?

Ans: An event consisting of the empty set is called impossible event.

Q. 15: Define sure event or certain event.

Ans: An event that consists of the sample space itself is called sure or certain event.

Q. 16: Define mutually exclusive events with example.

Ans: Two events A and B are said to be mutually exclusive if they can not occur together. Thus occurrence of head or tail in a single throw of a coin are mutually exclusive events.

Q. 17: Define not mutually exclusive events with example.

Ans: If two events can occur together, they are not mutually exclusive. If we draw a card from an ordinary deck of 52 cards, it can be both a queen and a card of spade.

Q. 18: Define equally likely events with example.

Ans: Two or more events are said to be equally likely events if they have same chance to occur. If we toss a coin, the head and tail have the same chance to be occur.

Q. 19: Define independent events with example.

(Lahore Board, 2008 & 2009)

(Sargodha Board, 2010)

Ans: Two events A and B are said to be independent if the events in no way affect each other. For example, suppose event A is the drawing of king from a pack of 52 cards and event B is head on tossing a fair coin. The events are independent because drawing of a card does not affect the head when tossing a coin.

Q. 20: Define dependent events.

Ans: Two events are said to be dependent when the happening of any one event is affected by the happening or non happening of the other.

Q. 21: Define the exhaustive events.

(Lahore Board, 2009)

Ans: Two or more events are said to be collectively exhaustive if their union generates the whole sample space.

Q. 22: Define favourable cases.

Ans: The cases which entail the occurrence of an event are said to be favourable cases to the event.

Q. 23: Are independent events also mutually exclusive?

Ans: No, mutually exclusive events are always independent.

Q. 24: What are two approaches to define probability?

Ans: Two approaches to define probability are:

(a) Subjective approach

(b) Objective approach

Q. 25: Give the classical priori definition of probability.

(Gujranwala Board, 2010)

Ans: If there are "n" equally likely, mutually exclusive and exhaustive sample points in a sample space and "m" out of them are favourable to an event A, then

$$P(A) = \frac{m}{n}$$

Q. 26: Give the relative frequency or posteriori definition of probability.

Ans: If a random experiment is repeated a large number of times say n under uniform conditions and the event A is observed m times, then probability of event A, is

$$P(A) = \lim_{n \rightarrow \infty} \frac{m}{n}$$

Q. 27: Give the axiomatic (mathematical approach) of probability.

Ans: The probability that an event A will occur is the number $P(A)$. The number $P(A)$ satisfy the following axioms.

(i) $0 \leq P(A) \leq 1$

(ii) $P(S) = 1$

(iii) If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Q. 28: When does probability become negative?

Ans: Probability can never be negative.

Q. 29: State the addition law of probability for two mutually exclusive events.

Ans: If A and B are two mutually exclusive events then the probability that anyone of them occur is given by sum of their respective probabilities, that is $P(A \cup B) = P(A) + P(B)$

Q. 30: State the addition law of probability for two not mutually exclusive events.

Ans: If A and B are not mutually exclusive events then the probability that at least one of them occur is given by the sum of their respective probabilities minus the probability of their joint occurrence that is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Q. 31: State the multiplication law of probability for two independent events.

Ans: If A and B are two independent events, then the probability that both occur is given as:

$$P(A \cap B) = P(A).P(B)$$

Q. 32: State multiplication law of probability for two dependent events.

Ans: If A and B are two dependents events, then

$$P(A \cap B) = P(A).P(B/A) \text{ provided } P(A) \neq 0 \text{ OR}$$

$$P(A \cap B) = P(B).P(A/B) \text{ provided } P(B) \neq 0$$

Q. 33: What is conditional probability?

Ans: If A and B are two events, and $P(B) \neq 0$, then conditional probability for event A given that the event B has occurred, denoted by $P(A/B)$, is defined as

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Q. 34: Define set.

(Lahore Board, 2010)

Ans: Well defined collection of distinct objects is called set. It is denoted by capital letters A, B, C, ...

Q. 35: What is empty set?

(Lahore Board, 2010)

Ans: A set consisting no element is called a null or empty set. It is denoted by $\emptyset$ or $\{\}$