

Wave Motion and Sound

8.1 VIBRATORY MOTION

In our surrounding we come across many things which undergo oscillatory or vibratory motion. Some examples are the motion of a pendulum, Prongs of a tuning fork when struck, sitar's string when plucked, etc. A weight attached to a stretched spring, once it is released, starts oscillating. The atoms in a solid possess vibratory motion. Similarly, atoms in molecules also vibrate relative to each other. The electrons in a radiating or receiving antenna are in rapid oscillations. An understanding of vibrational motion is essential for the discussion of wave phenomena.

8.2 MOTION UNDER ELASTIC RESTORING FORCE (HOOKE'S LAW)

An important type of motion occurs when the force acting on a body is directly proportional to the displacement of the body measured from its equilibrium position. Since this force acts only toward the equilibrium position, the result is a back-and-forth motion called simple harmonic motion. Consider a body of mass m , attached to a horizontal helical spring Fig.8.1. The whole system is placed on a horizontal, smooth surface. If the spring is stretched or compressed, a small distance from its equilibrium position, and then released, the spring will exert a force on the body given by

$$F_s = -kx \quad 8.1$$

Where x is the displacement of the body from its equilibrium position and k is a positive constant, known as the force constant of the spring. The above equation is the mathematical statement of

what is known as Hooke's law. The negative sign indicates that the force exerted by the spring on the body is always directed opposite to the displacement. For example, when x is greater than zero as in Fig.8.1 (a), the spring force is to the left that is negative. When x is less than zero as in Fig.8.1(c), the spring force is to the right that is positive. No doubt, when $x = 0$, as in Fig.8.1(b), the spring is neither stretched nor compressed and $F_s = 0$. As spring force always tends to restore than original condition of the spring, it is sometimes called a restoring force or more correctly elastic restoring force.

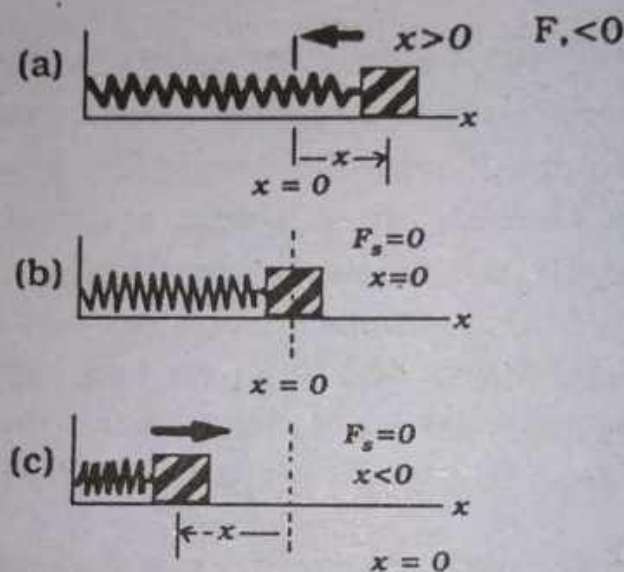


Fig. 8.1 The force of spring on a mass varies with the displacement of the mass from the equilibrium position $x = 0$.

(a) When x is positive (stretched spring), the spring force is to the left

(b) When x is zero (unstretched spring), the spring force is zero.

(c) When x is negative (compressed spring), the spring force is to the right.

Once the mass is displaced to some distance x_m (within elastic limit) from equilibrium position and then released, it will move from the position $-x_m$ through 0 to $+x_m$ position. This system will continue to oscillate back and forth about its equilibrium position.

During the oscillatory motion between two extreme positions when the body passes through the equilibrium position, the total energy is kinetic. The potential energy is zero because the spring is in the unstretched condition. As the body moves to the right or left from the equilibrium position its potential energy begins to increase at the cost of kinetic energy. When the body is at the extreme position, it stops there momentarily and kinetic energy is

zero. The total energy of the system is potential energy. Since the spring tries constantly to regain its original condition, the restoring force comes into play and the body begins to move towards the equilibrium position. During this motion from extreme position to the equilibrium position, the kinetic energy increases at the cost of potential energy till the total energy is kinetic, at the equilibrium, the potential energy being zero. Due to inertia, the body then moves to the extreme position from the equilibrium position and the process is repeated for infinitely long time provided there is no loss of energy due to any internal or external factors. Using Newton's second law of motion, $F = ma$, the restoring force of the spring is given as

$$F_s = ma = -kx$$

$$\text{or } a = -\frac{k}{m}x \quad (8.2)$$

$$= -\text{constant} \times x$$

$$\text{or } a \propto -x$$

Where the quantity (k/m) represents the constant of proportionality. Minus sign shows that the acceleration is always directed towards the equilibrium position. This back and forth (oscillatory) motion in which the instantaneous acceleration is proportional to the displacement of the oscillating body is called a simple harmonic motion. It is abbreviated as SHM

8.3 CHARACTERISTICS OF SHM

Before we derive expressions for displacement, velocity, acceleration and time period of a particle executing SHM, we compare SHM with uniform circular motion.

The connection between uniform circular motion and SHM

Many aspects of simple harmonic motion along a straight line can be better understood and visualized by showing their relationship to uniform circular motion. Consider a point mass, m , at a point P moving in a circle of radius x_0 with constant angular velocity ω Fig.8.2 (a). We call this circle as our reference circle for the

motion.

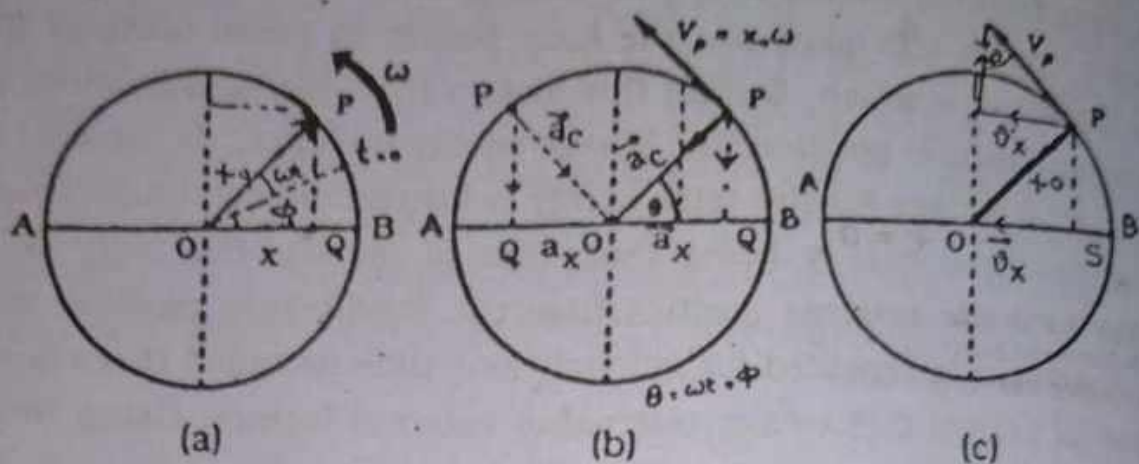


Fig. 8.2

As the particle at point P rotates along the circumference of a circle, the projection, Q, of the particle, P, moves back and forth along the diameter AOB. At some instant of time t , the angle between OP and the x-axis is $\omega t + \phi$, when ϕ is the angle which OP makes with the x-axis at time $t = 0$. This angle ϕ is known as the initial phase angle. We take this as our reference point for measuring angular displacement. As the particle P rotates on the circle, the angle that OP makes with the x-axis changes with time and the projection of P on the x-axis, moves back and forth along the diameter of the reference circle between the two extreme positions $x = \pm x_0$.

From the right angled triangle OPQ, the displacement of point Q from the mean position is given by

$$x = x_0 \cos (\omega t + \phi) \quad 8.3$$

Notice Fig.8.2 (a) that we take x to be positive when displacement is to the right; when the displacement is to the left, $\cos(\omega t + \phi) < 0$ and the necessary negative sign is automatically supplied.

In order to give physical significance to the constant appearing in Eq 8.3, it is convenient to plot x as a function of t as shown in Figure 8.3.

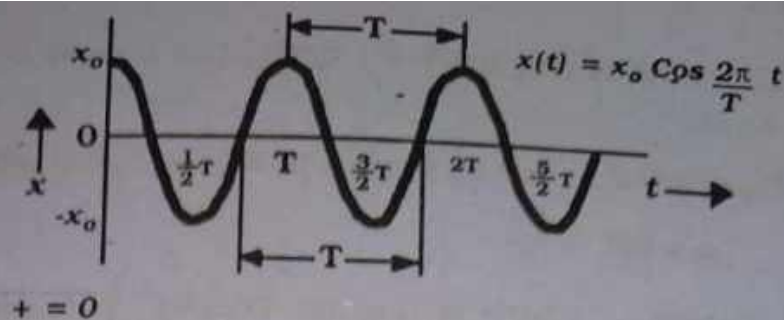


Fig. 8.3 Displacement as a function of time for a particle undergoing simple harmonic motion. There is a time interval T (The period) between any two successive corresponding point on the curve.

First we note that x_0 , called the amplitude of motion, is simply the maximum displacement of the particle, in either positive or negative direction of the axis of x . The constant angle ϕ is called the phase constant or phase angle. The constant ϕ tells us what the angular displacement was at the time $t = 0$. The quantity $(\omega t + \phi)$ is called the phase of the motion. We also note here that the function x is periodic and repeats itself when ωt increases by 2π radians.

The time T which the particle takes to go through one full cycle of its motion is called the period or time period. This means the value of x at time t is equal to the value of x at time $(t + T)$. We will show here that the period of the motion is given by $T = 2\pi/\omega$.

We know that the phase increases by 2π radians as t increases by T , then

$$\omega t + \phi + 2\pi = \omega(t + T) + \phi$$

$$\therefore \omega T = 2\pi \text{ or } T = \frac{2\pi}{\omega} \quad 8.4$$

The inverse of the period T , is known as the frequency of oscillatory motion, f . The frequency represents the number of oscillations / vibration which the particle makes in one second, i.e.

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad 8.5$$

The units of frequency are cycles per second or hertz abbreviated as Hz.

Equation 8.5, can be written in a somewhat different form

$$\omega = 2\pi f = \frac{2\pi}{T} \quad 8.6$$

Thus constant ω , which is angular velocity, has units of radians per second (rad/s).

The acceleration of the point P is directed towards the centre of the circle (i.e. the centripetal acceleration) along the line PO as shown in Fig.8.2 (b). The magnitude of this acceleration is given by

$$a_c = \frac{v_p^2}{x_o} = x_o \omega^2 \quad 8.7$$

Where $v_p = x_o \omega$ represents the linear speed of the point mass at P.

The acceleration of the point Q, (Q being the projection of point P) is equal to the component of the acceleration P along x-axis and is given by Fig.8.2 (b)

$$a_x = -x_o \omega^2 \cos\theta \quad 8.8(a)$$

The minus sign is needed because the acceleration, $\vec{a}_x$, of the point Q is toward the left (along negative x-axis). When Q is left of the centre, the acceleration of P is toward the right; but since $\cos\theta$ is negative at such point the minus sign is still needed.

$$\therefore x = +x_o \cos\theta$$

$$\therefore a_x = -\omega^2 x$$

8.8(b)

Here the acceleration of point Q is proportional to its displacement and is directed towards the centre. Hence the motion of the point Q is simple harmonic. Eq 8.8(b) shows that the acceleration is maximum at the extreme positions.

$$a_{\max} = \pm \omega^2 x_0$$

Comparing Eq 8.2 and Eq 8.8(b), we get

$$\omega = \sqrt{\frac{k}{m}} \quad 8.9(a)$$

we know that

$$\omega = 2\pi f = \frac{2\pi}{T}$$

The period of oscillation or time required for one complete trip is

$$T = 2\pi \sqrt{\frac{m}{k}} \quad 8.9(b)$$

The speed of the point Q is the component of the speed of the point P along diameter AOB as shown in Fig.8.2 (c)

$$\begin{aligned} v_x &= v_p \sin\theta \\ &= x_0 \omega \sin\theta \quad \because v_p = x_0 \omega \end{aligned} \quad 8.10(a)$$

Substituting for $\sin \theta$, we get

$$\begin{aligned} v_x &= x_0 \omega \sqrt{1 - \cos^2 \theta} \\ v_x &= \omega \sqrt{x_0^2 - x^2} \end{aligned} \quad 8.10(b)$$

$$v_x = \sqrt{\frac{k}{m}} \times \sqrt{x_0^2 - x^2} \quad 8.10(c)$$

Equation 8.10(b) shows that the velocity is maximum at the mean position O where $x = 0$ and is equal to ωx_0 ($v_{\max} = \omega x_0$), and it is minimum ($v_{\min} = 0$) at the extreme positions A and B.

8.4 ENERGY OF PARTICLE EXECUTING SHM

We often refer to a system that undergoes SHM as a simple harmonic oscillator. In an ideal oscillator there are no frictional

losses, so energy is conserved. At any instant of time, we have

$$E = \text{Total energy} = \text{P.E} + \text{K.E} = \text{constant}$$

To derive an expression for P.E we refer to fig. 8.1 and use Hooke's law i.e, $F = -kx$. Here the force is varying linearly with the displacement. The stretching force is zero when the displacement is zero and it is kx (only magnitude), when the displacement is x . Therefore the average force on the mass m during this displacement is $\frac{0 + kx}{2} = \frac{1}{2} kx$. Hence the work done on the mass by the force during displacement is $\frac{1}{2} kx \times x = \frac{1}{2} Kx^2$. This work done on mass is stored in it in the form of P.E. Thus the P.E of the oscillating mass at a displacement x from equilibrium position is given as

$$\text{P.E} = \frac{1}{2} kx^2 \quad 8.11 (a)$$

From the equation it is clear that the P.E is maximum at the two extreme positions where $x = \pm x_0$, i.e., $(\text{P.E})_{\max} = \frac{1}{2} kx_0^2$ and it is minimum at the equilibrium position i.e., where $x = 0$ and $(\text{P.E})_{\min} = 0$.

Using Eq 8.10(c), the expression for the K.E is given by

$$\begin{aligned} (\text{K.E}) &= \frac{1}{2} mv^2 \\ &= \frac{1}{2} m \left\{ \sqrt{\frac{k}{m}} \times \sqrt{x_0^2 - x^2} \right\}^2 \\ \text{K.E} &= \frac{1}{2} k (x_0^2 - x^2) \quad 8.11 (b) \end{aligned}$$

From the above Equation, it is clear that K.E is maximum at the equilibrium position where $x = 0$ and the maximum value is being given as $(\text{K.E})_{\max} = \frac{1}{2} kx_0^2$. It is maximum at the two extreme positions where $x = \pm x_0$. The minimum value is being given as $(\text{K.E})_{\min} = 0$

Therefore the total energy at any instant of the oscillating mass is given by

$$\text{Total energy} = \text{P.E} + \text{K.E} \quad 8.11(c)$$

$$E = \frac{1}{2} kx^2 + \frac{1}{2} k(x_0^2 - x^2)$$

$$E = \frac{1}{2} kx_0^2 \quad 8.11(d)$$

$$E = \text{Constant}$$

Eq. 8.11(d) shows that the total energy of the mass executing SHM is conserved throughout its displacement including mean and extreme positions

Finally, we shall now define some important terms used frequently when dealing with simple harmonic motion (SHM).

(i) Periodic motion:

A motion which repeats itself in equal interval of time is called periodic motion. If the motion is described by a periodic function $f(t)$ with period T , then

$$f(t) = f(t \pm nT)$$

Where $n = 0, 1, 2, 3, \dots$

and T is the time required to make one complete vibration/oscillation.

(ii) Frequency:

The number of vibrations made by a body in one second is called frequency. It is represented by a Greek letter (ν) read as 'nu'. It is expressed as vibrations/s or cycles/s or hertz (H_z). The reciprocal of frequency (ν) is equal to the time period (T)

8.5 EXAMPLES OF SHM

Simple pendulum

An ideal simple pendulum consists of a spherical bob sus-

pended from a light, flexible and inextensible string tied to a fixed rigid and frictionless support. When the bob is displaced from its equilibrium position, it begins to perform oscillatory motion. We will show that the bob will execute SHM, if the amplitude of oscillation is sufficiently small.

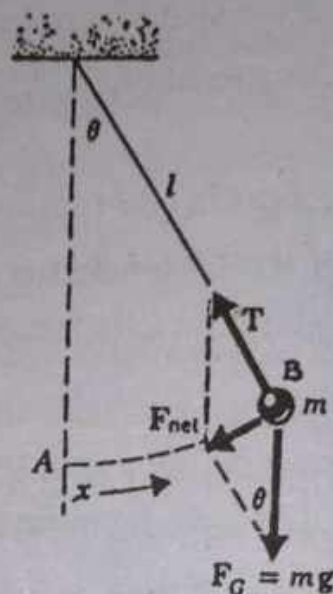


Fig. 8.4 A simple pendulum of length l . The net force on the pendulum bob is the vector sum of the gravitational force F_G and the tension in the string T .

Referring to Fig. 8.4 there are two forces acting on the pendulum the vertically downward gravitational force, $\vec{F}_G = m\vec{g}$ and the tension in the suspension string $\vec{T}$. The net force acting on the bob is given as

$$\vec{F}_{\text{net}} = \vec{F}_G + \vec{T}$$

We now resolve the force $\vec{F}_G$ into two components (i) one along the length of the string of the pendulum and (ii) the other perpendicular to the string. Thus we have

$$(F_G)_{||} = mg \cos\theta$$

$$(F_G)_{\perp} = mg \sin\theta$$

Where m is the mass of the bob. Since there is no motion along the string, the net force acting in the direction of the string is zero. This is possible if the component $mg \cos\theta$ balances the ten-

sion T. Hence the magnitude of the net force acting on the bob is

$$F_{\text{net}} = mg \sin\theta$$

This is the restoring force which is responsible for the oscillatory motion of the bob. The displacement, x , is the distance through which the bob moves along the arc (traced by the bob). Starting from A, the displacement of the bob at B is $x = l\theta$, where θ is measured in radians.

Using Newton's second law of motion and assuming $F_{\text{net}} = F$, the equation of motion of the bob is given as

$$F = ma = -mg \sin\theta$$

$$a = -g \sin\theta$$

The negative sign indicates that the force and hence acceleration are always directed towards the mean position. If θ is sufficiently small (less than about 5 degrees) then

$$\sin\theta \approx \theta$$

$$= \frac{x}{l}$$

substituting for $\sin\theta$, we write

$$a = -g \frac{x}{l}$$

$$a = -\frac{g}{l} x \quad 8.12$$

$$a \propto -x$$

where (g/l) is constant of proportionality

Thus the acceleration of the pendulum is directly proportional to its displacement and is directed towards its mean position. Hence the pendulum executes SHM.

Comparing Eq. 8.8(b) and Eq 8.12 and using Eq 8.9 (a), we get

$$\omega^2 = \frac{g}{l}$$

$$\therefore T = 2\pi \sqrt{\frac{l}{g}} \quad 8.13$$

Example:

Find the time period of a simple pendulum whose length is 88.2 cm. The value of acceleration due to gravity is 9.8 m/s^2 at the place where experiment is performed.

Solution

$$l = 88.2 \text{ cm} = 88.2 \times 10^{-2} \text{ m}$$

$$g = 9.8 \text{ m/s}^2$$

$$T = ?$$

$$\begin{aligned} T &= 2\pi \sqrt{\frac{l}{g}} \\ &= 2 \times 3.14 \sqrt{\frac{88.2 \times 10^{-2} \text{ (m)}}{9.8 \text{ (m/s}^2)}} \\ &= 1.885 \text{ s} \end{aligned}$$

8.6 GENERATION OF A WAVE PULSE.

A single isolated disturbance travelling through a system is known as a wave pulse. A simple way of generating a pulse in one dimensional medium is given below.

Consider a string whose one end is tied to a fixed support while the other end is held by the hand in a stretched position. This end is given a single up and down jerk which in turn produces a hump in the string. It (hump) travels along the string forming a wave pulse as shown in the Fig.8.5. The wave pulse retains its form only if there are no frictional losses and the height of the wave pulse is not so large as to affect the tension in the string applied by the hand. During the propagation of the wave pulse, the particles

of the medium (string) move up and down in a direction perpendicular to the direction of propagation of the wave pulse.

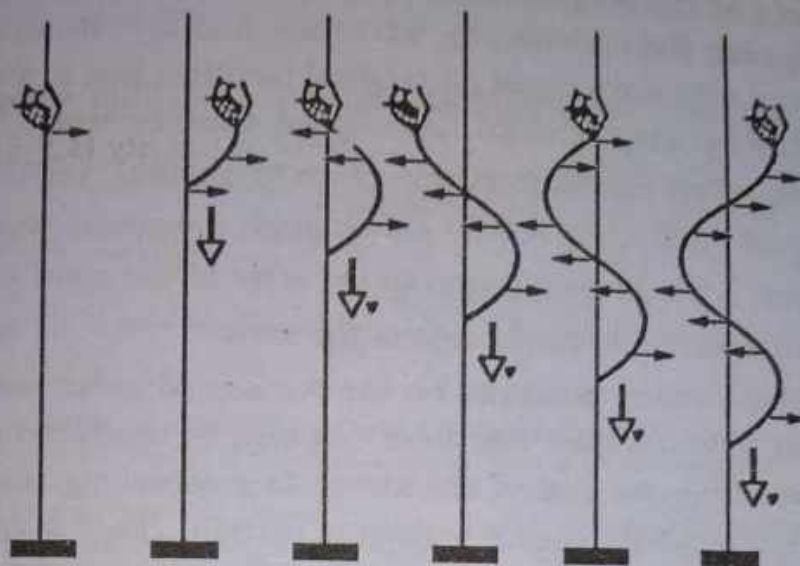


Fig. 8.5 in a transverse wave the particles of the medium (string) vibrate at right angles to the direction in which the wave itself is propagated.

The shape of the disturbed part of the string containing the wave pulse can be described at a given time mathematically by a wave function $f(x)$ such that

$$y = f(x)$$

Here y is the vertical displacement of a particle from its equilibrium position and x is the horizontal distance of the particle from the point where the displacement y is zero. Hence x, y are the coordinates of the particle. Since the wave pulse is moving along the string, its position is changing continuously with time. Hence the location of the wave pulse depends on time. Thus the shape of the wave pulse with its location can be given by the function $f(x, t)$ such that

$$y = f(x, t)$$

and is called wave function.

8.7 TRAVELLING WAVES

It is a common experience that disturbance is created in still

water of a pond, when a pebble is dropped into it. This disturbance produces waves on water, which move outward, finally reaching the shore of the pond. If we observe carefully the motion of a leaf floating near the disturbance, we would find that it moves up and down and side ways about its original position, but it does not undergo any net displacement. This is, the wave on water (or the disturbance which moves from one place to another, yet the water is not carried with it. Fig.8.6 (d) shows such a wave on water. In this case point A would correspond to the crest of the wave and point B would correspond to the trough of the wave.

These waves produced on the surface of water are known as travelling waves. A travelling wave can also be produced on a string by connecting one end of the string to a vibrating blade. As the blade oscillates with simple harmonic motion, every segment of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of vibration of the blade that drives the string.

A wave is characterized by three important physical concepts, the wave length, the frequency and the speed of the wave. The wave length is defined as the distance between any two points on a wave that behave identically. For example, in the case of waves on water, the wave-length is the distance between the adjacent crests or between adjacent troughs.

Every wave travels with a certain velocity which depends on the properties of the medium through which the wave is propagating. For examples, the sound waves travel through air with a speed about 344 m/s at 20°C, whereas the speed of sound through solid is higher than this value. A special class of waves which do not need a material medium for their propagation are electromagnetic waves, they travel through a vacuum with a speed of about 3×10^8 m/s.

The frequency and the time period have already been defined in section 8.4.

8.8 TRANSVERSE WAVE

Consider a long rope under tension as shown in Fig. 8.6 (b).

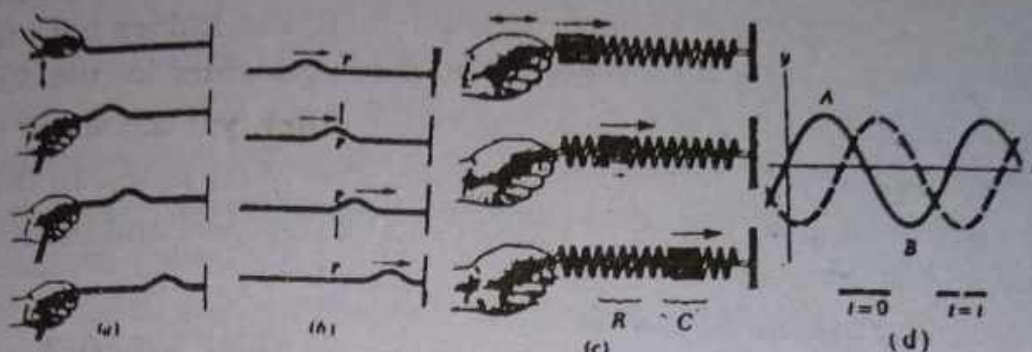


Fig. 8.6 (a) A wave pulse travelling down a stretched rope. (b) A pulse travelling on a stretched rope is a transverse wave. That is, any element P on the rope moves in a direction perpendicular to the wave motion. (c) A longitudinal pulse along a stretched spring. The disturbance of the medium (the displacement of the coils) is in the direction of the wave motion. For the starting extended region R . (d) A harmonic wave travelling to the right. The solid curve represents a snapshot of the wave at $t = 0$, and the dashed curve is a snapshot at some later time t .

one of its end is flipped to produce a wave in it. A portion of wave is produced this manner. The wave consists of a hump called pulse in the rope. This pulse travels to the right along the rope with a definite velocity. This type of disturbance is obviously a travelling wave. Fig. 8.6 (b) gives four consecutive snap shots of the travelling wave. In this case, the string is the medium through which the wave travels. We further assume that the shape of wave pulse remains unchanged while the wave travels along the rope.

It can be seen from the Fig 8.6 (c) that as the wave pulse travels along the rope, each segment of the rope, which is disturbed, moves in a direction perpendicular to the wave motion. Note that there is no motion in any part of rope in the direction of the wave propagation. A travelling wave such as this, in which the particles of the disturbed medium move perpendicular to the direction of propagation of the wave is called a transverse waves. Other examples of transverse waves are-electromagnetic waves, such as light, radio, and, television waves, etc.

8.9 ANALYTICAL TREATMENT OF TRAVELLING WAVES

We will give a simple mathematical treatment of one dimensional transverse travelling waves. Consider a wave pulse which is travelling to the right on a long stretched string with a constant speed v as shown in Fig.8.7. The pulse travels along the x -axis and the transverse displacement of the string is measured along y -axis. Fig.8.7 (a) gives the shape and position of the pulse at time t . The shape of the pulse is represented as $Y = f(x)$. Here y is a function of x . The maximum displacement denoted by $Y_{\max}$ is called the amplitude of the wave. The distance travelled by the pulse in time t

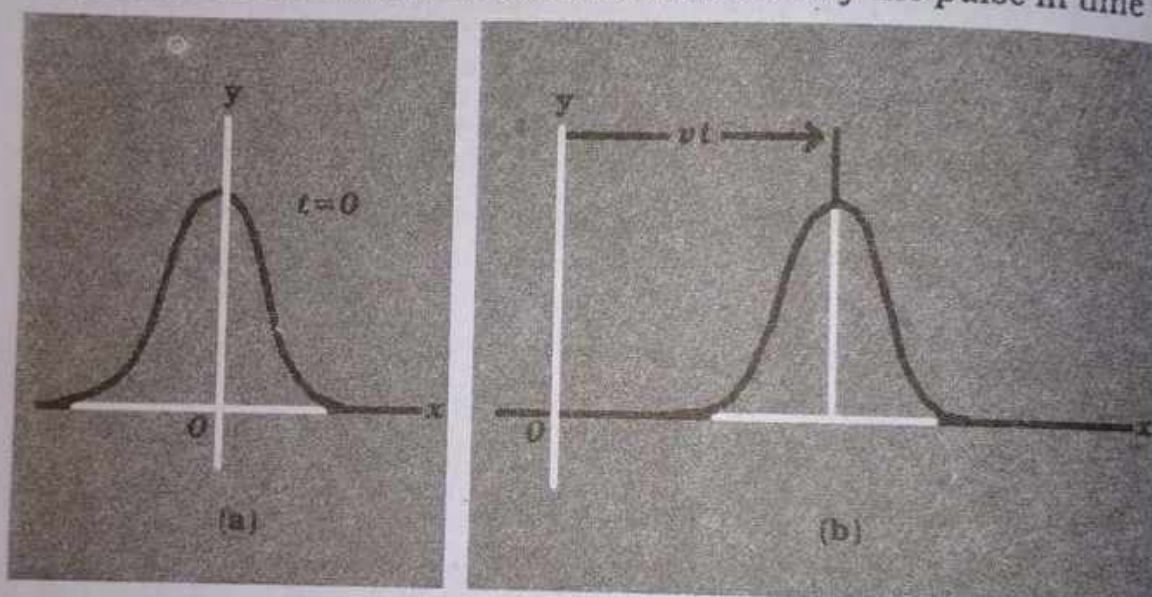


Fig. 8.7 (a) The shape of a string (in this case a pulse) at $t = 0$. (b) At a later time t the pulse has travelled to the right a distance $x = vt$.

is vt Fig. 8.7(b). Suppose the shape of the pulse does not change under this condition we can represent the displacement Y for all the later time measured in a stationary frame with the origin at O as

$$y = f(x + vt) \quad 8.14 (a)$$

Similarly if the wave pulse is travelling to the left, its displacement is given by

$$y = f(x - vt) \quad 8.14 (b)$$

Since y depends on two variables, x and t we often represent it by $y(x, t)$, which is read, " y as a function of x and t ". It is important to understand the meaning of y .

Consider a particular point P on the string, identified by a particular value of the co-ordinates x and t . As the wave passes the point P , the y coordinate of this point will increase, attain a maximum value and then decreases to zero. Therefore the wave function gives y co-ordinate of point at any time t . Furthermore, if t is fixed then the wave function y as a function of x , defines a curve representing actual shape of the pulse at this time. This is equivalent to a "snapshot" of the wave at this time.

To find the velocity of the pulse we can calculate how far its crest moves in a short time and then divide this distance by the interval. The crest of the pulse corresponds to that point for which y is maximum. In order to follow the motion of the crest, some particular value, say x_0 , must be substituted for $(x - vt)$. In order to stay with the crest, we must have $x - vt = x_0$, no matter how x and t change. This gives the equation of motion of the crest. Putting $t = 0$ in the above equation, $x = x_0$ and at later time dt , the crest is found at a distance $x = x_0 + vdt$. Thus the distance covered by the crest in time dt is clearly $dx = x_0 + vdt - x_0 = vdt$. Thus the wave speeds which is also often referred to as the phase velocity as given by

$$v = \frac{dx}{dt}$$

The phase velocity must not be confused with the transverse velocity of a particle in the medium.

Water waves in ripple tank:

When a stone is dropped into a smooth pond Fig.8.8 (a), a disturbance is produced. The disturbance extends over the surface

of water in the form of circles centered at the place where the stone was dropped. As a matter of fact when the stone hits the water surface, it forces the latter downwards and so produces a depression (through), with a hump (crest) all around it.

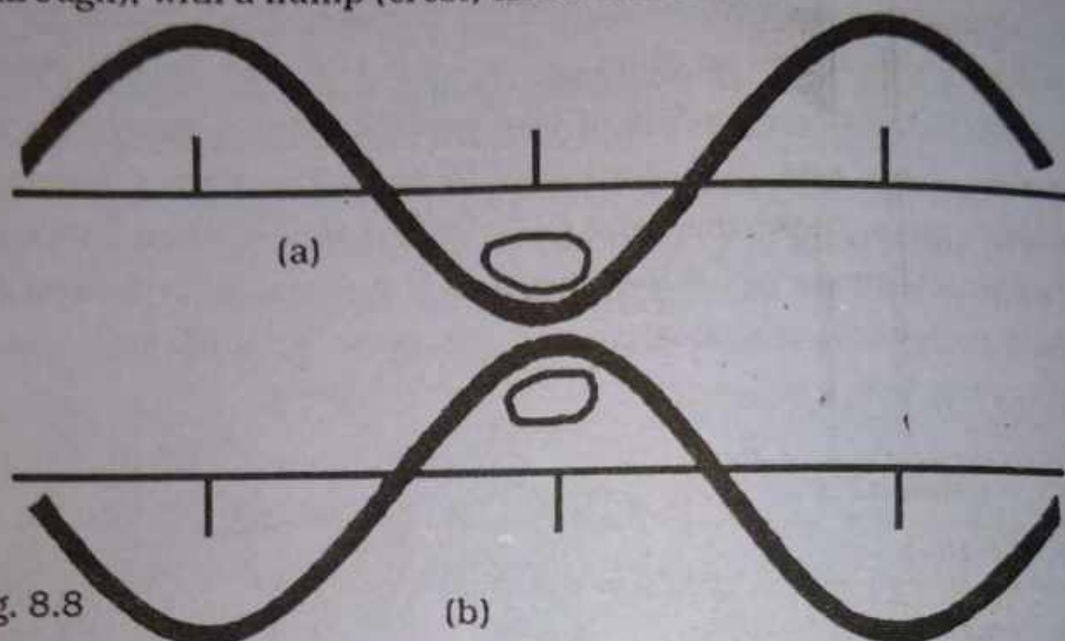


Fig. 8.8

The water regains its original position due to the difference of pressure so produced, but then instead of remaining at rest it overshoots this position because of its inertia, just as does a pendulum when pulled to one side of its equilibrium position Fig.8.8(b). Thus a circular ridge is formed. This expands into a large circle and is followed by a second circular ridge which expands and so on. The result is that the surface is soon covered with a series of circular crest, separated by circular troughs, all moving away from the centre of disturbance. Water waves can be produced by using a ripple tank show in Fig.8.9.

In its simplest form the ripple tank consists of a sheet of glass round the edges of which a beach is formed so that shallow layer of water may be held on the glass. The shelving of beach helps to reduce the unwanted reflection of the waves from the sides of the tank. A bright electric lamp is placed below the tank and this forms shadow of the waves on the screen if above the tank a mirror inclined at 45° is placed so as to reflect the light on to the screen.

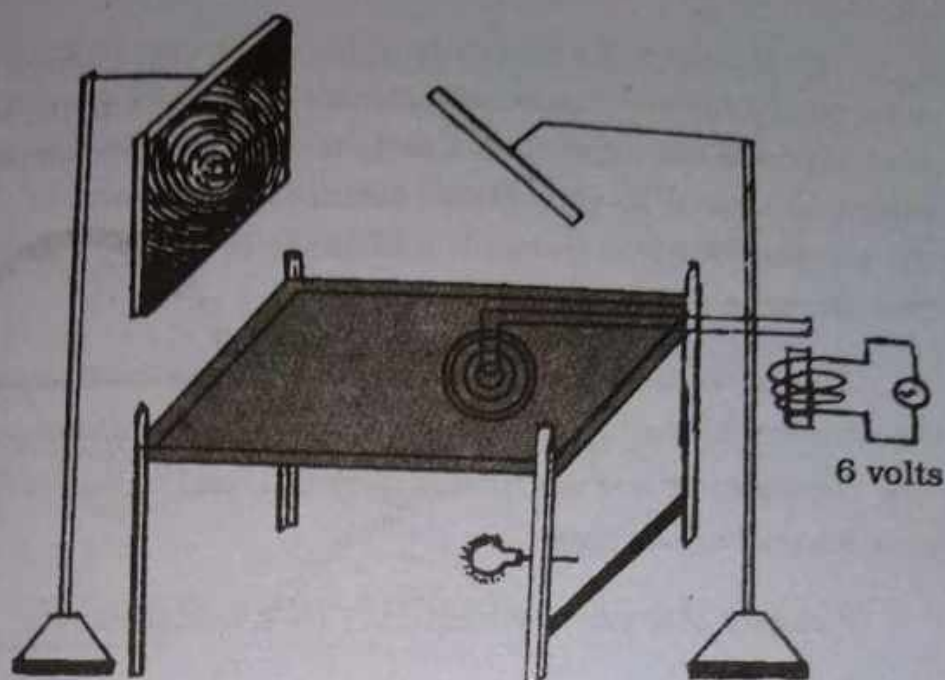


Fig. 8.9 The image of the waves is seen on the screen which is just a piece of white drawing paper placed under the ripple tank.

First we dip a finger or a pencil in the water and notice that the circular waves spread out from it whenever it moves slightly. If the finger is dipped in and out regularly at the same place, waves consisting of alternate crests and troughs follow one another at regular distances. The distance from one crest to next one is called the wave length.

If the finger is replaced by a dipper fixed to a vibrating steel strip, this will make the dippings occur at perfectly regular intervals and, further, these intervals can readily be altered by altering the length of the vibrating strip.

The state of affair set up on the water surface is called a wave motion, and three important terms are used to define it completely. These are velocity, wave length and frequency. We know that the frequency is the number of complete waves produced each second. In our example, it is the number of in and out movements made by the dipper in one second. One in and out movement of the dipper is called one complete vibration.

By the time the dipper is entering the water for the second time so as to produce a second trough, the trough produced by the first "dip" will have travelled a certain distance and this distance is clearly one wave length. Thus, during one vibration of the dipper the wavefronts travel outward a distance of one wavelength farther from the centre.

In one second the total distance a wavefront travels will be the wavelength multiplied by the number of vibrations per second. Now the distance a wave travels in one second is the velocity of the wave and hence we have

Velocity of wave(ϕ) = frequency (ν) $\times$ wavelength (λ).

$$\phi = \nu\lambda \quad (8.16)$$

knowing the frequency and wavelength, the velocity of wave can be calculated.

8.10 ENERGY IN WAVES

All mechanical waves travel through a medium, they carry energy with them and hence they are also called carriers of energy. This is easily demonstrated by having a weight on a stretched string and then sending a pulse down the string as shown in Fig. 8.10 (a). When the pulse meets the weight, the weight will be momentarily displaced as shown in Fig. 8.10 (b). Actually in this process energy is transferred to the weight since work must be done in moving it upward i.e. work is done at the cost of energy which is defined as the capability of doing work.

We describe here the state at which energy is carried along a string. We consider a sinusoidal wave (Harmonic wave) and calculate the power for this one dimensional wave.

Consider a harmonic wave travelling along a string Fig.8.10 (c). The source of the energy is some external agent at the left end

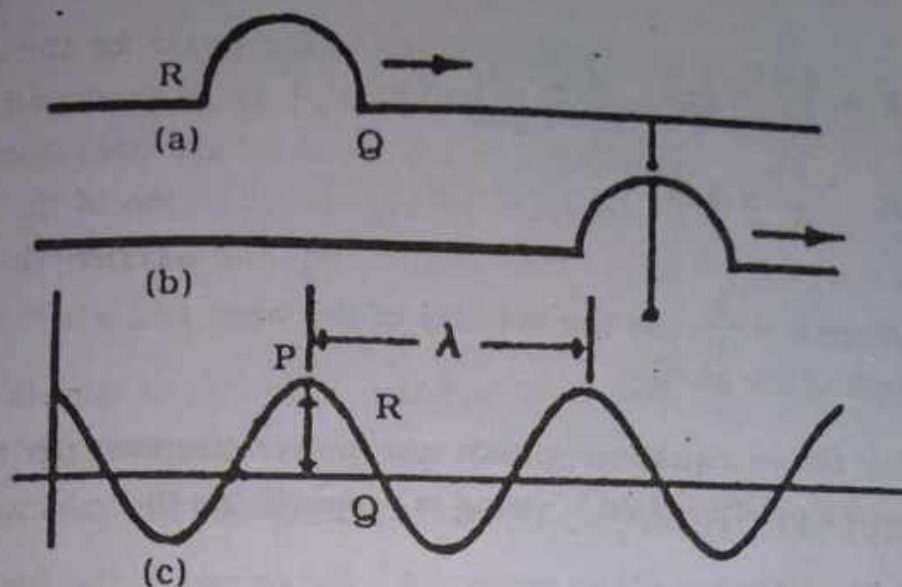


Fig. 8.10 The screen which is just a piece of white drawing paper placed under the ripple tank.

of the string, which does work in producing the oscillation in the string. The points P, Q and R represent various segments of the string which move vertically. The wave moves a distance equal to one wavelength λ , in a time T. To determine power transmitted by the wave, we first calculate the energy contained in one wavelength and divide the result by the time T.

We know that every point on the string moves vertically up or down. Thus every segment of equal mass has the same total energy. The energy of the segment at P is entirely potential energy since the segment is momentarily stationary. The energy of the segment at Q is entirely kinetic energy and segment at R has both kinetic and potential energies. Now consider the segment Q, which has a maximum transverse velocity $(\partial_y)_{\max}$ and mass Δm . The total energy of this segment is given by

$$\Delta E = \frac{1}{2} \Delta m (\partial_y)_{\max}^2 \quad 8.17$$

The power, P, of the wave is obtained by dividing the above expression for ΔE by the time period T.

$$P = \frac{\Delta E}{T} = \frac{1}{2} \mu \frac{\lambda}{T} \omega^2 y_0^2$$

$$P = \frac{1}{2} \mu v \omega^2 y_0^2$$

8.18

Where $v = \frac{\lambda}{T}$ is the velocity of the wave and μ is mass per unit length of the string

The above equation shows the power transmitted by harmonic waves produced on a string is proportional to

(i) the velocity of the wave (ii) the square of the frequency and (iii) the square of the amplitude. In fact, all the harmonic waves have the following general properties.

The power transmitted by any harmonic wave is proportional to the square of the frequency and to the square of the amplitude. It should be noted here that energy is not carried by only mechanical waves (waves in a string, water waves, sound waves etc). It is also carried by electro-magnetic waves like light waves, micro waves, radio waves, thermal radiation, etc. The energy carried by electro-magnetic waves also depends directly upon their frequencies. This means high frequency waves have more energy than low frequency waves. For example, energy carried by ultra violet light is much greater than those carried by infra-red light because the frequencies of ultra-violet light are much greater than those of infra-red light.

8.11 STANDING WAVES

If one end of a string, preferably a thick rubber, is fixed to a wall and the other end is held in the hand, wave can be set up along the string when stretched and flicked upward. This pulse which is in the form of a crest is reflected at the fixed end and passes back along the string. The crest is reversed on reflection and becomes a trough as shown in Fig.8.11.

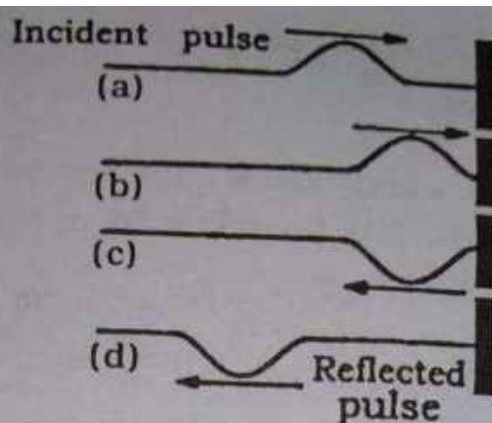


Fig. 8.11 The reflection of a travelling wave at the fixed end of a stretched string. Note that the reflected pulse is inverted, but its shape remains the same.

However, when the string is tightly stretched between two fixed supports and then plucked, the crest extends the whole distance between the supports. This distance is clearly half the wavelength of the transverse wave developed in the string as shown in Fig. 8.12. At each end, the wave suffers a phase change. The crest W_1 on reflection at Q becomes a trough W_2 and the trough W_2 becomes crest W_1 on reflection at P .

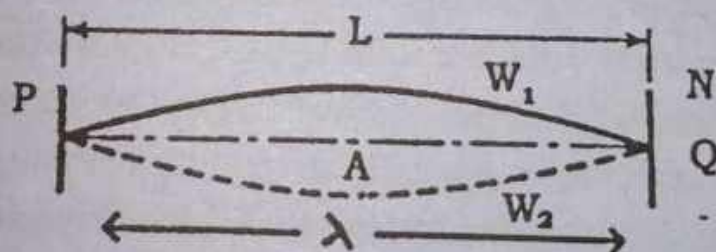


Fig. 8.12 Stationary waves set up in a stretched strings with its two clamped ends as nodes and the centre as an antinode are shown.

This simple vibration of a string consists of a transverse wave passing along the string and being reflected at each end in turn. At Q the incident and reflected waves are always equal in amplitude and opposite in phase and so the end is stationary and we get what we call a stationary wave or standing wave in the string.

The point where displacement is maximum is called anti-node denoted by A . And that where the displacement is minimum (zero) is called a node denoted by N .

The incident and reflected waves will combine according to the principle of superposition.

Consider two sinusoidal waves with the same amplitude, frequency and wavelength, but travelling in opposite directions. These waves can be written as.

$$y_1 = A_0 \sin(kx - \omega t) \text{ and } y_2 = A_0 \sin(kx + \omega t)$$

Where y_1 represents displacement of a wave travelling to the right taken as our incident wave and y_2 represents displacement of a wave travelling to the left, which is our reflected wave. The resultant wave function Y is given by

$$Y = y_1 + y_2 = A_0 \sin(kx - \omega t) + A_0 \sin(kx + \omega t)$$

Where $k = 2\pi/\lambda$ and $\omega = 2\pi f$. Using the trigonometric identity, we get

$$Y = (2A_0 \sin kx) \cos \omega t \quad 8.19$$

This expression represents the wave function of a standing wave which is entirely different from the expression which is used to describe a travelling harmonic wave. From this expression, it is obvious that a standing wave has an angular velocity ω (a constant) and an amplitude given by $2A_0 \sin kx$. This means every particle of the string vibrates in SHM with same frequency ω . However the amplitude of the motion of a given particle depends on $\sin kx$. This is in contrast to the situation involving a travelling harmonic wave, in which all particles oscillate with the same amplitude and same frequency.

As the amplitude of the standing wave for any value of x is equal to $2A_0 \sin kx$ therefore the maximum amplitude has the value $2A_0$. It happens when $\sin kx = \pm 1$, i.e.,

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \quad 8.20(a)$$

Since $k = \frac{2\pi}{\lambda}$ the maximum amplitude points called anti-nodes (also mentioned earlier), given by

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots, \frac{n\lambda}{4} \quad 8.20(b)$$

Where $n = 1, 3, 5 \dots$ It is to be noted that anti-nodes are separated by a distance of $\lambda/2$. Similarly the standing wave has a minimum amplitude points when $\sin kx = 0$ or when

$$kx = 0, \pi, 2\pi, 3\pi, \dots, n\pi \quad 8.21(a)$$

substituting for $k = \frac{2\pi}{\lambda}$ in Eq. 8.21 (a) we get

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots, \frac{n\lambda}{2} \dots \dots \dots 8.21(b)$$

Where $n = 1, 2, 3, \dots$. These points of zero amplitude called nodes, are also separated by $\frac{\lambda}{2}$. The distance between a node and an antinode is $\frac{\lambda}{4}$.

8.12 FUNDAMENTAL FREQUENCY AND HARMONICS

We consider a stretched string of length L fixed at both ends as shown in Fig. 8.13 (a). Standing waves are set up by a continuous superposition of waves incident and reflected from the ends. The string has a number of natural patterns of vibrations called normal modes (as shown in Fig. 8.13 a, b, c, and d). Each of these modes has a characteristic frequency. The frequencies are easily calculated.

It is to be noted that the ends of the string must always be nodes because they are fixed. If the string is displaced at the middle point and released, the vibration is as shown in Fig. 8.13 (a). In this case, the middle point of the string has the maximum displacement, called anti-node. For this normal mode of vibration, the length of the string is equal to $\frac{\lambda_1}{2}$ which is distance between two neighbouring nodes. Since the length of the string is L , so we have

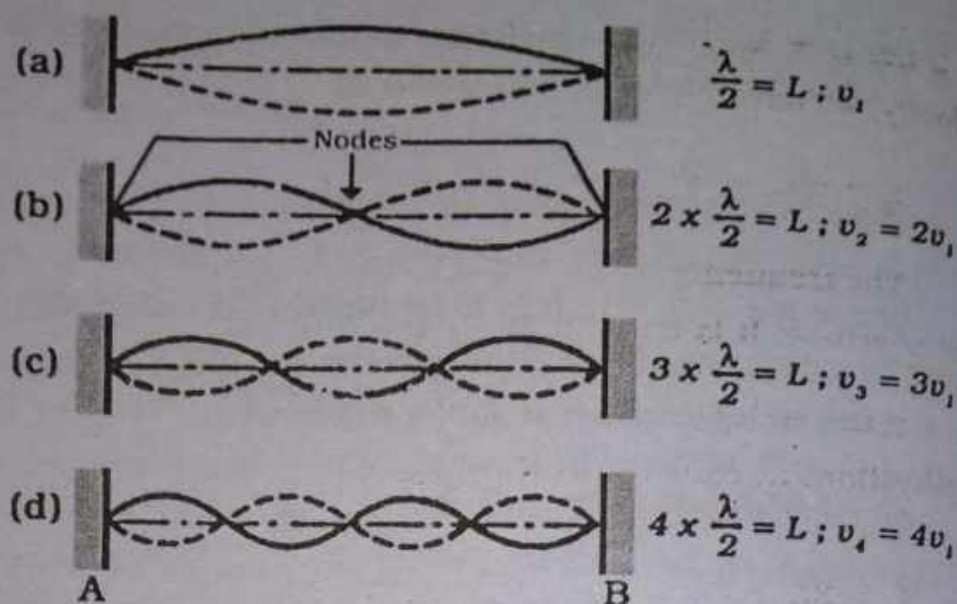


Fig.8.13 Standing waves on a stretched string. All such waves must have nodes at the termination points.

$$L = \lambda_1/2$$

$$\text{or } \lambda_1 = 2L$$

If f is the frequency of fundamental mode of vibration and v is velocity of the wave then we have

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

The frequency so obtained is called the fundamental frequency or the first harmonic of the standing wave.

The same string may be made to give out next higher order frequency instead of the fundamental, without altering the length of the string. This is done as follows:

A feather or a finger is very lightly pressed at a point half way between the middle point and the end of the string. The mode of vibration of the string is given in Fig. 8.13 (b). The string consists of two loops. The two ends of the string and its mid point are nodes whereas there are two anti-nodes each lying between two neighbouring nodes. Let λ_2 be the wavelength of this mode of vibration of the string and is equal

to L i.e. $L = \lambda_2$. If f_2 is the frequency of this mode and v is the velocity, we have

$$f_2 = v/\lambda_2 = v/L = 2f_1$$

The frequency so obtained is called the second harmonic or first overtone. It is equal to twice the fundamental frequency.

If the string is made to vibrate in three loops, the frequency of vibration, f_3 , called third harmonic or second overtone, is given by

$$f_3 = 3f_1$$

In general if the string vibrates in n loops, the corresponding frequency f_n is given by

$$f_n = nf_1 \quad 8.22$$

8.13 SONOMETER

A sonometer is a practical application of stretched strings for vibrating a portion of the string into desired number of loop. It consists essentially of a thin metallic wire stretched across two bridges A and B, on the top of a hollow, wooden sounding box about one metre long. One end of the wire is fastened to a peg at one end of the box. The other end passes over a smooth frictionless pulley fixed at the other end of the box. The pulley carries a scale pan, S, so that it can be loaded to have any desired tension in the string. In between the two bridges A and B, there is a movable bridge C sliding over a scale to adjust the length of the vibrating portion of the string. Here the bridges A, B, C always form nodes at their respective positions. If there is no bridge between the bridges A and B, then the simplest mode of vibration is one in which the portion of the string between A and B vibrates in one loop only.

The frequency of vibration of the string is minimum in this case. This corresponds to the fundamental frequency. This is denoted by f_1 . Now place the bridge C just in the middle of A and B.

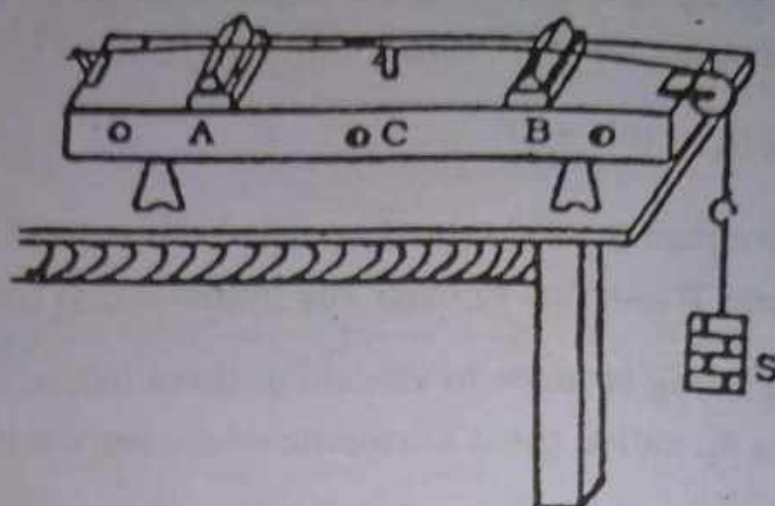


Fig. 8.14

The string will vibrate in two loops with a frequency $f_2 = 2f_1$. If the bridge is placed in such a position that the string vibrates in three loops, then the frequency excited in the string will be $3f_1$, and so on.

All the laws of transverse vibration of string can be verified using sonometer. If L is the length of vibrating segment of the string, T is the tension and μ is the mass per unit length of the wire, then the frequency produced in the string is given by

$$f_1 = \frac{n\vartheta}{\lambda} = \frac{n}{2L} \sqrt{\frac{T}{\mu}} \quad \therefore \quad \vartheta = \sqrt{\frac{T}{\mu}} \quad 8.23$$

Where $n = 1, 2, 3, \dots$ i.e. the frequencies are integral multiple of the fundamental frequency. The $\vartheta = \sqrt{T/\mu}$ represents the speed of the wave on the string.

The above equation shows that

(i) the frequency produced in the string for a given tension is inversely proportional to its length i.e.

$$f \propto \frac{1}{L}$$

(ii) for a string of given length and material, the frequency varies directly as a square root of the tension T i.e.

$$f \propto \sqrt{T}$$

(iii) The frequency of vibration of strings of the same length and subjected to the same tension varies inversely as the square root of the mass per unit length, μ , of the string i.e.

$$f \propto \frac{1}{\sqrt{\mu}}$$

8.14 LONGITUDINAL WAVES

A second type of waves, called longitudinal waves are the waves in which the particles of the disturbed medium undergo displacement in a direction parallel to the direction of wave motion. These waves are produced in substances which are elastic and compressible like gases and wire springs. Sound waves are longitudinal which result from the disturbance of the medium. The disturbance corresponds to a series of high and low pressure regions that travel through air or through any other material medium with a certain velocity. A longitudinal pulse can be easily produced in a spring. Fig.8.15 represents a spring whose turns are large and one of whose ends is supported from a hook in the wall. If you compress a few turns of the spring near its left end, these move slightly and compress those just ahead and these in turn squeeze together the turns still further and thus a pulse, or compression wave, goes along the spring.

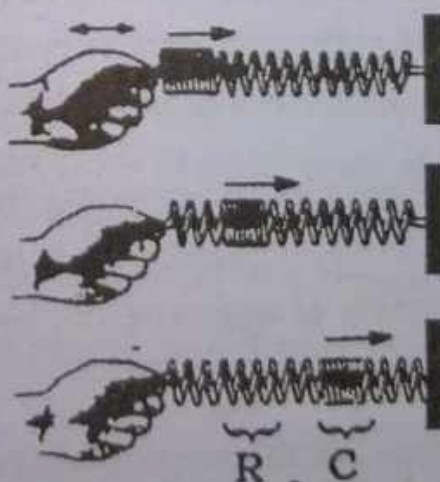


Fig. 8.15 A longitudinal pulse along a stretched spring. The disturbance of the medium (the displacement of the coils) is in the direction of the wave motion. For the starting motion described in the text, the compressed region C is followed by an extended region, R.

Next let the left end of the spring be given a quick pull so that the turns nearly are drawn apart for an instant. Then the adjacent turns will be pulled apart, one after another, until this disturbance reaches the right end. Thus it is clear that any push or pull given to the spring at one end is transmitted as a push or pull to the other end. Waves of this sort in which the particles of the transmitted medium move back and forth in the direction of propagation of the waves are called compressional, longitudinal or pressure waves.

Sound waves are the most important example of longitudinal waves. They can travel through any material medium i.e. gases, solids and liquids with a speed which depends upon the properties of the medium. As sound waves travel through a medium, the particles of the medium vibrate along the direction of propagation of the wave motion. This is in contrast to a transverse wave motion in which the particles of the medium vibrate in a direction perpendicular to the direction of wave motion. The displacement that occurs as a result of sound waves involve the longitudinal displacement of the individual molecules from the equilibrium position. This results in a series of high and low pressure regions called compressions and rarefactions, respectively. Graphically the longitudinal wave is represented like transverse wave.

There are three categories of longitudinal mechanical waves which cover different regions of frequency (i) Audible waves (ii) Infrasonic waves and (iii) Ultrasonic waves.

Audible waves are the sound waves that lie within the range of sensitivity of human ear, typically, 20 Hz to $20,000 \text{ Hz}$. They can be generated in a variety of ways such as by musical instruments, human vocal chord and loud speakers.

Infrasonic waves, are the longitudinal waves with frequencies below audible range that is below 20 Hz . Earth quake waves are an example of infrasonic waves.

Ultrasonic waves are the longitudinal waves with frequencies above the audible range that is above 20 KHz. They can be generated by inducing vibrations in a quartz crystal with an applied alternating electric field.

8.15 SPEED OF SOUND WAVES

Newton's Formula for the speed of sound waves:

As we know, sound waves are compressional waves which propagate through a compressible medium such as air. The speed of such compressional waves depends upon the compressibility and the inertia of the medium. If the compressible medium has a bulk modulus denoted by B and density (inertial property) denoted by ρ , the speed of sound in that medium is given by

$$v = \sqrt{B/\rho} \quad 8.24$$

The expression given by Eq. 8.24 is known as Newton's formula for the speed of sound waves.

Note that the bulk modulus, B , is the ratio of the change in pressure, Δp , to the resulting fractional change in volume, $-\Delta v/v$

$$B = -\Delta p / \left(\frac{\Delta v}{v} \right) \quad 8.25$$

Here Δv is the change in original volume V . The ratio $(\Delta p/\Delta v)$ is always negative because Δv decreases as Δp increases and vice versa. This shows that B is always positive. In fact the speed of all mechanical waves can be expressed in a general form,

$$v = \sqrt{\text{elastic property} / \text{inertial property}}$$

Table 8.1
speed of sound in various media

Medium	v (m/s)
GASES	
Air (0°C)	331
Air (100°C)	336
Hydrogen (0°C)	1286
Oxygen (0°C)	317
Helium (0°C)	972
LIQUIDS at 25°C	
Water	1493
Methyl alcohol	1143
Sea water	1533
SOLIDS	
Aluminium	5100
Copper	3560
Iron	5130
Lead	1322
Vulcanized rubber	54

8.16 LAPLACE'S CORRECTION

The Newton's formula (Eq 8.24) was obtained on the assumption that the compressions and rare-factions take place at constant temperature. This kind of process is called an isothermal process and Boyle's Law is obeyed throughout the change from the initial pressure and volume to the final pressure and volume. Under this condition the bulk modulus B is equal to the pressure of gas.

Therefore

$$v = \sqrt{P/\rho}$$

However, in a sound wave, the wave motion is so rapid and the heat conductivity is so low that there is insufficient time for the heat produced in the compressed regions to be conducted to the rarefied regions. This means that the process is no more isothermal. Therefore compressions and rare-factions occur adiabatically

(A process in which heat does not flow into or out of the system) and not isothermally as was assumed by Newton.

For such a case, the bulk modulus of the gas is γ times the pressure of the gas where γ is the ratio of molar specific heat of gas at constant pressure (C_p) to the molar specific heat at constant volume (C_v). The speed of sound is thus.

$$v = \sqrt{\gamma p / \rho} \quad 8.26$$

This is called Laplace's correction.

If we use the ideal gas law $PV = nRT$ then we can rewrite Eq 8.26 in the form

$$v = \sqrt{\frac{\gamma RT}{M}} \quad 8.27$$

Where M is the molecular mass of the gas in units of kg/mole, n is number of moles R is universal gas constant and has value 8.314J/mol-K and T is temperature expressed on kelvin scale.

Using Eq 8.27 we can calculate the velocity of sound in air at 0°C . We know air consists of approximately 80% of Nitrogen and 20% Oxygen. The masses of Nitrogen and Oxygen molecules are 28 a.m.u and 32 a.m.u respectively. Therefore the mean molecular mass for air is:

$$\text{Mean molecular mass for air} = 0.8 \times 28 \text{ a.m.u} + 0.2 \times 32 \text{ a.m.u} = 28.8 \text{ a.m.u}$$

$$\text{Then } M = 28.8 \text{ g/mole} = 0.0288 \text{ kg/mole}$$

The velocity of sound at 0°C is

$$v = \sqrt{1.40 \times (8.314\text{J/mole} - \text{K}) \times (273\text{K}) / 0.0288 \text{ kg mole}}$$

$$v = 332 \text{ m/s at } 0^\circ\text{C}$$

At any other temperature T , the speed of sound in air can be obtained by multiplying this result by $\sqrt{T/273}$. For example, at an altitude of 10000 ft (3.05km), the temperature is about -50°C or 223K therefore

$$v = (332 \text{ m/s}) \times \sqrt{\frac{223}{273}} = 300 \text{ m/s}$$

8.17 MUSICAL SOUND AND NOISE

We perceive some sound as noise and some sound as music. For example the slamming of a door or the rumbling of a truck is considered as a noise, whereas, the sound produced by a piano or a guitar is regarded as musical. These two types of sound are represented in form of graphical patterns as shown in Fig.8.16 (a) and (b). The Fig.8.16 (a) represents an irregular, non symmetric, random fluctuations producing disagreeable sensation hence called noise: The Fig 8.16(b) shows a regular, symmetric and periodic fluctuation, which produce a smooth pleasant sensation hence called music. The physical difference between these two types of sound is better understood by studying their spectra. The spectrum of noise and the spectrum of musical sound are sketched in Fig. 18.16 (c) and Fig. 18.16 (d) respectively the spectrum of noise contains frequency components which are not harmonically related i.e. the frequency components are not integral multiple of the fundamental. Besides this the amplitude of each frequency components are mostly not symmetric.

The spectrum of musical sound contains frequency components which are harmonically related i.e. the frequency components are integral multiple of the fundamental. Besides this the fluctuations in amplitude are symmetric.

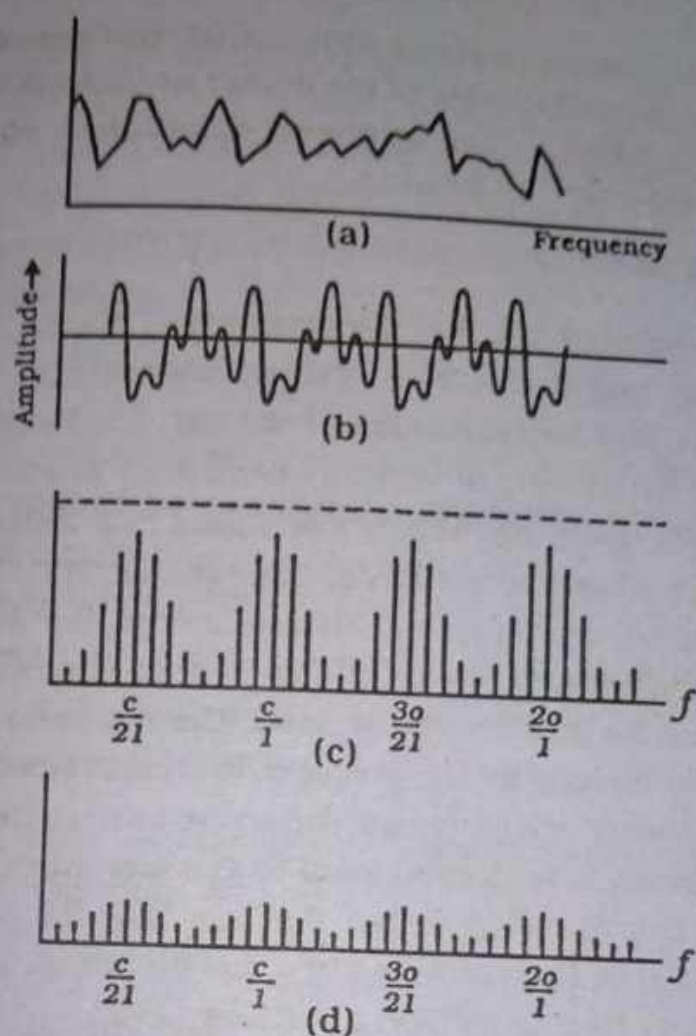


Fig. 8.16 (a) , (b) The wave pattern of a violin (c) Power spectrum of noise (d) power spectrum of musical signal.

Characteristics of a musical sound

Musical sounds or tones can be distinguished from one another by the following characteristics:

- (i) Intensity (loudness)
- (ii) Pitch or frequency
- (iii) Quality

Intensity of sound is measured in terms of the energy of the sound wave, passing every second through a unit area perpendicular to the direction of propagation of sound and is measured in watt/m^2 . For example, if one talks as loudly as possible, the average speech power is about 10^{-3} W/m^2 ; and if one speaks in as

weak a voice as possible (without whispering), the power drops to 10^{-7} W/m^2 . The intensity levels of the sound at various frequencies are shown in Fig 8.16. The figure also shows that threshold of pain and threshold of hearing.

(i) Loudness

It may be pointed out that the intensity of sound is purely a physical quantity and it can be measured accurately. It does not depend upon the auditory sensation of the ear. On the other hand Loudness is the magnitude of auditory sensation produced by the sound. It depends upon the intensity of sound and as well as it depends upon the auditory sensation of the ear. A normal human ear is very sensitive detector of sound. It can record the least intense sound (10^{-12} W/m^2) which is one billionth of the maximum sound intensity that can be heard without pain. The loudness of a sound does not increase directly as the power delivered to the ear increases but seems to vary roughly as the logarithm of the power. It means the loudness, L , is proportional to the logarithm of intensity I i.e. $L \propto \log I$. This is called Weber Fechner law. If I and I_0 represent the intensities of two sound waves, the difference in loudness, known as intensity level is defined by the equation

$$\text{intensity level} = P = \log \frac{I}{I_0} \quad 8.28(a)$$

where I_0 is equal to 10^{-12} Wm^{-2} corresponding roughly to the faintest sound that can be heard. This is regarded as arbitrary reference level. If I is taken ten times of I_0 ($I = 10 I_0$), then the intensity level is equal to one unit. This unit of intensity level is called a bel (B).

A bel is a rather large unit, so we usually refer to sound intensity levels in terms of the decibel (dB): $1 \text{ dB} = \frac{1}{10} \text{ B}$, thus

$$\beta = 10 \log \frac{I}{I_0} \quad 8.28(b)$$

$$\text{When } \frac{I}{I_0} = 1, \beta = 0 \text{ dB}$$

$$\frac{I}{I_0} = 10, \beta = 10 \text{ dB}$$

$$\frac{I}{I_0} = 10^2, \beta = 20 \text{ dB}$$

The loudness of a sound is roughly correlated with the intensity level of the note. For a long time it was accepted that the correlation was perfect. But there is now no doubt that decibel scale of intensity level no more correlates with the subjective sensation of loudness. A power law between loudness and intensity level of the note has been proposed,

$$L = K \left(\frac{I}{I_0} \right)^{0.3}$$

Where K is an arbitrary constant which can be evaluated after defining the unit of loudness 'sone'. The sone is defined as

$$1 \text{ sone} = 40 \text{ dB at } 1000 \text{ Hz}$$

$$\frac{I}{I_0} = 10^4 \text{ when } \beta = 40 \text{ dB, therefore } K = 1/16.$$

$$\therefore L = \frac{1}{16} \left(\frac{I}{I_0} \right)^{0.3} \quad 8.28(c)$$

The sone scale of loudness has been recognised internationally, and being practised.

(ii) Pitch

It is defined as the sensation that sound produces in the ear of a listener and is clearly related to the frequency of sound. Frequency and pitch are both measured in Hertz (Hz). Thus greater the frequency the greater the pitch and lower the frequency lower the pitch. The pitch of sound produced by various physical instruments usually depends upon the natural resonant frequency of the instrument. This is our common experience that sound of a sparrow is shrill because of its high pitch since it has a high frequency. Whereas the sound of a lamb is grave due to its low pitch depending upon its low frequency.

(iii) Quality

Quality or timbre is a characteristic of a musical sound. It is quality which enables us to distinguish between notes of the same pitch and intensity when played on different instruments or sung by different voices. Even instruments of the same kind may yield

notes of different quality. For example, it is the quality of the tones produced by two violins which makes great difference in value between the instruments. We recognise the voice of a friend over the telephone by quality.

Helmholtz was the first to discover the cause of difference in musical tones which is called quality.

The difference in the sound produced by two notes of the same pitch and intensity is due to the difference in their resultant waveforms. The resultant wave form of any sound is obtained by

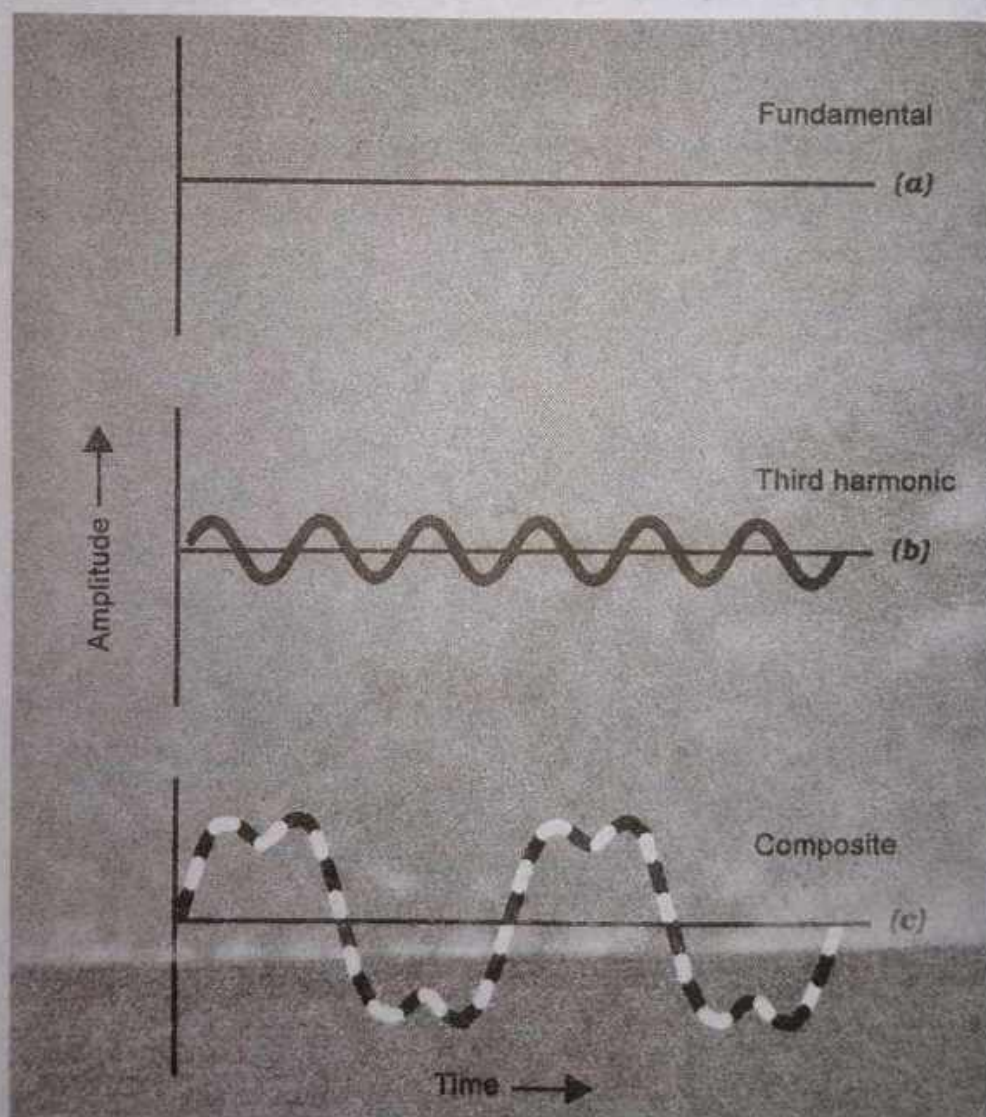


Fig. 8.17 The combination or superposition of two waves. (a) the fundamental and (b) the third harmonic.

combining the amplitudes of fundamental and the harmonics of the given sound.

The resultant waveforms are formed due to combination of fundamental and harmonics as shown in Fig.8.17 (a) and 8.17(b). In Fig: 8.17 (c) , the fundamental and third harmonic are combined together to give a resultant wave form.

8.18 SUPERPOSITION OF SOUND WAVES

According to principle of superposition when two or more waves in the same (linear) medium travel the net displacement of the medium caused by the resultant wave at any point is equal to the algebraic sum of the displacements of all the waves. We shall apply this principle to two harmonic waves travelling in the same direction in a medium. The two waves are travelling to the right and have the same frequency, same wavelength and same amplitude, but they differ in phase, we can express their individual wave function displacements as

$$y_1 = A_0 \sin (kx - \omega t) \text{ and } y_2 = A_0 \sin (kx - \omega t - \phi)$$

Hence the resultant wave, function displacement is given by

$$y = y_1 + y_2 = A_0 [\sin (kx - \omega t) + \sin (kx - \omega t - \phi)]$$

We make use of following trigonometric identity to simplify it

$$\sin \alpha + \sin \beta = 2 \cos \left(\frac{\alpha - \beta}{2} \right) \sin \left(\frac{\alpha + \beta}{2} \right)$$

Let $\alpha = kx - \omega t$ and $\beta = kx - \omega t - \phi$, we get

$$y = (2A_0 \cos \frac{\phi}{2}) \sin (kx - \omega t - \frac{\phi}{2}) \quad 8.29$$

There are several important features of this result. It can be easily seen that the resultant wave function y is also harmonic and has the same frequency and wavelength as the individual waves. The amplitude of the resultant wave is $2A_0 \cos \frac{\phi}{2}$ and its phase is equal to $\frac{\phi}{2}$. If the phase constant ϕ is zero, then $\cos \frac{\phi}{2}$ is unity

and the amplitude of the resultant wave is $2A_0$. This means that the amplitude of the resultant wave is twice as large as that of either of individual wave having the same wave length. In this case, the waves are said to interfere constructively that is the crests of one fall on the crests of the other and similar is the case for troughs as shown by the dotted lines in Fig.8.18 (a).

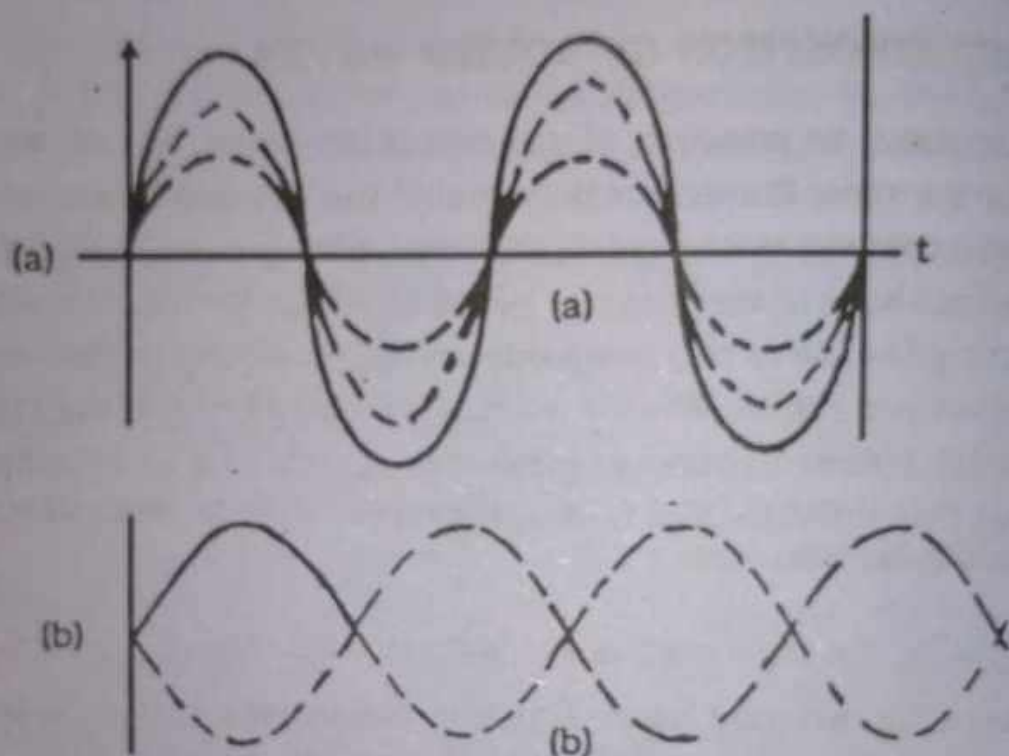


Fig. 8.18 (a), (b)

In general, constructive interference takes place where $\cos \frac{\phi}{2} = \pm 1$, or where $\phi = 0, 2\pi, 4\pi, \dots, 2n\pi$ Where n is an integer, on the other hand if $\phi = \pi$ radians (or any odd multiple of π) then $\frac{\phi}{2} = \cos \frac{\pi}{2} = 0$ and the resultant wave has zero amplitude every where. In this case, the two waves are said to interfere destructively i.e. the crests of one wave coincide with the troughs of the second and vice versa as shown in Fig.8.18 (b) and their displacement cancel at every point.

8.19 BEATS

So far we have been discussing the interference or superposition of two waves having the same frequency and travelling in op-

posite direction. We now consider another type of interference effect which results from the superposition of two waves having slightly different frequencies and travelling in the same direction. Under this condition when two waves are observed at a given point, they are periodically in and out of phase, i.e. there is an alternation in time between constructive and destructive interference. Thus, we refer to this phenomenon as interference in time. For example if two tuning forks of slightly different frequencies are struck on rubber pad, one hears a sound of changing intensity, called beats. Hence we can define beats as the periodic variation in intensity at a given point due to superposition of two wave having slightly different frequencies. The number of beats that one hears per second (or beats) is equal to the difference in frequency between two sources. The maximum beat frequency that the human ear can detect is about 7 beats per second. When the beat frequency (number of beats produced per second) is greater than seven, we can not hear them clearly. One can use this effect to tune instrument such as piano, by beating a note against a reference tone of known frequency. The string can then be adjusted to equal the frequency of the reference by tightening or lossening it until no beats are heard.

Now consider two waves with equal amplitude travelling through a medium in the same direction having slightly different frequencies f_1 and f_2 . We can represent the displacement that each wave would produce at a point as:

$$y_1 = A_0 \cos 2\pi f_1 t \text{ and } y_2 = A_0 \cos 2\pi f_2 t$$

$$y = y_1 + y_2 = A_0 (\cos 2\pi f_1 t + \cos 2\pi f_2 t)$$

$$= 2A_0 \cos 2\pi t \left(\frac{f_1 - f_2}{2} \right) \cos 2\pi t \left(\frac{f_1 + f_2}{2} \right) \quad 8.30$$

Here y_1 and y_2 are the instantaneous displacements caused by the individual wave as shown in Fig. 8.19 (a) and y is the resultant displacement caused by the two waves together as shown in Fig. 8.19 (b)

The resultant displacement has an effective frequency equal to the average frequency, $(f_1 + f_2)/2$, and an amplitude given by

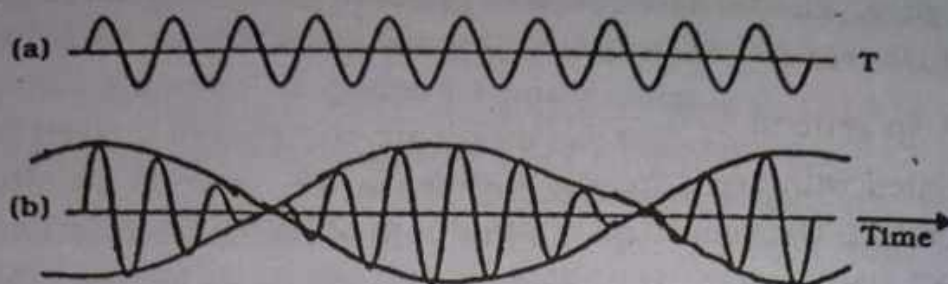


Fig. 8.19 Two waves with slightly different frequencies combine to produce beats.

$$A = 2A_0 \cos 2\pi \left(\frac{f_1 - f_2}{2} \right) t \quad 8.31$$

That is, the amplitude varies in time with frequency given by $(f_1 - f_2)/2$. When f_1 is closed to f_2 , the amplitude variation is shown as illustrated by the envelope (broken line) of the resultant wave form, Fig. 8.19 (b)

Note that a beat, for a maximum in amplitude, will be detected whenever

$$\cos 2\pi \left(\frac{f_1 - f_2}{2} \right) t = \pm 1$$

That is, there will be two maxima in each cycle. Since the amplitude varies with the frequency as $(f_1 - f_2)/2$, the number of beats per second, or the beat frequency f_b , is twice this value. That is

$$f_b = f_1 - f_2 \quad 8.32$$

For example if two tuning forks vibrate individually at frequencies of 439 Hz and 443 Hz, the resultant sound wave of combination would have a frequency of 441 Hz and a beat frequency of 4 Hz. (The listener would hear the 441 Hz sound wave go through an intensity maximum four times every second.)

8.20 ACOUSTICS

The study of production and properties of sound is called

acoustics. The term is also used to describe the way in which sound is reproduced in practical situations.

In general by acoustics we mean its second statement which is related with reproduced sound so we limit our discussion to this statement. The reproduced sound may be indistinct and confusing. It may also be clear and distinct. The quality of reproduced sound depends on several factors which we will enlist later on.

To give a clear concept of acoustics let us suppose that sound waves are produced in a big hall. These sound waves spread out, scatter and strike the surfaces of the walls, the floor and the ceiling of the hall and also the surfaces of various objects present in the hall. Some of the sound energy is absorbed by these surfaces and some is reflected. The reflected parts (consisting of many reflected sound waves) travel back and recombine to form undesirable echoes. These echoes interfere with the original sound waves coming directly from the source. The result is that we have an indistinct and unintelligible sound and hence give rise to bad acoustics.

Conditions for good acoustics

- (1) The loudness of each separate syllable should be sufficiently large.
- (2) The decay period of each syllable should be small so that the succeeding syllable can be heard clearly.
- (3) Echoes should be just sufficient to maintain the continuity of sound.
- (4) The hall should have some open windows. Sound absorbing soft porous materials like cloth, cork, asbestos etc or heavy curtains should be placed in the hall at various places so as to avoid much reflection.
- (5) For good acoustics of a hall reverberation should not be too small otherwise that will be away instantaneously and will give a dead effect to the hall.

8.21 DOPPLER EFFECT

When a source of sound or a listener, or both are in motion relative to the medium (air), the frequency and hence the pitch of the sound, as heard by the listener, is in general not the same as when listener and source are at rest. This phenomenon is referred to as the Doppler Effect. A common example is a train while whistling passes near you. A considerable change in the pitch of the sound is heard. When the train is approaching, the pitch of the sound increases whereas the pitch of the sound decreases when the train is moving away. A similar change in the pitch also occurs when moving listener passes a stationary automobile horn or siren.

Obviously, there are three general possibilities to discuss the Doppler effect, namely:

- I. When the listener is moving and the source is at rest.
- II. When the source is moving and the listener is at rest.
- III. When both the source and the listener are moving.

I(a). Suppose the listener is moving toward a stationary source as shown in Fig.8.20 with speed v_o . Suppose further that the source emits a wave with frequency v and wavelength $\lambda = v/v$. The Fig.8.20 shows several wave crests separated by equal distances 1λ (1 wavelength). The waves approaching the moving listener have a speed of propagation relative to the listener $v + v_o$. Thus the frequency v' heard by the listener is

$$\begin{aligned}v' &= \frac{v + v_o}{\lambda} = \frac{v + v_o}{v/v} \\&= \left(\frac{v + v_o}{v}\right)v = \left(1 + \frac{v_o}{v}\right)v \\v' &= v + \left(\frac{v_o}{v}\right)v\end{aligned}\tag{8.33}$$

Therefore the listener moving toward a source at rest, detects a larger frequency and hence higher pitch. Consequently, the change in pitch in this case is

$$v' - v = \left(\frac{v_0}{v} \right) v$$

8.34

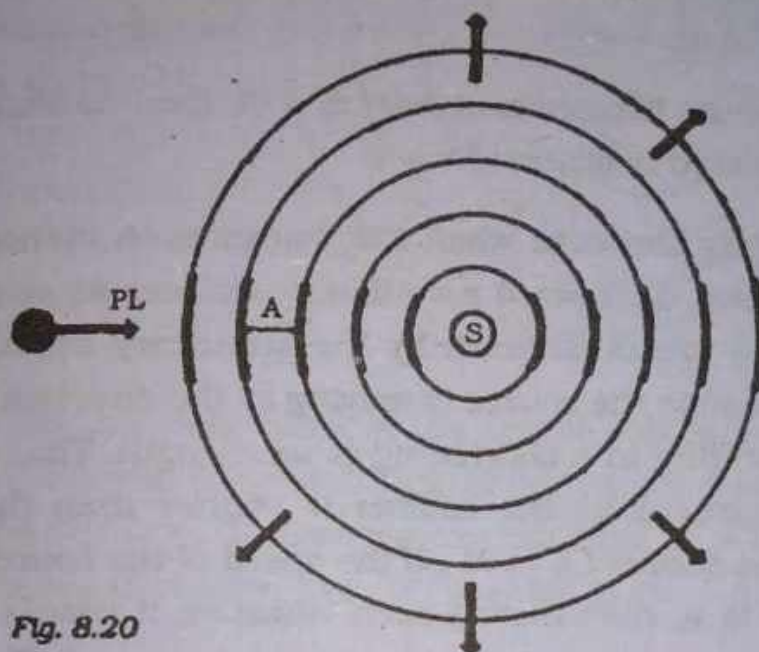


Fig. 8.20

1 (b). Similarly, a listener moving away from the stationary source ($v_0 < 0$) hears a lower pitch and the frequency detected by the listener in this case is

$$v' = \left(1 - \frac{v_0}{v} \right) v$$

$$v' = v - \left(\frac{v_0}{v} \right) v$$

8.35

Consequently, the change in pitch in this case is

$$v' - v = - \left(\frac{v_0}{v} \right) v$$

Hence, the general relation holding when the source is at

rest with respect to the medium and observer is moving through it, is given by

$$v' = v \pm \left(\frac{v_o}{v} \right) v = \left(\frac{v \pm v_o}{v} \right) v \quad 8.36$$

Where the positive sign refers to the motion toward the source and negative sign refers to the motion away from the source.

Notice when the listener is at rest ($v_o = 0$), then no change in the frequency of sound is observed $v' = v$

II (a). Now consider the case when the source is in motion and moving with a speed, v_s , toward a stationary listener as shown in Fig.8.21. The wave crests detected by the stationary listener are closer together because the source is moving in the direction of the outgoing wave resulting in a shortening of wavelength. That is, the wavelength λ' measured by the listener is shorter than the true wavelength λ of the source ($\lambda' < \lambda$). If the speed of the source is v_s and its frequency is v , then during each vibration it travels a distance $\frac{v_s}{v}$ and consequently each wavelength is shortened. Thus the wavelength of the sound arriving at the listener at rest is

$$\lambda' = \frac{v}{v} - \frac{v_s}{v} = \frac{1}{v} (v - v_s)$$

therefore, the frequency of the sound heard by the listener (at rest) is increased and is given by

$$\begin{aligned} v' &= \frac{v}{\lambda'} = \frac{(v v)}{(v - v_s)} \\ v' &= \frac{v}{1 - \left(\frac{v_s}{v} \right)} \end{aligned} \quad 8.37$$

The Eq. 8.37 indicates an increase in the frequency of the sound heard by stationary listener.

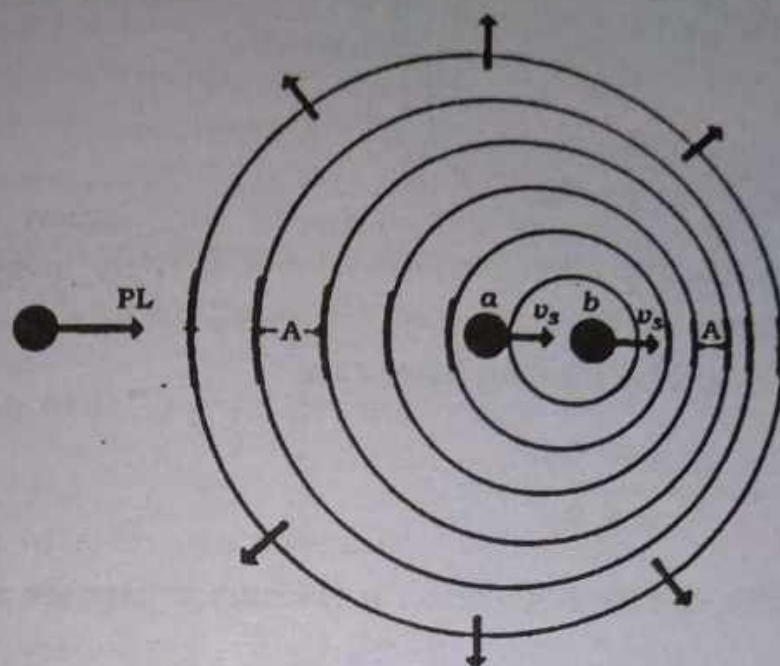


Fig. 8.21

II(b). On the other hand, if the source moves away from the stationary listener, the wavelength of the sound arriving at the listener is greater than the true wavelength λ (i.e. $\lambda' > \lambda$) and the listener detects a decreased frequency which is given by

$$v' = \frac{v \cdot \phi}{(\phi + \phi_s)} = \frac{v}{\left(1 + \frac{\phi_s}{\phi}\right)} \quad 8.38$$

The Eq.8.38 indicates a decrease in the frequency of the sound heard by the stationary listener. Thus in general relation valid for the listener at rest and for the source in motion with respect to the medium is given as

$$v' = \left(\frac{\phi}{\phi \pm \phi_s}\right) v = \frac{v}{\left(1 \pm \frac{\phi_s}{\phi}\right)} \quad 8.39$$

where the minus sign refers to the motion of the source toward the stationary observer and plus sign indicates the motion of the source away from the stationary observer.

Notice that when the source is at rest ($v_s = 0$), then no change the frequency of sound is observed (i.e. $v' = v$)

III(a). If the source and the observer are approaching along the line joining the two in the direction toward the each other, then the frequency heard by moving listener is

$$v' = \left(\frac{v + v_o}{v - v_s} \right) v \quad 8.40$$

III(b). On the other hand, if source and the observer are moving away from each other along the line joining the two, then the frequency heard by moving listener is

$$v' = \left(\frac{v - v_o}{v + v_s} \right) v \quad 8.41$$

Finally, the Eq.8.40 and Eq.8.41 are expressed together as

$$v' = \left(\frac{v \pm v_o}{v \mp v_s} \right) v \quad 8.42$$

Notice that Eq.8.42 reduces to Eq.8.36 when $v_s = 0$ (i.e. when the source is at rest) and when $v_o = 0$ reduces to Eq.8.39. Thus the validity of the Eq.8.42 is checked.

Notice when both the listener and source are at rest ($v_o = 0$, $v_s = 0$), then no change in the frequency of sound is observed ($v' = v$).

Fig.8.20 shows what happens when the speed of source (v_s) through the medium is less than the speed of the sound wave (v) in that medium i.e. $v_s < v$. Now we would like to examine what happens when the speed of the source v_s is comparable in magnitude to v ($v_s \approx v$)? In this case the source keeps pace with the outgoing spherical waves in the direction of medium since $v_s \approx v$. These waves can never move ahead of the source and they continue to pile up. As the process continues (i.e. wave are formed) the pile region extends farther and farther from the source in the direction perpendicular to its motion. This happens because the linear relationship between the restoring force and displacement (assumed

uptil now) does not remain true. The speed of the wave propagation (characteristics of the medium) is no longer the normal phase velocity. Components of the motion perpendicular to the line joining source and observer also contributes to the Doppler effect at these high speeds. Consequently, a wavefront that is a sheet or plane is formed as shown in Fig. 8.22.

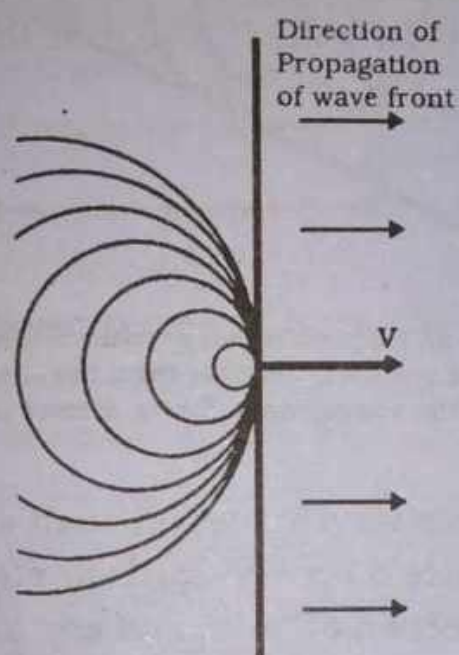


Fig. 8.22 When the source moves with a speed exactly equal to the wave speed, the waves pile up and form a plane that extends perpendicular to the direction of motion of the source.

Next we would like to see what happens when v_s exceeds the wave speed i.e. $v_s > v$. Obviously, in this case, the source runs ahead of the outgoing waves and in such a case the pile up of the waves produces wavefront which takes the shape of a cone with the moving object at its apex as shown in Fig 8.23.

The situation is best explained by considering the source at S_0 (Fig.8.23) at time $t = 0$, and some later time t , the spherical wavefront centered at S_0 travels a distance vt (the radius of the wavefront). Whereas the source reaches at S_n after travelling a distance $v_s t$. At that instant the source is at S_n and the waves just beginning to be generated at this point have wavefronts of zero radius. The line joining S_n and the wavefront centered on S_0 is tangent

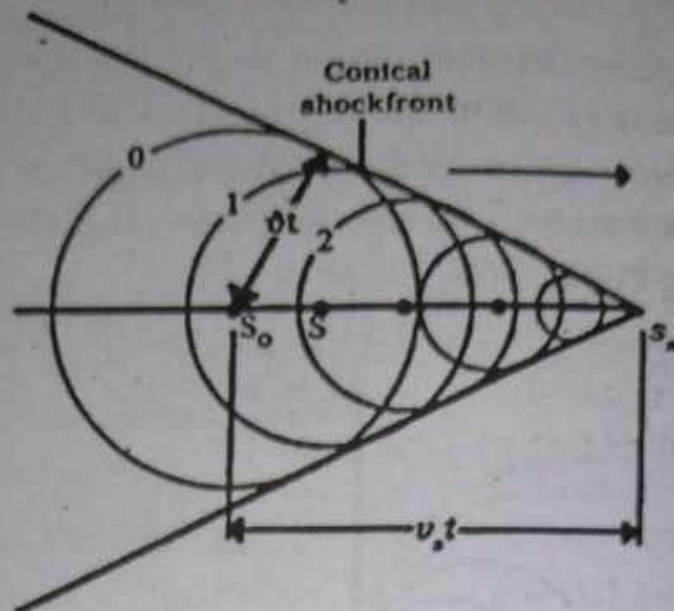


Fig.8.23 Representation of a shock wave produced when a source moves from S_0 to S_n with a speed v_s that is greater than the wave speed, v , in that medium. The envelope of the wavefronts forms a cone whose half-angle is given by $\sin \theta = \phi / \phi_s$.

to all other intermediate wavefronts. All such tangent lines lie on a surface of a cone. hence a cone-shaped wavefront is formed. The angle θ between the direction of travel and any tangent line is given by

$$\sin \theta = \frac{\phi t}{\phi_s t} = \frac{\phi}{\phi_s}$$

In aerodynamics the ratio ϕ / ϕ_s is referred to as the Mach number.

Along the cone-shaped wavefront that is produced by a supersonic object, the air is highly compressed. This moving sheet of high pressure air is called a shock wave. Some examples are the bow wave from a speed boat on the water when the boat's speed exceeds the speed of water waves. Jet aircraft (F-16) travelling at a supersonic speed ($\phi_s > \phi$) produces a shock wave which results in loud explosion or sonic boom. These shock waves produced by supersonic jet can damage building when flying at low altitudes. In supersonic jets for example F-16, Mirage Fighter, Concorde airliner, a double boom is produced because two shock waves are formed (within time interval of 0.02s) one from the front and one from the rear of the aircraft.

APPLICATIONS OF THE DOPPLER EFFECT

- (i) The Doppler effect provides a method for tracking a satellite. Suppose the satellite is emitting a radio signal (i.e. an electromagnetic wave) of constant frequency f_s . The frequency f_L of the signal received on the earth decreases as the satellite is passing. The received signal is combined with a constant signal generated in the receiver to produce beat. The beat frequency produces an audible note whose pitch changes as the satellite passes overhead.
- (ii) The Doppler effect is used in measuring the speed of automobile by traffic police. A "radar gun" is fixed on police car. An electromagnetic signal is emitted by the radar gun in the direction of the automobile whose speed is to be checked. The wave is reflected from the moving automobile and received back. The reflected wave is then mixed with the locally generated original signal and beats are produced. The frequency shift is measured using beats and hence the speed of the automobile is determined.
- (iii) Radars (Radio detection and ranging) are commonly used for civil and military purposes at civilian airports and military air-bases respectively, to detect the presence of any aircraft (foe or friend) in the airspace by evaluating the range. The Doppler Radar based on the principle of the Doppler Effect, is extensively used in the detection of aircraft speed and direction.
- (iv) VOR (Very high frequency omni Range) is a guiding system usually installed at the airports to guide the incoming aircrafts toward the location of the airport. Nowadays big and modern airports for example Quaid-e-Azam International Airport, Islamabad Airport, are equipped with Doppler VOR whose principle is once again based on the Doppler effect and provides air effective and better guiding system to the

aircraft. The electromagnetic signal used here has frequency in the VHF range (30MHz - 300MHz).

- (v) X-rays have been a major diagnostic tool of medicine, but recent years have seen the emergence of an alternate tool which is inherently safer than x-rays: ultra sound. Unlike x-rays, ultra sound radiations have not been reported to damage living cells so far. Measurements of internal reflections of ultra sound have facilitated the diagnosis of breast cancer and taking the heartbeats of fetuses and newborns.

An ultrasonic instrument called Homosonde uses the doppler effect of ultrasonic waves reflected from moving masses in the patient. The device is very sensitive for detecting blood flow and measures faint heartbeats in a very noisy environment where the use of stethoscope may not be reliable.

- (vi) In similar manner, we can detect the motion of an objects under-water (for example a submarine) by employing sonar (ultrasonic waves) based on effective use of Doppler effect.
- (vii) The Doppler effect for light is important in astronomy. Spectral analysis of light emitted by the elements in distant stars shows shift in the wavelength compared to light from the same element on earth. These shifts can be interpreted as Doppler shifts due to the motion of the stars. The shift is nearly always toward the longer wavelength, or red end of the spectrum, and is therefore called the red shift. Such observations have provided practically all the evidence for "expanding universe" cosmological theories, which represents the universe as having evolved from a great explosion several billion years ago in a relatively small region of space.

Example 8.1

A block with a mass of 0.1 kg is attached to a spring and placed on a horizontal frictionless table. The spring is stretched 20 cm when a force of 5N is applied. (i) Calculate spring constant (ii) Find the characteristic frequency and period of oscillation when the mass is set in motion.

- (i) The applied force and displacement are related by

$$F = kx, \text{ therefore}$$

$$K = \frac{F}{x} = \frac{5\text{N}}{0.2\text{m}} = 25\text{N m}^{-1}$$

- (ii) The characteristic frequency is

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{25\text{N m}^{-1}}{0.1\text{kg}}} = 2.52 \text{ Hz}$$

The period of oscillation is $T = \frac{1}{f}$

$$T = \frac{1}{2.52 \text{ Hz}} = 0.397\text{s}$$

Example 8.2

A body of mass 2kg attached to a spring is displaced through 0.04 m from its equilibrium position and then released. If the spring constant is 200 N m^{-1} , (i) find the period and frequency of vibration:

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{2\text{kg}}{200\text{N m}^{-1}}} = 0.628\text{s}$$

$$f = \frac{1}{T} = \frac{1}{0.628 \text{ s}} = 1.59 \text{ Hz}$$

$$\omega = 2\pi f = \sqrt{\frac{K}{m}} = \sqrt{\frac{200\text{N m}^{-1}}{2\text{kg}}} = 10\text{s}^{-1}$$

(ii) Calculate the maximum velocity attained by the vibrating body:

The maximum velocity occurs at the equilibrium position i.e. when $x=0$, for any x ,

$$v_x = \pm \omega (x_0^2 - x^2)^{1/2}$$

So when $x = 0$,

$$\begin{aligned} v_x &= v_{\max} = \pm \omega x_0 \\ &= \pm (10 \text{ s}^{-1}) (0.04) = \pm 0.4 \text{ m s}^{-1} \end{aligned}$$

(iii) Compute the maximum acceleration

We know $a_x = -\omega^2 x$

the maximum acceleration occurs at the end of the path, i.e. when $x = \pm x_0$, therefore

$$a_{\max} = \pm \omega^2 x_0 = \pm (10 \text{ s}^{-1})^2 (0.04) = \pm 4.0 \text{ m.s}^{-2}$$

Example 8.3

A body of mass 0.025 kg attached to a spring is displaced through 0.1m to the right of equilibrium position. If the spring constant is 0.4 Nm^{-1} and its velocity at the end of this displacement be 0.4 ms^{-1} , calculate (i) The total energy (ii) the amplitude of its motion (i.e. maximum displacement)

(i) Total energy:

The total energy at any instant during the motion is equal to the sum of potential and kinetic energy at that instant

$$\begin{aligned} E &= \frac{1}{2} kx^2 + \frac{1}{2} mv^2 \\ &= \frac{1}{2} (0.4 \text{ Nm}^{-1}) (0.1 \text{ m})^2 + \frac{1}{2} (0.025 \text{ kg}) (0.4 \text{ ms}^{-1})^2 \\ &= 4 \times 10^{-3} \text{ J} \end{aligned}$$

(ii) Amplitude:

Total energy is equal to the maximum potential energy. The maximum potential energy occurs at the end of motion where the kinetic energy is zero (the body is momentarily at rest).

$$E = K.E + P.E$$

$$E = 0 + (P.E)_{\max} = \frac{1}{2} kx_0^2$$

$$\frac{1}{2} kx_0^2 = 4 \times 10^{-3} \text{ J}$$

$$x_0^2 = \sqrt{\frac{2 \times 4 \times 10^{-3} \text{ J}}{0.4 \text{ Nm}^{-1}}} = 0.1414 \text{ m}$$

$$x_0 = 0.1414 \text{ m}$$

Example 8.4

A simple pendulum completes one oscillation in 2 s. Calculate its length when $g = 9.8 \text{ ms}^{-2}$, the time period of simple pendulum is given by

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$l = \frac{T^2}{4\pi^2} \times g = \frac{4 \times 9.8 \text{ ms}^{-2}}{4 \times (3.141)^2} = 0.992 \text{ m}$$

$$l = 0.992 \text{ m}$$

Example 8.5

A string 2 m long and of mass 0.004 kg is stretched horizontally by passing one end over a pulley and attaching a 1 kg mass to it. Find the speed of the transverse waves on the string and the frequency of the second and the fourth harmonics to which the string will resonate.

The speed of a transverse wave is given by equation

$$v = \sqrt{\frac{T}{m/l}} = \sqrt{\frac{Mg}{m/l}}$$

where $T = Mg$ (Tension in the string)

$M =$ the Total mass suspended from the string $= 1 \text{ kg}$

$g =$ the acceleration due to gravity $= 9.8 \text{ m s}^{-2}$

Substituting the various values, we get

$$v = \sqrt{\frac{1 \times 9.8}{0.004/2}}$$

$$v = 70 \text{ ms}^{-1}$$

For resonance frequencies we know that

$$v_n = n \frac{v}{2l}$$

For second harmonic (v_2), $n = 2$

$$\therefore v_2 = \frac{2 \times 70}{2 \times 2} = 35 \text{ hertz}$$

For fourth harmonic (v_4), $n = 4$

$$\therefore v_4 = \frac{4 \times 70}{2 \times 2} = 70 \text{ hertz}$$

Example 8.6

A certain string resonates to several frequencies, the lowest of which is 50 hertz. What are the next four higher frequencies to which it resonates?

We have $v_n = n \frac{v}{2l}$ where $n = 1, 2, 3, \dots$

Therefore all the higher frequencies are integral multiples of the lowest frequency which corresponds to $n=1$.

$$v_1 = 50 \text{ hertz}$$

$$v_2 = 2 \times v_1 = 2 \times 50 = 100 \text{ hertz}$$

$$v_3 = 3 \times v_1 = 3 \times 50 = 150 \text{ "}$$

$$v_4 = 4 \times v_1 = 4 \times 50 = 200 \text{ "}$$

$$v_5 = 5 \times v_1 = 5 \times 50 = 250 \text{ "}$$

Example 8.7

The speed of a wave on a particular string is 24 m/s. If the string is 6 m long, to what frequencies will it resonate? Draw a picture of the string for each resonant frequency.

As $\lambda_n = 2l/n$ where $n = 1, 2, 3, 4, 5, \dots$

$$\text{we have } \lambda_1 = \frac{2 \times 6}{1} = 12\text{m}$$

$$\lambda_2 = \frac{2 \times 6}{2} = 6\text{m}$$

$$\lambda_3 = \frac{2 \times 6}{3} = 4\text{m}$$

$$\lambda_4 = \frac{2 \times 6}{4} = 3\text{m}$$

As the frequency v_n is given by

$$v_n = \frac{v}{\lambda_n}$$

$$\therefore v_1 = \frac{v}{\lambda_1} = \frac{24}{12} = 2 \text{ hertz}$$

$$v_2 = \frac{v}{\lambda_2} = \frac{24}{6} = 4 \text{ hertz}$$

$$v_3 = \frac{v}{\lambda_3} = \frac{24}{4} = 6 \text{ hertz}$$

$$v_4 = \frac{v}{\lambda_4} = \frac{24}{3} = 8 \text{ hertz}$$

Example 8.8

A spring 4 m long resonates in four segments (node at both ends). The frequency of driving system on the spring is 20 hertz. Calculate the speed of the wave in the spring.

As the spring is vibrating in four segments, $l = 2\lambda$

$$\lambda = l/2 = \frac{4}{2} = 2\text{m}$$

$$\text{Again } v = \nu \times \lambda$$

$$= 20 \times 2$$

$$v = 40 \text{ m/s}$$

We can obtain the same result from the equation

$$v = n \frac{v}{2l}$$

$$v = \frac{2v \times l}{n}$$

$$v = \frac{2 \times 4 \times 20}{4} = 40 \text{ m/s}$$

Thus speed of the wave in the spring = 40 m/s.

Example 8.9

Compute and compare the speed of sound in air and that in steel rod.

The speed of a longitudinal wave in a steel rod is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

where Y = Young's modulus and for steel $Y = 20 \times 10^{10} \text{ Nm}^{-2}$
 density of steel $= \rho = 7.8 \times 10^3 \text{ kgm}^{-3}$
 therefore

$$v = \sqrt{\frac{20 \times 10^{10} (\text{Nm}^{-2})}{7.8 \times 10^3 (\text{kgm}^{-3})}} = 5.06 \times 10^3 \text{ ms}^{-1}$$

The speed of sound in air is given by

$$v = \sqrt{\frac{\gamma B}{\rho}}$$

where B = Bulk modulus and for air

$B = 1.01 \times 10^5 \text{ Nm}^{-2}$ and density of air ρ .

$\rho = 1.29 \text{ kg m}^{-3}$ $\gamma = 1.41$

therefore

$$v = \sqrt{\frac{1.01 \times 10^5 (\text{Nm}^{-2}) \times 1.41}{1.29 (\text{kgm}^{-3})}} = 3.2 \times 10^3 \text{ ms}^{-1}$$

Example 8.10

Compute the wavelength of the wave transmitted by a radio station that broadcasts on an assigned frequency of 1000 KHz.

All electromagnetic, including light, radio waves, television, etc travel with a speed of $3 \times 10^8 \text{ ms}^{-1}$ which is expressed in terms of wavelength and frequency by $v = v \lambda$

therefore

$$\lambda = \frac{v}{v} = \frac{3 \times 10^8 \text{ ms}^{-1}}{1000 \text{ kHz}} = \frac{3 \times 10^8 \text{ ms}^{-1}}{10^6 \text{ Hz}} = 300\text{m}$$

$$\lambda = 300\text{m}$$

Example 8.11

A note of frequency 500 Hz is emitted from an ambulance. (a)
 If the ambulance moves toward you at a speed of 60 km/h, what is

the frequency you will detect? (b) After the ambulance passes away from you its speed is 60 km/h. what frequency will you detect? The speed of sound is 340 ms^{-1} .

The speed of ambulance = $60 \text{ km/h} = 16.7 \text{ m/s}$

- (a) The frequency heard by the stationary listener is given by Eq. 8.37

$$v' = \frac{v}{v - v_s} v$$

where $v_s = 16.7 \text{ ms}^{-1}$, $v = 500 \text{ Hz}$ and the velocity of sound $v = 340 \text{ ms}^{-1}$

therefore

$$v' = \left(\frac{340 \text{ ms}^{-1}}{340 \text{ ms}^{-1} - 16.7 \text{ ms}^{-1}} \right) 500 \text{ Hz} = 525.8 \text{ Hz}$$

- (b) When the ambulance is moving away, the frequency heard by you is

$$v' = \frac{v}{v + v_s} = \left(\frac{340 \text{ m/s}}{340 \text{ m/s} + 16.7 \text{ m/s}} \right) 500 \text{ Hz} \\ = 476.6 \text{ Hz}$$

Problems

1. An object is connected to one end of a horizontal spring whose other end is fixed. The object is pulled to the right (in the positive x-direction) by an externally applied force of magnitude 20N causing the spring to stretch through a displacement of 1 cm (a) Determine the value of force constant if the mass of the object is 4kg (b) Determine the period of oscillation when the applied force is suddenly removed.

Ans (a) $2 \times 10^3 \text{ N/m}$.

(b) 0.28108s

2. A body hanging from a spring is set into motion and the peri-

od of oscillation is found to be 0.50 s. After the body has come to rest, it is removed. How much shorter will the spring be when it comes to rest?

(Ans. 6.21 cm)

3. A pipe has a length of 2.46m. (a) Determine the frequencies of the fundamental mode and the first two overtones if the pipe is open at both ends. Take $v = 344$ m/s as the speed of sound in air. (b) What are the frequencies determined in (a) if the pipe is closed at one end? (c) For the case of open pipe, how many harmonics are present in the normal human hearing range (20 to 20000 Hz)?

Ans. (a) 70 Hz, 140 Hz, 210 Hz
(b) 35 Hz, 105 Hz, 175 Hz
(c) 285 harmonics.

4. A standing wave is established in a 120 cm long string fixed at both ends. The string vibrates in four segments when driven at 120 Hz (a) Determine the wavelength (b) What is the fundamental frequency?

Ans. (a) 0.60 m
(b) 30Hz

5. Calculate the speed of sound in air at atmospheric pressure $p = 1.01 \times 10^5$ N/m², taking $\gamma = 1.40$ and $\rho = 1.2$ kg/m³.

Ans. 343 m/s

6. A sound wave propagating in air has a frequency of 4000Hz. Calculate the percent change in wave length when the wave front, initially in a region where $T = 27^\circ\text{C}$, enters a region where the air temperature decreases to 10°C .

Ans. 2.87%

7. The frequency of the second harmonic of an open pipe (open at both ends) is equal to the frequency of the second harmonic of a closed pipe (open at one end). (a) Find the ratio of the length of the closed pipe to the length of the open pipe. (b) If the fundamental frequency of the open pipe is 256Hz, what is the length of pipe? (Use $v = 340$ m/s/

Ans. (a) $3/4$

(b) $L_{\text{open}} = 0.664\text{m}$

$L_{\text{closed}} = 0.498\text{m}$

8. A 256 Hz tuning fork produces four beats per second when sounded with another fork of unknown frequency. What are two possible values for the unknown frequency?

Ans. 252Hz, 260Hz.

9. An ambulance travels down a highway at a speed of 75 mi/h. Its siren emits sound at a frequency of 400Hz. What is the frequency heard by a person in a car travelling at 55 mi/h in the opposite direction as the car approaches the ambulance and as the car moves away from the ambulance.

Ans. 477Hz, 337Hz.

10. A car has siren sounding a 2 kHz tone. What frequency will be detected as stationary observer as the car approaches him at 80 km/h? Speed of sound = 1200 km/h.

Ans. 2143 Hz