

Geometrical Optics

10.1 LENSES

A lens is a piece of transparent material that can focus a transmitted beam of light. This is usually bounded by two spherical surfaces, or a spherical and a plane surface. For our convenience, we will deal with thin lenses in this chapter. Basically lenses fall into two categories, converging or convex lenses, and diverging or concave lenses.

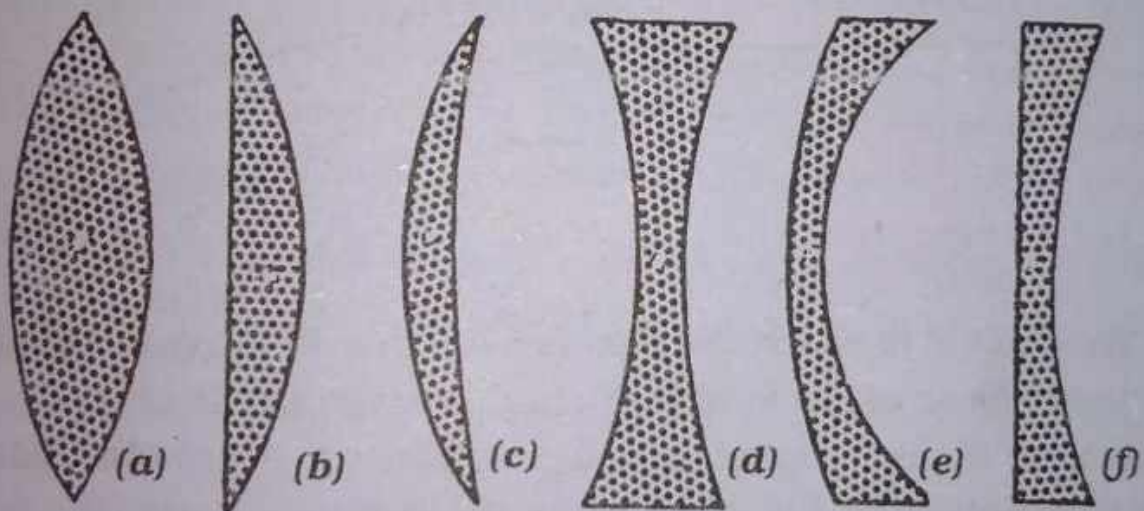


Fig: 10.1 (a) Double convex lens (b) Plano-convex (c) concavo-convex (d) Double concave (e) convexo-concave (f) Plano-concave

A convex lens is thicker in the middle and thinner at the edges, converges light rays towards its optical axis, (the line through its centre of curvature), so that a beam of parallel rays converges at a point F (Fig: 10.2). For example in bright sun light, a convex lens may produce a spot of light intense enough to ignite paper. A concave lens which is thinner in the middle and thicker at the edges bends rays outward from its optical axis Fig. 10.3.

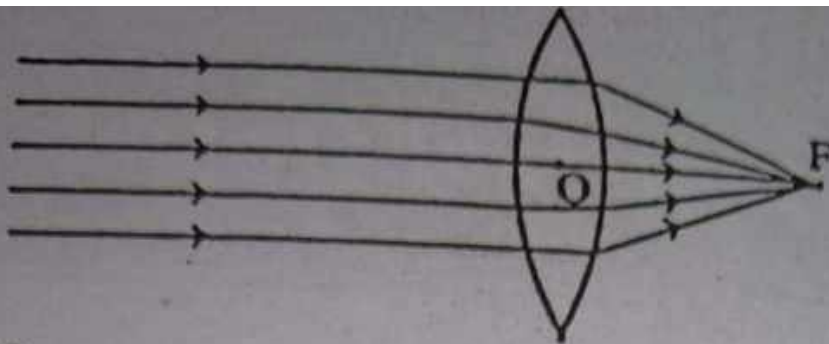


Fig: 10.2

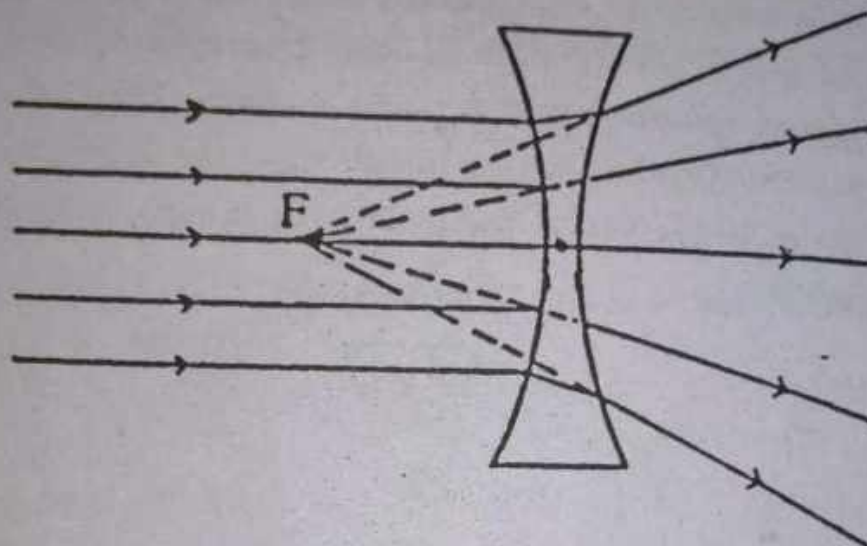


Fig: 10.3

The point F to which the rays are brought to focus is called the principal focus or the focal point, and the distance between the optical centre of the lens and its principal focus is called the focal length. Conventionally f is taken to be positive for convex lenses and negative for concave lenses.

10.2 IMAGE FORMATION

We have seen that in case of a convex lens light rays from a very-distant point on its axis arrive parallel to the axis and meet to form an image at the principal focus. Rays from other points form images whose locations can be found graphically, if the focal length of the lens is known. However we consider only three rays out of them whose intersection determine the location of the image as

shown in Fig. 10.4 . That is, the three rays which are numbered.

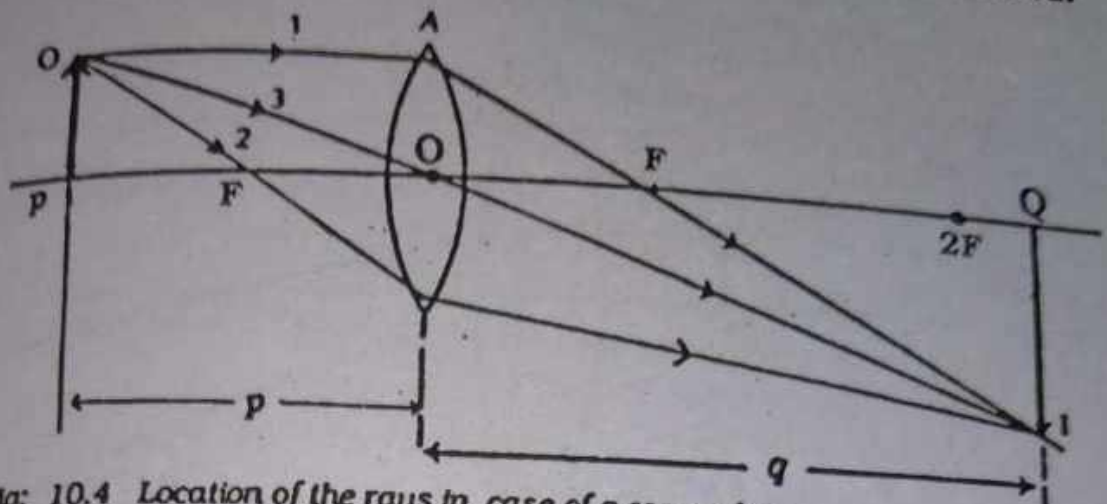


Fig: 10.4 Location of the rays in case of a convex lens

Fig. 10.4 (location of the rays in case of a convex lens) in ray diagram. Three rays are drawn from the tip of the object as follows:

- (i) The ray 1 leaving the tip of the object parallel to the axis is refracted by the lens so that it passes through the principal focus F on the other side of the lens.
- (ii) The ray 2 passing through the principal focus F emerges from the lens parallel to the axis.
- (iii) The ray 3 passing through the optical centre of the lens remains unchanged in the direction.

The location of the image when an object is placed before a concave lens is shown in Fig. 10.5 . Convex lenses yield real image except for the case when the object is placed between the principal focus and the lens as shown in Fig. 10.6. Concave lenses yield

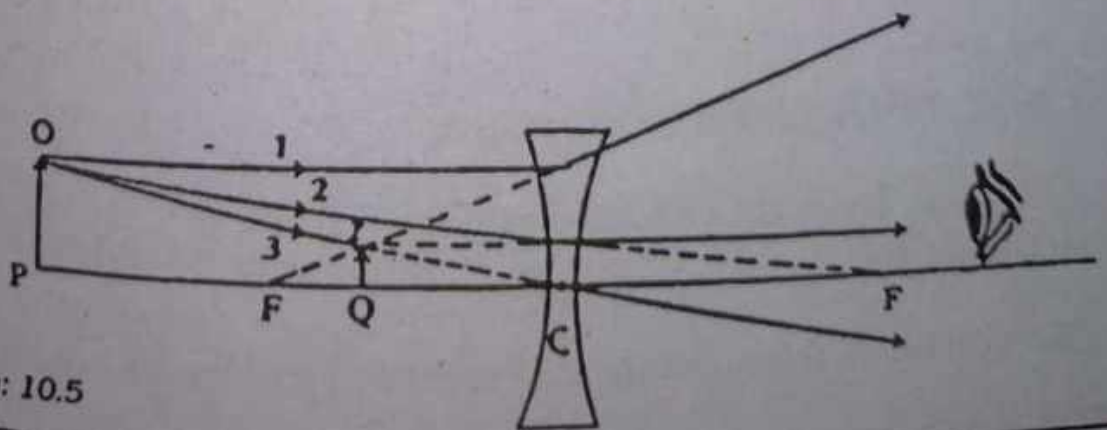


Fig: 10.5

only virtual images regardless of the position of the object. Real images are always inverted with respect to the object and virtual images are always erect.

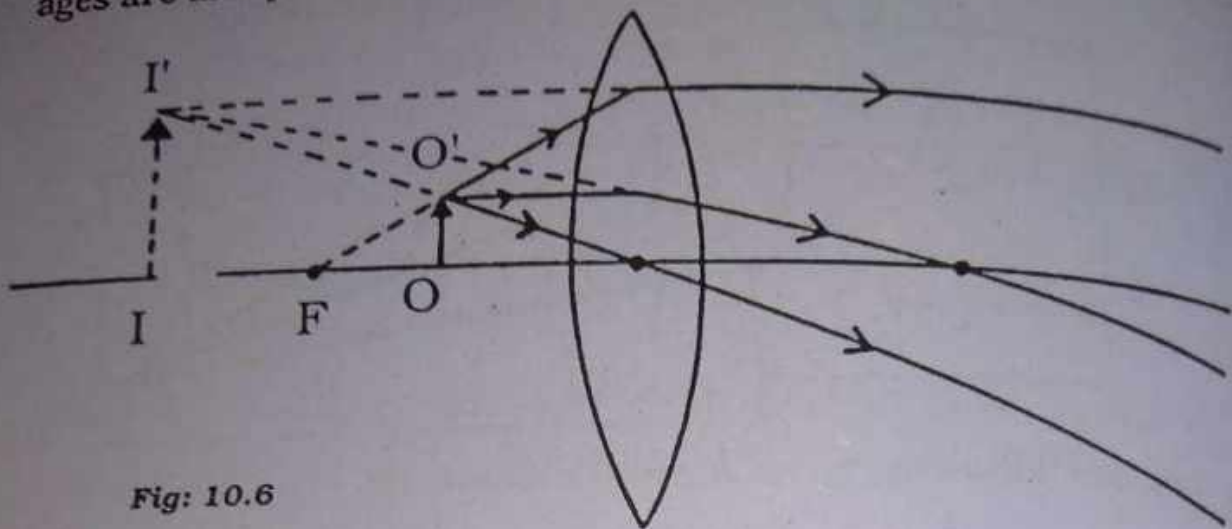


Fig: 10.6

The different forms of image formed, with respect to the object placed at the different positions are shown in Fig.10.7.

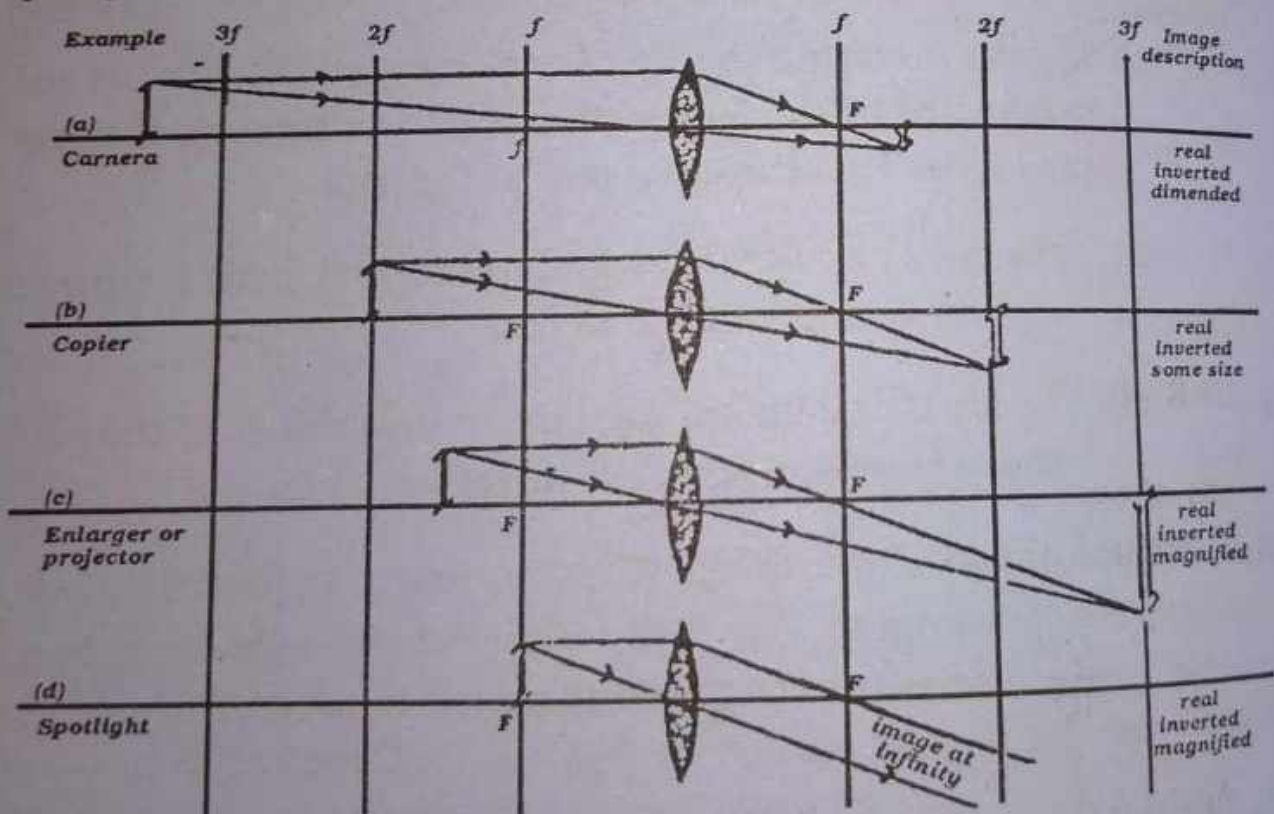


Fig: 10.7

10.3 THE THIN LENS FORMULA

Thin lens formula can be developed from the ray diagram shown in Fig. 10.8.

The distance from the optical centre of the lens to the object

is denoted by p , and the distance from the optical centre to the image is denoted by q . The following sign conventions are used.

Object distance p is $\begin{cases} +ve \text{ for any real object.} \\ -ve \text{ for any virtual object.} \end{cases}$

Image distance q is $\begin{cases} +ve \text{ for a real image.} \\ -ve \text{ for a virtual image.} \end{cases}$

Focal length f is $\begin{cases} +ve \text{ for a convex lens} \\ -ve \text{ for a concave lens} \end{cases}$

Consider an object whose real and inverted image is formed by a thin convex lens as shown in Fig 10.8.

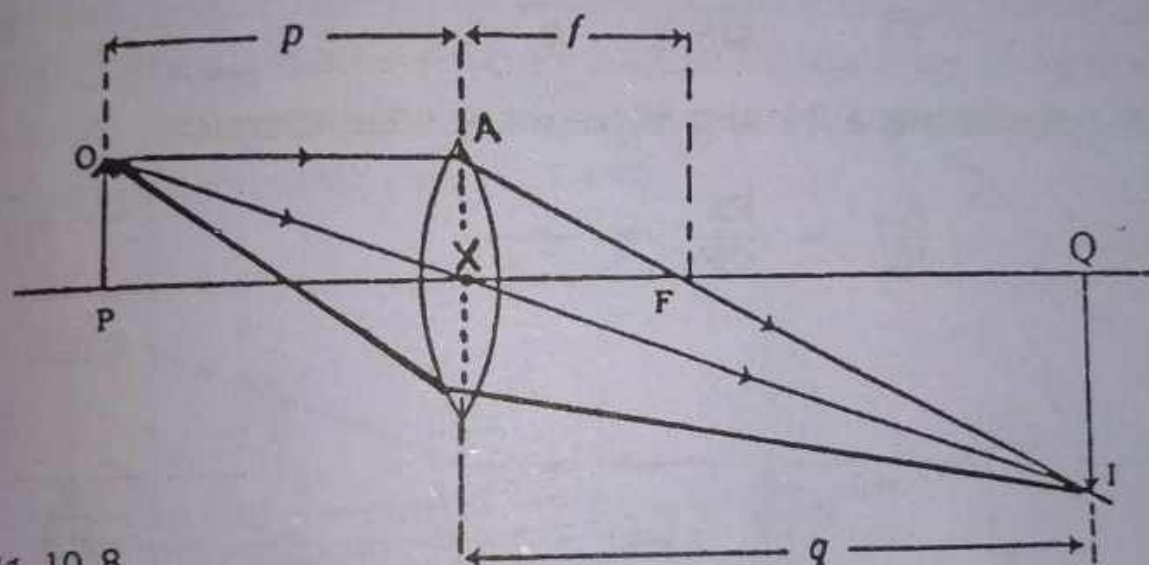


Fig. 10.8

As shown in the figure the right angled triangles OPX and IQX are similar, therefore we can write

$$\frac{OP}{IQ} = \frac{PX}{QX} = \frac{p}{q} \quad 10.1$$

Again ΔAXF and ΔIQF are also similar.

$$\therefore \frac{AX}{IQ} = \frac{XF}{QF} = \frac{f}{q-f} \quad 10.2$$

Since $AX = OP$

$$\therefore \frac{OP}{IQ} = \frac{XF}{QF}$$

$$\text{or } \frac{p}{q} = \frac{f}{q-f}$$

simplifying the relation, we find that

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

10.3 (a)

10.3. (b)

This is known as the lens equation or lens formula. In the same manner, we can obtain lens formula in case of a concave lens.

Consider the Fig. 10.9, the triangles OPX and IQX are similar, therefore

$$\frac{OP}{IQ} = \frac{PX}{QX} = \frac{p}{q}$$

10.4

Similarly ΔAXF and ΔIQX are similar, therefore

$$\frac{AX}{IQ} = \frac{FX}{QF} = \frac{f}{f-q}$$

10.5

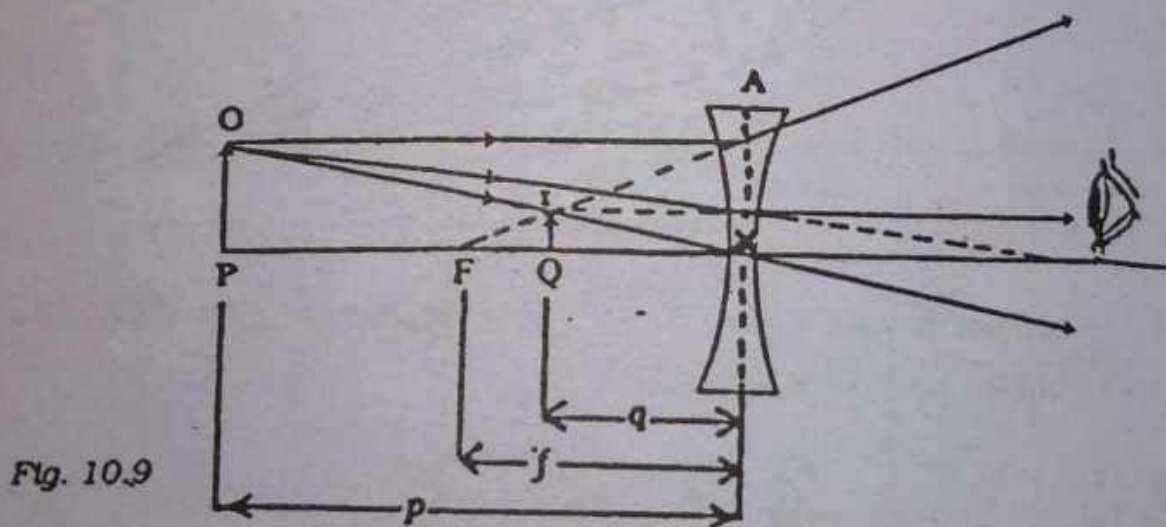


Fig. 10.9

Since $AX = OP$

$$\therefore \frac{OP}{IQ} = \frac{FX}{QF}$$

$$\frac{p}{q} = \frac{f}{f-q}$$

10.6

simplifying the above relation, we get

$$\frac{1}{p} - \frac{1}{q} = -\frac{1}{f}$$

10.7

Applying sign convention, the above equation becomes

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

10.4 COMBINATION OF THIN LENSES

In most of the optical instruments two or more lenses are used in combination. The location, size and nature of the final image can be determined by using the lens formula or ray diagram. In either case, we locate first the image formed by the first lens. Using that image as the object for the second lens, the final image formed by the second lens can be located. If there are more than two lenses, this process is continued; the object for each lens is the image for the preceding lens Fig 10.10

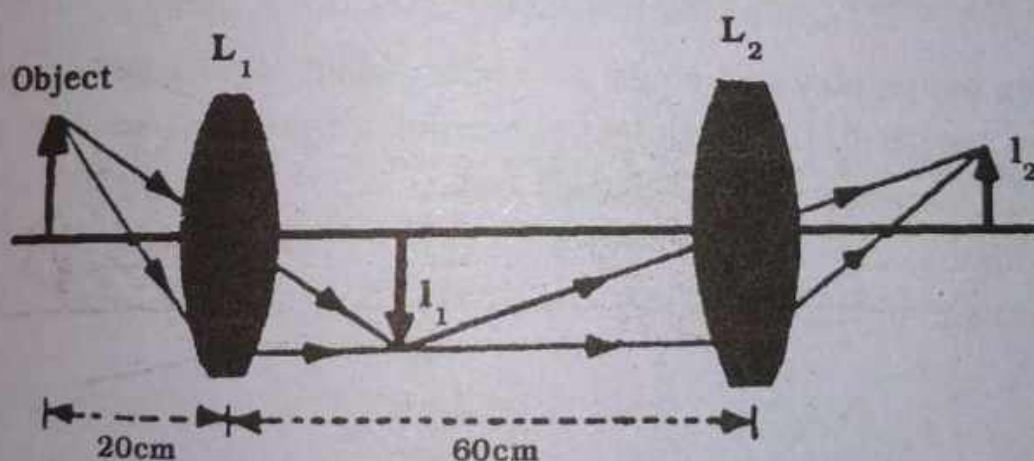


Fig. 10.10

Referring Fig. 10.10 we can see that lens L_1 forms an image I_1 . This image acts as a real object for the Lens L_2 , which forms a real image I_2 . Notice that I_2 is inverted with respect to I_1 and erect with respect to the object.

We shall now consider the case when the two thin lenses are in contact, which means their separation is very small as compared to their focal lengths. This is illustrated in Fig. 10.11

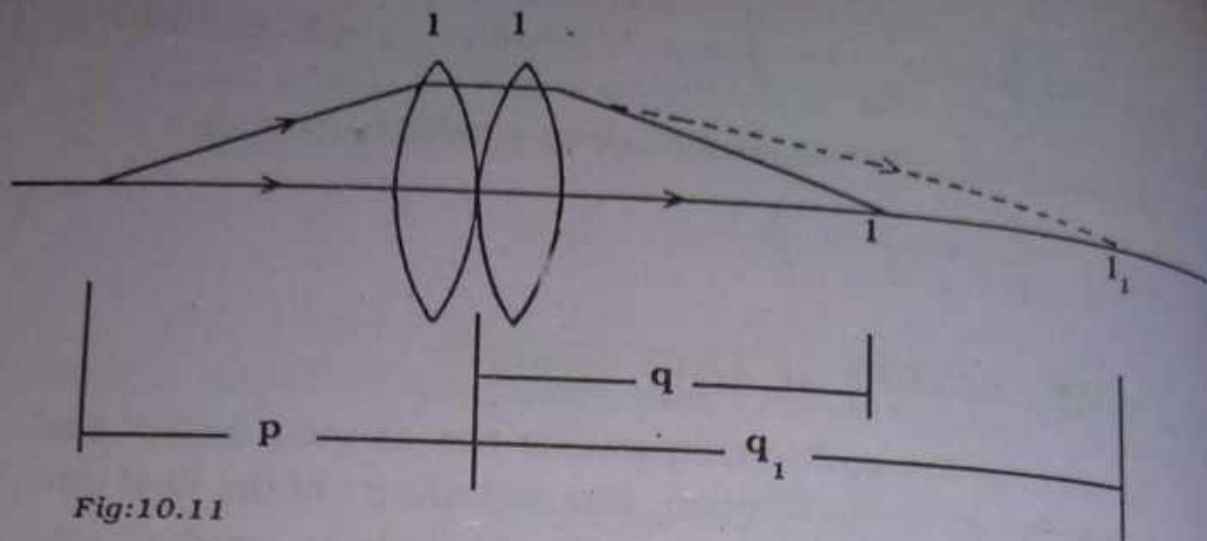


Fig:10.11

Let a point object O be placed at a distance p from the lens L_1 whose real image I_1 is formed by it at a distance q_1 . From the lens formula

$$\frac{1}{p} + \frac{1}{q_1} = \frac{1}{f_1} \quad 10.8$$

Where f_1 is the focal length of lens L_1 .

This image now serves as a virtual object for the second lens L_2 of focal length f_2 . If we neglect the small separation between the lenses, the distance of this virtual object from lens L_2 will be the same as its distance from the lens L_1 . If the lens L_2 forms an image I of this virtual object at a distance q

$$-\frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_2} \quad 10.9$$

As the object is virtual for lens L_2 , i.e. $P_2 = -q_1$

Adding Eq. 10.8 and 10.9, we get

$$\frac{1}{p} + \frac{1}{q_1} - \frac{1}{q_1} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2} \quad 10.10$$

$$\text{or } \frac{1}{p} + \frac{1}{q} = \frac{1}{f_1} + \frac{1}{f_2} \quad 10.11$$

Now if we replace the two lenses of focal lengths f_1 and f_2 by a single lens of focal length f such that it forms an image at a dis-

tance q of an object placed at a distance p from it as shown in Fig. 10.19, such a lens is called equivalent lens, and its focal length is known as equivalent focal length. For equivalent lens L , we have

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad 10.12$$

Comparing Eq (10.11) and (10.12), we get

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \quad 10.13$$

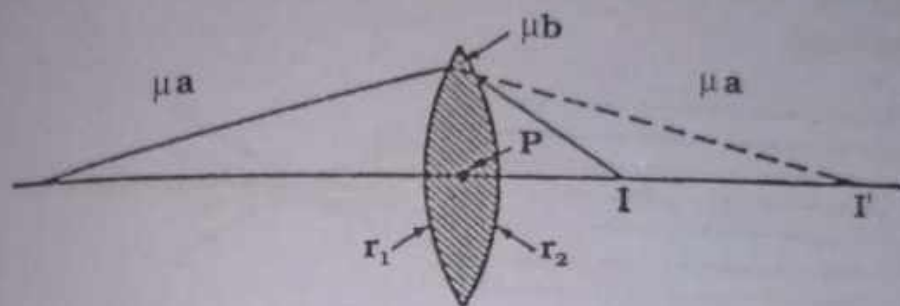


Fig: 10.12

This close combination behaves as a single lens whose focal length is given by the above relation.

The Eq. 10.13 shows that for a pair of lenses in contact the sum of the reciprocals of their individual focal lengths is equal to the reciprocal of the focal length of the combination.

10.5 POWER OF LENSES

If two lenses with focal lengths f_1 and f_2 placed in contact with each other are equivalent to a single lens with a focal length f satisfying $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ the power of the pair of lenses is $P = P_1 + P_2$. Thus the powers of lenses in contact are simply added to find the resultant power of the combination.

10.6 LENS ABERRATION

There are two main defects in the image formed by the lenses i.e. spherical aberration and chromatic aberration.

Spherical aberration is due to the fact that the focal points of light rays far from the optical axis of a spherical lens are different from those of rays passing through the centre. Fig. 10.13. Illustrates the spherical aberration for parallel rays passing through a converging lens. Rays near the middle of the lens have longer focal lengths than rays at the edges. Hence there is no single focal length for a lens. Many cameras are equipped with an adjustable aperture to reduce spherical aberration when possible (An aperture is an opening that controls the amount of light transmitted through the lens).

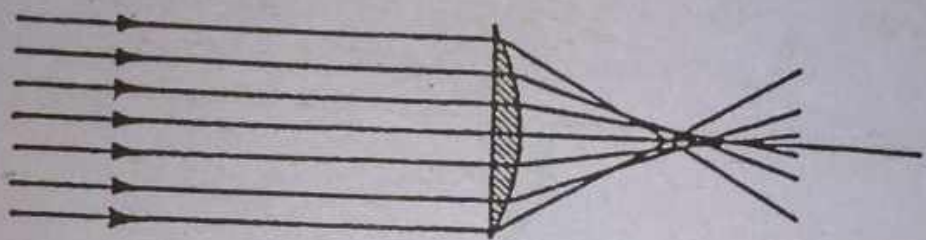


Fig: 10.13 (Spherical aberration)

The fact that different wavelengths of light refracted by lens focus at different points give rise to chromatic aberration.

From the lower diagram we see that the focal length of violet light is less than that of red light. Other wave lengths (not shown in figure) would have different focal points. This defect is called chromatic aberration. This aberration can be reduced to a greater

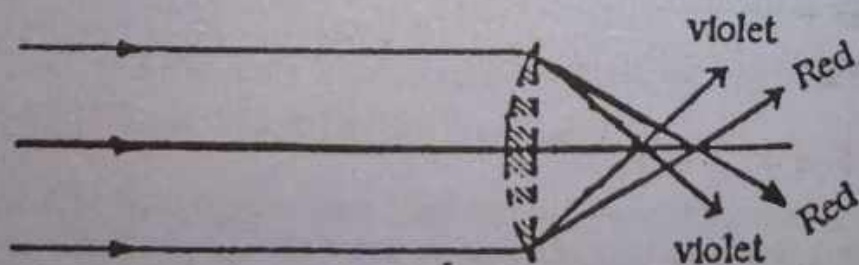


Fig 10.14 chromatic aberration

extent by the combination of convex and concave lenses made from the different type of glass.

10.7 LINEAR MAGNIFICATION

The Linear magnification, M , produced by a lens is defined as the ratio of the size of the image to the size of object. From Fig 10.15,

$$\text{Magnification} = \frac{\text{size of image}}{\text{size of object}}$$

$$M = \frac{IQ}{OP} = \frac{y'}{y}$$

A typical object and image linked by a ray through the pole is shown in fig 10.15. The $\triangle OPX$ and $\triangle IQX$ are similar and therefore,

$$\frac{IQ}{OP} = \frac{XQ}{XO} = \frac{q}{p}$$

$$\therefore \text{Magnification} = \frac{q}{p}$$

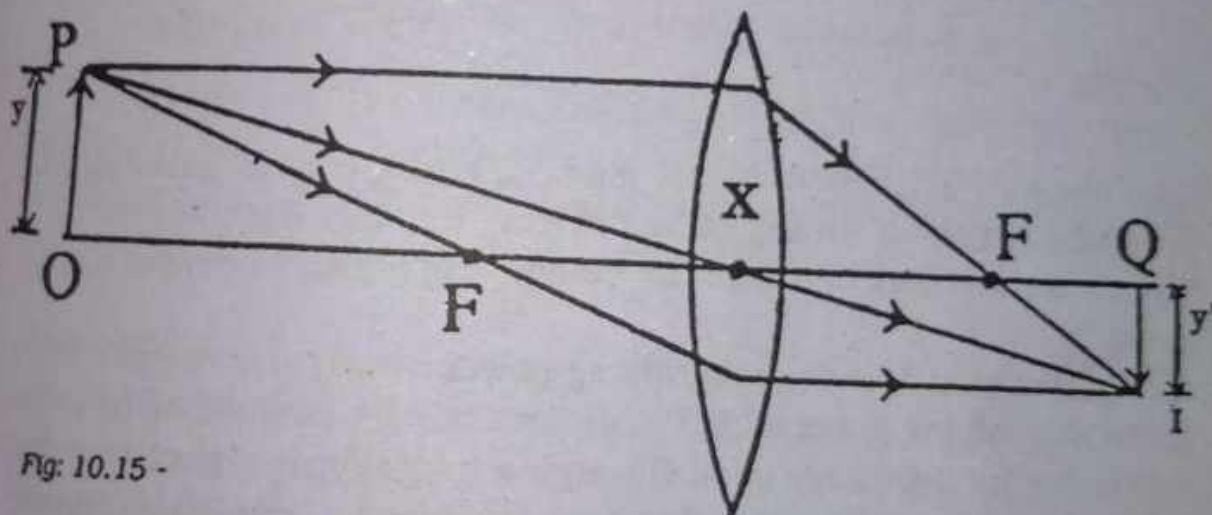


Fig: 10.15 -

If the image is inverted and if p and q are both positive, y' and y have opposite signs. then the formula for magnification becomes:

$$M_i = - \frac{q}{p}$$

In fact, this is just one of a series of sign convention associated with the lens equation and similar expressions.

10.8 MAGNIFYING GLASS

The apparent size of an object depends upon the angle subtended by it at the eye. This angle is called the visual angle. The

greater the visual angle, the greater is the apparent size of the object. This fact is illustrated in Fig 10.16 where the same object OP at two different positions will appear to be of different sizes at the eye. The smaller the distance of the object from the eye, the greater will be the visual angle consequently it will appear larger if we wish to see the fine details of a small object, we bring it as close to the eye as possible, thus increasing the visual angle and getting a large and real image on the retina of the eye. But we know that a normal person cannot see clearly an object if it is closer than the

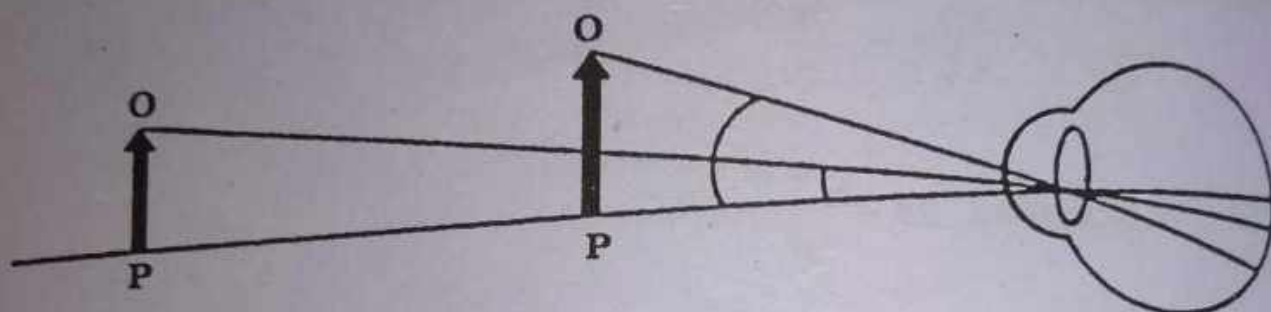


Fig: 10.16

least distance of distinct vision, d i.e. 25 cm. A convex lens helps us to see the details of an object by bringing it closer than 25cm. Such a convex lens is known as magnifying glass, or simply a magnifier.

Let us now calculate the magnifying power or angular magnification of the magnifying glass which is defined as the ratio of visual angle subtended by the image seen through a magnifying glass to the visual angle subtended by the object when placed at the least distance of distinct vision, when seen through naked eye.

Consider a small object OP which is placed at a distance p within the focal length of a magnifying glass 'L' such that its erect, virtual and magnified image IQ is produced at the least distance of distinct vision ' d ' as shown in Fig. 10.17(b). The magnifying power of the magnifying glass is given by

$$M = \frac{\beta}{\alpha}$$

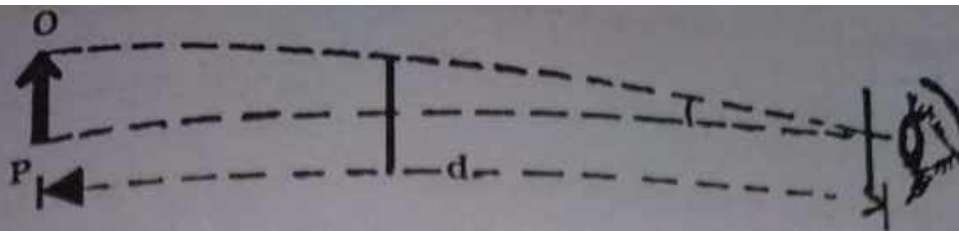
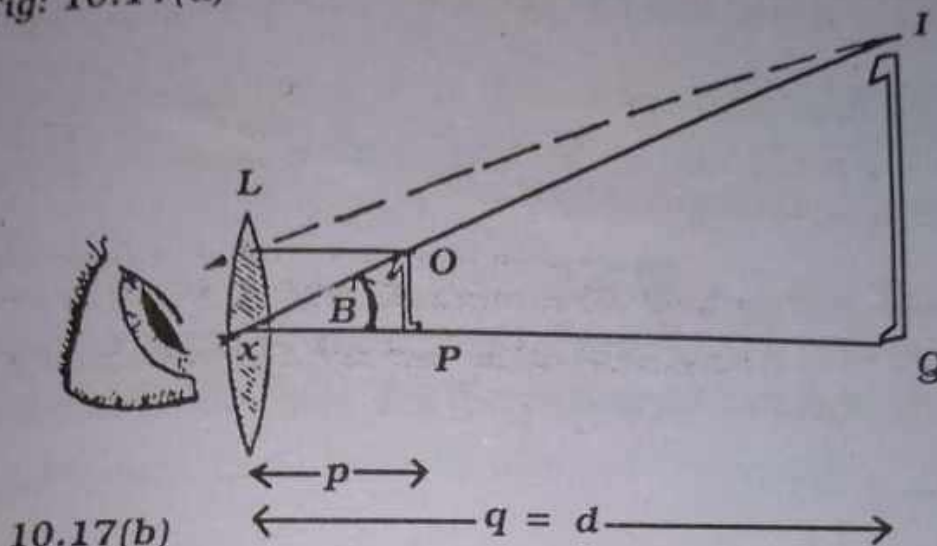


Fig: 10.17(a)



10.17(b)

Where α is the visual angle subtended by the object when placed at least distance of distinct vision, when seen through unaided eye and β is the visual angle subtended by the image seen through the magnifying glass.

therefore

$$\tan \alpha = \frac{OP}{d} \quad \text{since } \alpha \text{ is small}$$

$$\therefore \tan \alpha = \alpha$$

$$\therefore \alpha = \frac{OP}{d} \quad 10.15$$

In right angled $\triangle OPX$ in Fig. 10.17(b), we have

$$\beta = \tan \beta$$

$$10.16$$

$$\therefore \beta = \frac{IQ}{d} = \frac{OP}{p}$$

By substituting values of α and β from Eq. 10.15 and 10.16 in Eq. 10.14, the angular magnification is

$$M = \frac{\beta}{\alpha} = \frac{d}{p} = \frac{IQ}{OP} = \frac{\text{Image Size}}{\text{Object Size}} \quad 10.17$$

From the lens formula, we have

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

for the magnifying glass, we have

$$p = +p, \quad q = -d \text{ and } f = +f$$

$$\therefore \frac{1}{p} - \frac{1}{d} = \frac{1}{f}$$

multiplying throughout by d , we get

$$\frac{d}{p} = \frac{d}{f} = +1 = \frac{25}{f} + 1$$

Substituting this value of $\frac{d}{p}$ in Eq. 10.17, we get

$$M = \frac{d}{f} + 1 \quad 10.18$$

Example 10.1

A chess piece 4cm high is located 10 cm, from the converging lens whose focal length is 20 cm. What is the nature size and location of the image?

Solution:-

The object is between the focal point and the lens. The image distance is found by using the lens equation

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{20} = \frac{1}{10} + \frac{1}{q}$$

$$q = -20$$

Since q is negative, the image is virtual and therefore erect.

The magnification is given by

$$M = -\frac{q}{p} = \frac{20}{10} = +2$$

$$\therefore M = \frac{y'}{y} = -\frac{q}{p}$$

$$\text{or } y' = My = 2 \times 4 = 8 \text{ cm}$$

The magnification is 2 in this case, with final size being 8 cm. The positive value of image size means that the image is erect.

Example 10.2

How far from a convex lens whose focal length is + 20 cm must a specimen of a Red Admiral butterfly be placed if its image is to be a real one, three times as large as the object?

Solution:-

Although we neither know the distance of object nor that of the image, we can relate p and q from the magnification formula. Since the real image is inverted,

i.e. $y' = 3y$, the magnification is given

$$\text{by } M = \frac{y'}{y} = \frac{-3y}{y} = -3$$

$$\text{Hence } M = \frac{-q}{p} \quad \text{or } q = +3p$$

The -ve sign is necessary since the image is to be real and therefore inverted. Now using the lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\text{or } \frac{1}{p} + \frac{1}{3p} = \frac{1}{20}$$

$$\therefore \frac{3+1}{3p} = \frac{1}{20}$$

$$\text{or } 3p = 4 \times 20 = 80 \text{ cm}$$

$$p = 26.66$$

Example 10.3

A convex lens of focal length 20 cm, is used to form an erect image which is twice as large as the object. Find the position of the object?

Solution:-

Magnification = + 2, as the image is erect

$$\text{Thus } -\frac{q}{p} = +2 \quad \text{or } q = -2p$$

$$\text{But } \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\text{Here } q = -2p, f = 20 \text{ cm}$$

$$\text{Thus } \frac{1}{p} - \frac{1}{2p} = \frac{1}{20} \text{ cm}$$

$$\text{or } \frac{1}{2p} = \frac{1}{20}$$

$$p = 10 \text{ cm}$$

The object is 10 cm from the lens.

Example 10.4

An object is placed at a distance of 60 cm from a concave lens of focal length 30 cm. Find the position and nature of the image?

Solution:-

Here $p = 60 \text{ cm}$, and $f = -30 \text{ cm}$, as lens is concave

By the lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{60} + \frac{1}{q} = -\frac{1}{30}$$

$$\text{or } \frac{1}{q} = -\frac{1}{30} - \frac{1}{60} = -\frac{3}{60}$$

$$\therefore q = -\frac{60}{3} = -20\text{cm}$$

$$\text{Also magnification} = -\frac{q}{p} = -\frac{(-20)}{60} = +\frac{1}{3}$$

The negative sign of q shows that the image is virtual. The positive sign of magnification shows that the image is erect. Hence the image is erect, virtual and diminished.

Example 10.5

A diverging lens of focal length $f = -16$ cm is held 8 cm from a rare stamp. Where is the image of this stamp located and what is the magnification of the lens?

Solution:-

$$\text{Given } f = -16\text{cm}$$

$$p = 8\text{ cm}$$

$$q = ?$$

Using lens equation

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\therefore \frac{1}{q} = -\frac{1}{16} - \frac{1}{8} = -\frac{3}{16}$$

$$\text{or } q = -\frac{16}{3} = -5.33\text{cm}$$

The -ve sign shows that the image is virtual, hence erect and to the left of the lens. The magnification is

$$M = \frac{-q}{p} = -\frac{(-5.33)}{8} = +0.666$$

The positive value of magnification indicates that the image is erect. Since the magnification is less than 1, it means that the image is actually smaller than the object.

10.9 COMPOUND MICROSCOPE

A compound microscope is an optical instrument which is used to see small object with very high magnification. It consists of two convex lenses- an objective (a lens placed before the object) of very short focal length f_1 , and an eye-piece (a lens placed before the eye) of relatively long focal length f_2 . The objective forms a real, inverted and magnified image of the object just placed beyond its focus. The eye piece is used as a magnifying glass to see the image formed by the objective. The final image seen by the eye through the microscope is virtual and very much magnified. Fig. 10.18 shows the path of rays through the microscope.

In order to derive an expression for the magnifying power of microscope, consider a small object OP placed at the distance p just beyond the focus of the objective lens L_1 , whose real, inverted and magnified image IQ is formed at a distance q from the objective lens.

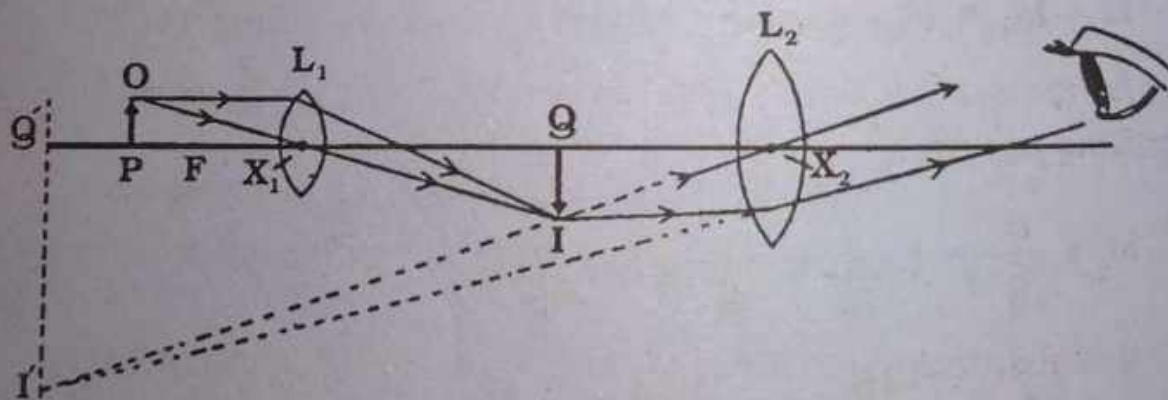


Fig 10.18 Compound Microscope

Magnifying power of the microscope is given by

$$M = \frac{\beta}{\alpha}$$

Where α and β are the visual angles subtended by the object and image respectively when formed at the least distance of distinct vision

$$\therefore \alpha = \frac{OP}{d} \quad \text{and} \quad \beta = \frac{I'Q'}{d}$$

Substituting the values of α and β in Eq: 10.19, we get

$$\begin{aligned} \therefore M &= \frac{I'Q'}{OP} \\ \text{or } M &= \frac{IQ}{OP} \times \frac{I'Q'}{IQ} \end{aligned} \quad 10.20$$

The magnifying power of objective L_1 is given by

$$M_1 = \frac{IQ}{OP} = \frac{q}{p} \quad (\because \Delta OPX_1 \text{ and } \Delta IQX_1 \text{ are similar})$$

The magnifying power of eye-piece is given by

$$M_2 = \frac{I'Q'}{IQ} \quad (\because \Delta IQX_2 \text{ and } \Delta I'Q'X_2 \text{ are similar})$$

The Eq. 10.20 can also be written as

$$M = M_1 \times M_2 \quad 10.21$$

As the eye-piece acts here as a magnifying glass, hence its magnifying power can also be written as

$$M_2 = \frac{d}{f_2} + 1$$

By substituting values of M_1 and M_2 in Eq: 10.21 we get

$$M = \frac{q}{p} \left(\frac{d}{f_2} + 1 \right) \quad 10.22$$

As the object OP lies just outside the focus of the objective L_1

$$\therefore p \approx f_1$$

Also the image IQ is formed very close to the eye piece L_2

$$\therefore QX_1 \approx X_1 X_2$$

$$\text{or } q \approx L$$

Where L is the distance between objective and eye-piece, which is also called the length of the microscope. Hence the magnifying power of the compound microscope is found to be

$$M \approx \frac{L}{f_1} \left(\frac{d}{f_2} + 1 \right) \quad 10.23$$

Example 10.6

A microscope has an objective of 10.0mm focal length and eye piece of 25 mm focal length. What is the distance between the lenses and what is the magnification if the object is in sharp focus when it is 10.5mm from the objective?

Solution:

For the objective $p = 10.5\text{mm}$, $f_1 = 10\text{mm}$, $q = ?$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_1}$$

$$\frac{1}{10.5} + \frac{1}{q} = \frac{1}{10}$$

$$\therefore q = 210 \text{ mm}$$

For the eye piece $f_2 = 25 \text{ mm}$, $q = -250 \text{ mm}$ ($\because$ the final image should be at the least distance of distinct vision)

$$p = p_1 = ?$$

$$\frac{1}{p_1} - \frac{1}{250} = \frac{1}{25}$$

$$p_1 = 22.7 \text{ mm}$$

Distance between the lenses

$$q + p_1 = 210 + 22.7 = 232.7 \text{ mm}$$

$$\text{Magnification by objective} = -\frac{q}{p} = \frac{210}{10.5} = -20$$

$$\text{Magnification by eye-piece} = \left(\frac{d}{f_2} + 1\right) = \left(\frac{250}{25} + 1\right) = 11$$

$$\text{Total magnification} = 20 \times 11 = 220$$

10.10 TELESCOPE

Telescopes are used to see distant objects. The image of a distant object formed by a telescope is smaller than the actual object; because it is much nearer to the eye and has a greater visual angle, the object looks larger when viewed through the telescope.

Astronomical Telescope

An astronomical telescope is needed to see the heavenly bodies i.e. planets and stars. It consists of two convex lenses an objective, L_1 of long focal length f_1 and an eye piece L_2 of short focal length f_2 . Fig 10.19 shows the path of rays through an astronomical telescope.

In order to derive an expression for the magnifying power of the astronomical telescope consider a distant object whose real, inverted and diminished image IQ is formed by the objective L_1 at its focus.

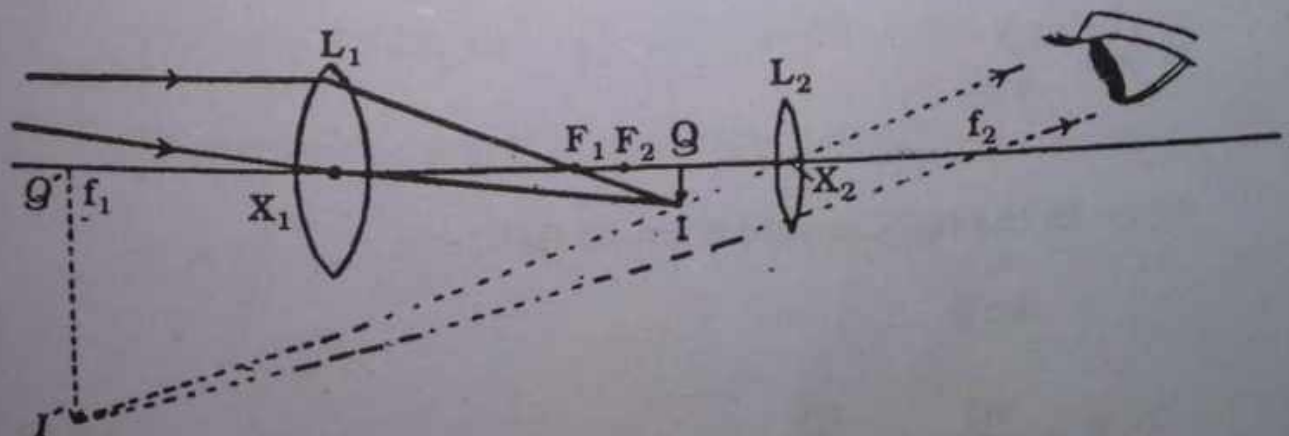


Fig. 10.19 Astronomical telescope.

The eye piece L_2 is so adjusted that IQ lies just inside its focus, hence a virtual and magnified image $I'Q'$ is formed by it at infinity.

$$\therefore QX_2 = f_2$$

Thus an astronomical telescope forms an inverted, virtual and highly magnified image of a distant object as shown in the Fig. 10.19 The distance between objective and eye piece is called the length of the telescope, which is given as

$$L = X_1 X_2$$

$$\text{or } L = QX_1 + QX_2$$

$$\text{Since } QX_1 = f_1 \text{ and } QX_2 = f_2$$

$$\therefore L = f_1 + f_2$$

The magnifying power of the telescope is given by

$$M = \frac{\beta}{\alpha} \quad 10.25$$

Where α and β are the visual angles subtended by the object and image respectively.

In the right angled triangle IQX_1 , we have

$$\alpha = \tan \alpha \text{ (}\because \alpha \text{ is small)}$$

$$\therefore \alpha = \frac{IQ}{QX_1} \quad 10.26$$

Again in the right angled triangle IQX_2

$$\beta = \tan \beta$$

$$\therefore \beta = \frac{IQ}{QX_2} = \frac{IQ}{f_2} \quad 10.27$$

Substituting the values of α and β from Eq.10.26 and 10.27 in Eq.10.25 we get

$$M = \frac{f_1}{f_2} = \frac{\text{focal length of objective}}{\text{focal length of eye piece}}$$

From the above relation, it is clear that for high magnification the focal length of the objective should be very large as compared to that of the eye piece

Example 10.7

An astronomical telescope has an objective lens whose power is 2 dioptres. This lens is placed 60 cm from the eye piece. When the telescope is adjusted for minimum eye strain. Calculate the angular magnification of the telescope.

Solution:-

Our first step is to determine the focal lengths of both lenses. For the objective we are given a power in dioptres

$$f_1 \text{ (in metres)} = \frac{1}{\text{power (in dioptres)}} = \frac{1}{2} \text{ m} = 50 \text{ cm}$$

The sum of the focal lengths $f_1 + f_2$ equals the separation of the two lenses, therefore

$$f_1 + f_2 = 60 \text{ cm and } f_2 = 60 - f_1 = 60 - 50 = 10 \text{ cm}$$

The magnifying power of this telescope is

$$M = \frac{f_1}{f_2} = \frac{50 \text{ cm}}{10 \text{ cm}} = 5$$

This is a low-power telescope that is used to examine a large area of the sky. We would require a high-power telescope to examine individual bodies, such as moon and planets in closed detail.

10.11 GALILEAN TELESCOPE

Galilean telescope is an optical instrument which is used to see the objects on earth. It was developed by Galileo. A Galilean Telescope consists of a convex lens L_1 as an objective and a concave lens L_2 as an eye piece. The virtual and erect image of distant object when seen through the eye piece L_2 is formed at the focus of the objective L_1 .

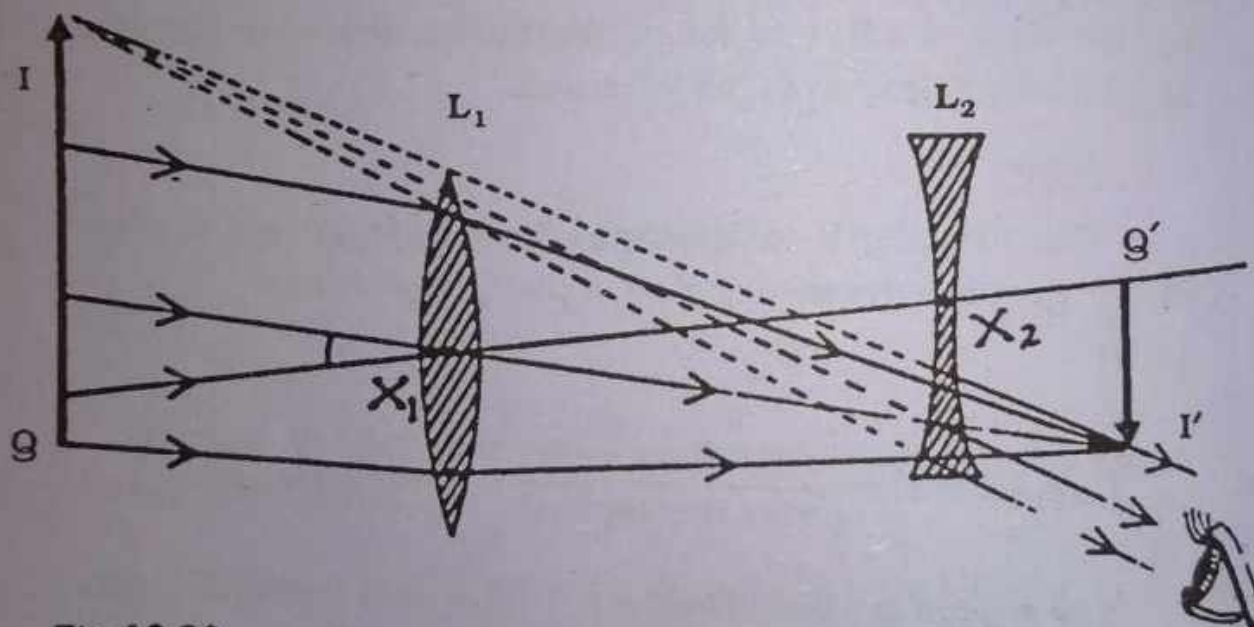


Fig 10.20

$$\therefore QX_1 = f_1 \quad 10.28$$

The eye piece L_2 is introduced between L_1 and IQ such that IQ lies very close to its focus.

$$\therefore QX_2 = f_2 \quad 10.29$$

Thus erect, virtual and highly magnified image $I'Q'$ is produced as shown in the Fig.10.20 The distance between the objec-

tive and the eye piece is called length of the telescope, which is given by

$$L = X_1 X_2 = QX_1 - QX_2$$

$$\therefore L = f_1 - f_2$$

Expression for magnifying power

If α and β are visual angles subtended by the object and the image respectively, then magnifying power can be obtained as

$$M = \frac{\beta}{\alpha} \quad 10.30$$

In the right angled $\triangle IQX_1$, we have

$$\alpha = \tan \alpha \quad (\because \text{the angle is small})$$

$$\therefore \alpha = \frac{IQ}{IX_1}$$

$$\text{or } \alpha = \frac{IQ}{f_1} \quad 10.31$$

Again in right angled $\triangle IQX_2$, we have

$$\beta = \tan \beta$$

$$\therefore \beta = \frac{IQ}{QX_2}$$

$$\text{or } \beta = \frac{IQ}{f_2} \quad 10.32$$

By substituting the values of α and β from Eq. 10.39 and 10.40 in Eq. 10.38, we get

$$M = \frac{f_1}{f_2} = \frac{\text{focal length objective}}{\text{focal length of eye piece}}$$

10.12 TERRESTRIAL TELESCOPE

When telescope is used for astronomical purpose, i.e. when we see the heavenly bodies like moon and stars, their image is seen inverted. But when terrestrial objects are to be viewed, it is necessary to have an erect final image. The erection can be accomplished by introducing a third lens between the objective and eye piece of telescope. The arrangement is shown schematically in Fig. 10.21

The function of erecting lens is clear from Fig. 10.21. It only

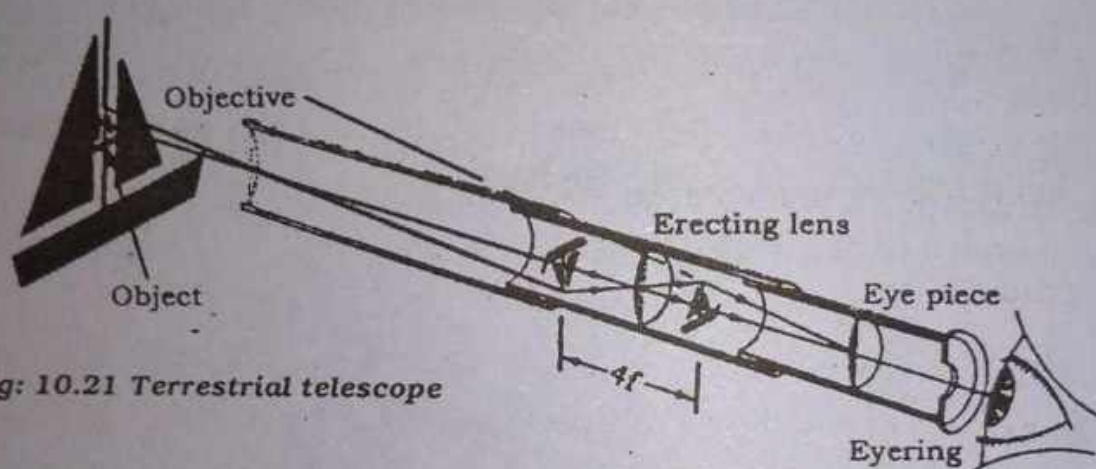


Fig: 10.21 Terrestrial telescope

inverts IQ to $I'Q'$ and does not change the magnifying power which is same as in the case of an astronomical telescope.

10.13 SPECTROMETER

It is an instrument which is used to study the spectrum of luminous bodies. The essential parts of this instrument are (a) collimator (b) telescope and (c) turn table as shown in the Fig 10.22

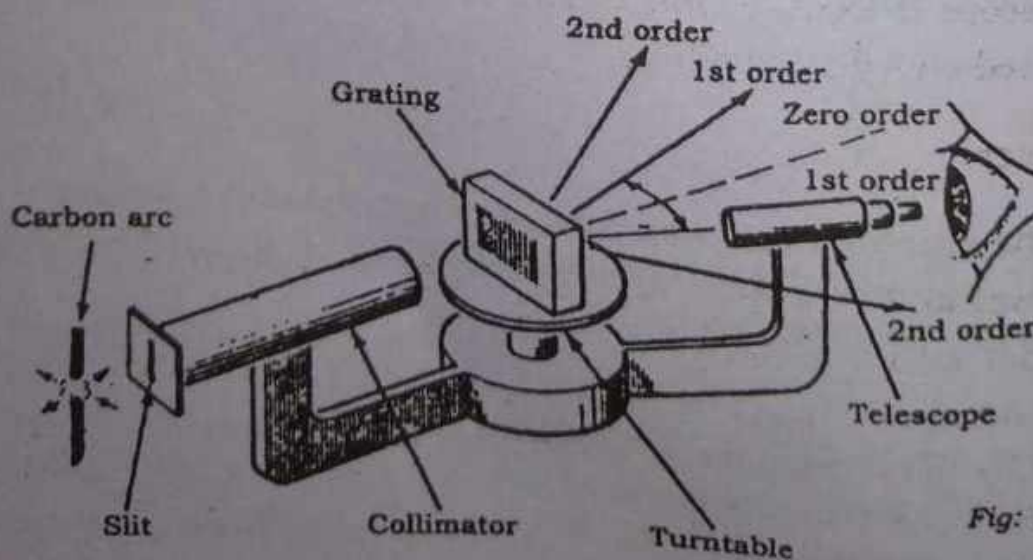


Fig: 10.22

(a) Collimator

It consists of a metal tube provided with a convex lens at its one end and an adjustable slit S at its other end. The length of the tube is altered by a screw to be equal to the focal length of the convex lens so that the collimator produces a parallel beam of light. The collimator is fixed to the base of the instrument while turn table and telescope can rotate about a common axis.

(b) Telescope

The telescope is simple astronomical telescope which is used for making measurements, or for examining the spectrum. The adjustment for focussing the telescope is made with the help of screw S_2 . The telescope can be rotated about an axis.

(c) Turn Table

It is a circular metallic plate which can be rotated about an axis. Its height is also adjustable and this can be levelled by means of three screws L, M and N.

There are arrangements for fine motion of the telescope and turn table. A vernier scale is provided to measure the angle with great accuracy, i.e. in degrees and minutes.

Before using, the collimator is adjusted for parallel rays and the telescope is focussed for parallel rays or for infinity. For this it is focussed on a distant object.

Uses

The spectrometer is an analysing instrument used primarily to discover and measure the wave lengths of a given light. When light from an incandescent solid, liquid or gas is examined by a spectrometer, an image of the slit is formed for each wave length emitted by the source. Such a spectrum is called an emission spectrum. If all the visible wave lengths are present in the light which is analysed, the images overlap and form a continuous spectrum. If

the source emits only a few definite wave lengths, the images of the slit are separated from one another and appear as a series of bright lines. Such a spectrum is called a line spectrum. A third type of emission spectrum is called bandspectrum which shows band instead of lines. These bands are found to emit of closely spaced lines arranged in an orderly manner. The line spectrum of a mercury light is shown in Fig.10.23.

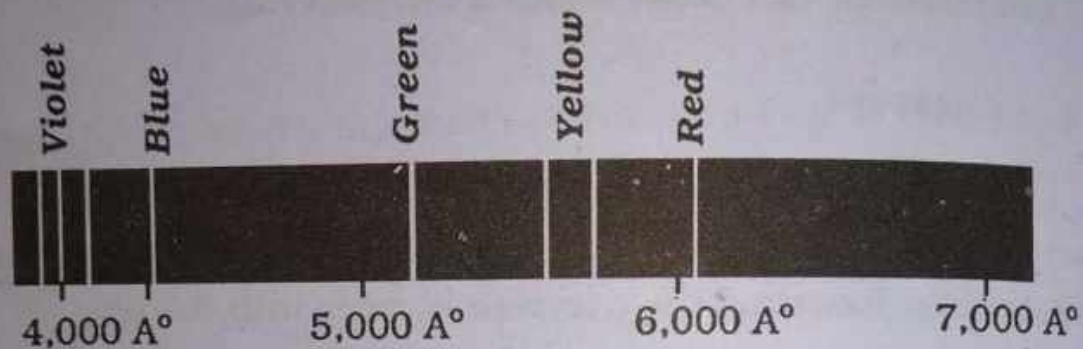


Fig: 10.23 Spectrum of mercury light

10.14 THE EYE

The eye is very important optical system which has much common with the camera. Like the camera the eye gathers light and produces a sharp image.

Fig.10.25 shows the essential parts of the eye. The front of the eye is covered by a transparent membrane called the cornea. This is followed by a clear liquid region, a variable aperture (iris and pupil) and the crystalline lens. Most of the refraction occurs in the cornea, since the liquid medium surrounding the lens has an average index of refraction close that of the lens. The iris is a muscular diaphragm that controls the size of the pupil. It regulates the

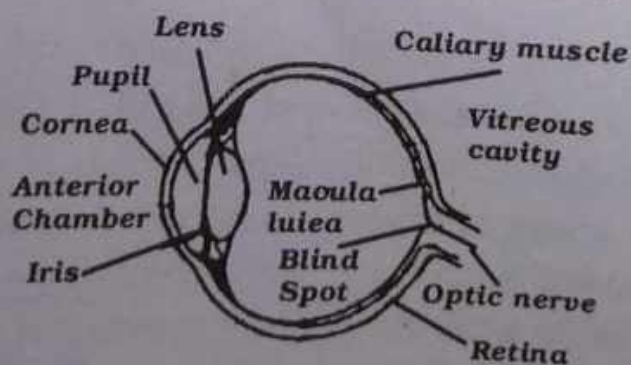


Fig: 10.24

amount of the light entering the eye by dilating the pupil in light of low intensity light. Light entering the eye is focused by the cornea-lens system on the back surface of the eye called the retina. The retina contains nerve fibres which branch out into millions of sensing structures called rods and cones. Optical image received by the retina is transmitted to brain via the optic nerve.

Although the eye is one of the most remarkable creation in nature, it often does not function perfectly. The eye may have several abnormalities, which can some times be corrected by eye glasses, contact lenses, or surgery. When the relaxed eye produces an image of a distant object behind the retina, as in Fig 10.25(a) the abnormality is known as myopia and the person is said to be farsighted. This condition is corrected with a converging lens, as shown in Fig. 10.25 (b). When an image of a distant object is focussed in front of the retina, as in Fig. 10.26(a) the abnormality is known as hyperopia or short sightedness. This condition can be corrected with a diverging lens as in Fig 10.26(a), (b).

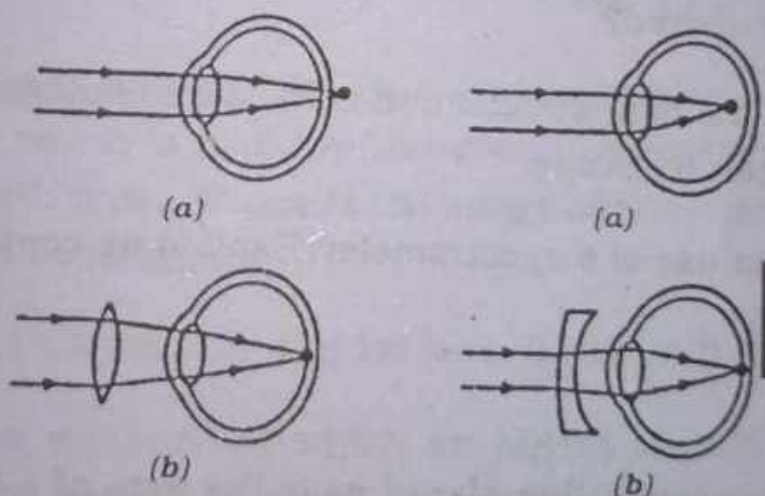


Fig: 10.25 (a) A hyperopic eye (far sighted) is slightly shorter than normal and so the image of a distant object focuses behind the retina. (b) The condition can be corrected with a converging lens.

Fig: 10.26 (b). (a) A myopic eye (near sighted) is slightly longer than normal and so the image of a distant object forms in front of the retina. (b) The condition can be corrected with a diverging lens.

QUESTIONS

1. How can a real image be distinguished from a virtual image? Can each type of image be projected on screen? Explain.

2. Show how the position, nature and size of the image are formed by a (a) convex lens (b) concave lens.
3. Under what conditions does a converging lens act as a diverging lens?
4. Why is a convex lens of small focal length preferred for a magnifying glass?
5. Define the construction, working and magnifying power of a compound microscope.
6. How is the magnifying power of a (i) telescope and (ii) microscope affected by increasing the focal length of their objectives?
7. What is the difference between astronomical and terrestrial telescopes?
8. Explain the defects which occur in the lenses, and how they can be removed?
9. Explain the construction and calculate the magnifying power of Galilean telescope.
10. What is the use of a spectrometer? Explain its construction and working.

PROBLEMS

1. An object 4cm tall is placed near the axis of a thin converging lens. If the focal length of the lens is 25 cm, where will the image be formed and what will be the size of the image? Sketch the principal ray diagram.
(Ans. 100 cm behind 12 cm)
2. A convex lens has a focal length of 10 cm. Determine the image distances when an object is placed at the following distances from the lens.
50 cm, 20 cm, 15 cm, 10 cm, and 5 cm
(Ans. 12.5 cm, 20 cm, 30 cm, infinity and -10 cm)

3. Two converging lenses of focal lengths 40 cm and 50 cm are placed in contact. What is the focal length of this lens combination? What is the power of the combination in diopters?
(Ans. 22.2 cm, 4.5 diopters)
4. A converging lens of focal length 20 cm is placed in front of a converging lens of focal length 4 cm. What is the distance between the lenses if parallel rays entering the first lens leave the second lens as parallel rays?
(Ans. 24 cm)
5. A parallel light beam is diverged by a concave lens of focal length -12.5 cm and then made parallel once more by a convex lens of focal length 50 cm. How far are the two lenses apart?
(Ans. 37.5 cm)
6. Two lenses are in contact, a converging one of focal length 30 cm and a diverging one of focal length -10 cm. What is the focal length and power of the combination?
(Ans. 15 cm, -6.7 diopters)
7. Moon light passes through a converging lens of focal length 19 cm, which is 20.5 cm from a second converging lens of focal length 2 cm. Where is the image of the moon produced by the lens combination?
(Ans. 14.5 cm from the 1st lens, 6 cm from the second lens)
8. Find the distance at which an object should be placed in front of a convex lens of focal length 10 cm to obtain an image of double its size?
(Ans. 15 cm for producing a real image, 5 cm for producing virtual image).
9. A compound microscope has a 12 mm focal length objective and a 75 mm focal length eye piece, and the two lenses are mounted 200 mm apart. If the final image is 225 mm from the eye piece, what is the magnification produced?
(Ans. 44)

10. An astronomical telescope of angular magnification 1000 has an objective of 15 m focal length. What is the focal length of the eye piece?

(Ans. 15 mm)

11. A Galilean telescope has an objective of 120 mm focal length and an eye piece of 50 mm focal length. If the image seen by the eye is 300 mm from the eye piece, what is angular magnification?

(Ans: 2.66)

12. A compound microscope has an objective with a focal length of 10 mm and a tube 100 mm long. An image is produced 250 mm from the eye piece when the object is 12 mm from the objective. What is the angular magnification?

(Ans. 31)

13. A converging lens of 4 dioptres is combined with a diverging lens of -2 dioptres. Find the power and focal length of the combination.

(Ans. Power of combination = 2 dioptres; focal length of combination = 50 cm).

14. A convex lens forms image of an object on a fixed screen 20 cm from the lens. On moving the lens 60 cm towards the object, the image is again formed on the screen. What is the focal length of the lens?

(Ans: 16 cm)

15. Two converging lenses are 25 cm apart. Focal length of each is 10 cm. An object is placed in front of one lens at 50 cm. Find the distance between the object and the final image?

(Ans. 125 cm)