

EXERCISE NO. 5**SET - A****-:5.1:-**If $f(x) = 5x + 3$, find $f(1)$, $f(2)$, $f(3)$ **SOLUTION**

$$f(x) = 5x + 3$$

$$f(1) = 5(1) + 3 = 4 + 3 = 8$$

$$f(2) = 5(2) + 3 = 10 + 3 = 13$$

$$f(3) = 5(3) + 3 = 15 + 3 = 18$$

-:5.2:-If $f(x) = 2x - 5$, find $f(1)$, $f(10)$, $f(100)$ **SOLUTION**

$$f(x) = 2x - 5$$

$$f(1) = 2(1) - 5 = 2 - 5 = -3$$

$$f(10) = 2(10) - 5 = 20 - 5 = 15$$

$$f(100) = 2(100) - 5 = 200 - 5 = 195$$

-:5.3:-If $f(t) = 6t + 4$, find $f(-1/2)$, $f(1/2)$, $f(3/2)$ **SOLUTION**

$$f(t) = 6t + 4$$

$$f(-1/2) = 6(-1/2) + 4 = -3 + 4 = 1$$

$$f(1/2) = 6(1/2) + 4 = 3 + 4 = 7$$

$$f(3/2) = 6(3/2) + 4 = 9 + 4 = 13$$

-:5.4:-If $y = 2x + 3$, find y when $x = 1, 2, 3, 4$ **SOLUTION**

$$y = 2x + 3$$

$$y = 2(1) + 3 = 2 + 3 = 5$$

$$y = 2(2) + 3 = 4 + 3 = 7$$

$$y = 2(3) + 3 = 6 + 3 = 9$$

$$y = 2(4) + 3 = 8 + 3 = 11$$

-:5.5:-

If $y = -3x + 5$, find y when $x = -1, -2, -3, -4$ **SOLUTION**

$$y = -3x + 5$$

$$y = -3(-1) + 5 = 3 + 5 = 8$$

$$y = -3(-2) + 5 = 6 + 5 = 11$$

$$y = -3(-3) + 5 = 9 + 5 = 14$$

$$y = -3(-4) + 5 = 12 + 5 = 17$$

-:5.6:-

For the line $y = 4x + 2$, compute the following table.

x	0	1	2	3	4	5	6	7	8	9	10
y											

SOLUTION

$$y = 4x + 2$$

$$y = 4(0) + 2 = 0 + 2 = 2, \quad y = 4(1) + 2 = 4 + 2 = 6$$

$$y = 4(2) + 2 = 8 + 2 = 10, \quad y = 4(3) + 2 = 12 + 2 = 14$$

$$y = 4(4) + 2 = 16 + 2 = 18, \quad y = 4(5) + 2 = 20 + 2 = 22$$

$$y = 4(6) + 2 = 24 + 2 = 26, \quad y = 4(7) + 2 = 28 + 2 = 30$$

$$y = 4(8) + 2 = 32 + 2 = 34, \quad y = 4(9) + 2 = 36 + 2 = 38$$

$$y = 4(10) + 2 = 40 + 2 = 42$$

X	0	1	2	3	4	5	6	7	8	9	10
Y	2	6	10	14	18	22	26	30	34	38	42

-:5.7:-

Find the equation to the straight line passing through $(0, 0)$ and $(2, -2)$ **SOLUTION**

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Here $x_1 = 0, y_1 = 0, x_2 = 2, y_2 = -2$

$$(y - 0) = \frac{-2 - 0}{2 - 0} (x - 0)$$

$$y = -1(x - 0)$$

$$y = -x$$

∴5.8:-

Find the equation to the straight line passing through (3, 4) and (5, 6)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Here $x_1 = 3$, $y_1 = 4$, $x_2 = 5$, $y_2 = 6$

$$(y - 4) = \frac{6 - 4}{5 - 3} (x - 3)$$

$$y - 4 = \frac{2}{2} (x - 3)$$

$$y - 4 = x - 3$$

$$y = x - 3 + 4$$

$$y = x + 1$$

∴5.9:-

Find the equation to the straight line passing through (1, 3) and (-7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Here $x_1 = 1$, $y_1 = 3$, $x_2 = -7$, $y_2 = 8$

$$(y - 3) = \frac{8 - 3}{-7 - 1} (x - 1)$$

$$y - 3 = \frac{5}{-8} (x - 1)$$

$$8(y - 3) = -5(x - 1)$$

$$8y - 24 = -5x + 5$$

$$8y = -5x + 5 + 24$$

$$8y = -5x + 29$$

:-5.10:-

Find the equation to the straight line passing through (-1, 3) and (7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Here $x_1 = -1$, $y_1 = 3$, $x_2 = 7$, $y_2 = 8$

$$(y - 3) = \frac{8 - 3}{7 + 1} (x + 1)$$

$$y - 3 = \frac{5}{8} (x + 1)$$

$$8(y - 3) = 5(x + 1)$$

$$8y - 24 = 5x + 5$$

$$8y = 5x + 5 + 24$$

$$8y = 5x + 29$$

:-5.11:-

Find the equation to the straight line passing through (-1, -3) and (-7, 8)

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Here $x_1 = -1$, $y_1 = -3$, $x_2 = -7$, $y_2 = 8$

$$(y + 3) = \frac{8 - (-3)}{-7 + 1} (x + 1)$$

$$y + 3 = \frac{11}{-6} (x + 1)$$

$$-6(y + 3) = 11(x + 1)$$

$$-6y - 18 = 11x + 11$$

$$-6y = 11x + 11 + 18$$

$$-6y = 11x + 29$$

$$6y = -11x - 29$$

-.5.12:-

Find the equation to the straight line passing through $(0, -a)$ and $(b, 0)$

SOLUTION

Equation to straight line passing through two points is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

Here $x_1 = 0, y_1 = -a, x_2 = b, y_2 = 0$

$$(y + a) = \frac{0 + a}{b - 0} (x - 0)$$

$$y + a = \frac{a}{b} (x)$$

$$by + ab = ax$$

$$by = ax - ab$$

-.5.13:-

Find the equation to the straight line passing through $(2, 7)$ and its slope is $3/5$.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{3}{5}(x - 2)$$

$$5(y - 7) = 3(x - 2)$$

$$5y - 35 = 3x - 6$$

$$5y = 3x - 6 + 35$$

$$5y = 3x + 29$$

-.5.14:-

Find the equation to the straight line passing through $(-2, -3)$ and its slope is -3 .

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$

$$y + 3 = -3(x + 2)$$

$$y = -3x - 6 - 3$$

$$y = -3x - 9$$

-:5.15:-

Find the equation to the straight line passing through $(-3, 4)$ and its slope is $2/3$.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{2}{3}(x + 3)$$

$$3(y - 4) = 2(x + 3)$$

$$3y - 12 = 2x + 6$$

$$3y = 2x + 6 + 12$$

$$3y = 2x + 18$$

-:5.16:-

Find the equation to the straight line passing through $(2, -3)$ and its slope is -2 .

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$

$$y + 3 = -2(x - 2)$$

$$y + 3 = -2x + 4$$

$$y = -2x + 4 - 3$$

$$y = -2x + 1$$

-:5.17:-

Find the equation to the straight line passing through $(0, 2)$ and its slope is $-2/3$.

SOLUTION

Equation to straight line by point slope equation is

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{2}{3}(x - 0)$$

$$3(y - 2) = -2x$$

$$3y - 6 = -2x$$

$$3y = -2x + 6$$

-.5.18:-

Find the equation to the straight line in which y-intercept is 2 and slope is $\frac{2}{3}$.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$

$$y = \frac{2}{3}x + 2 = \frac{2x + 6}{3}$$

$$3y = 2x + 6$$

-.5.19:-

Find the equation to the straight line in which y-intercept is -3 and slope is $\frac{3}{4}$.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$

$$y = \frac{3}{4}x - 3 = \frac{3x - 12}{4}$$

$$4y = 3x - 12$$

-.5.20:-

Find the equation to the straight line in which y-intercept is $\frac{5}{2}$ and slope is $-\frac{2}{5}$.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$

$$y = -\frac{2}{5}x + \frac{5}{2} = \frac{-4x + 25}{10} \Rightarrow 10y = -4x + 25$$

-:5.21:-

Find the equation to the straight line in which y-intercept is -4 and slope is -1.

SOLUTION

The slope, intercept equation is

$$y = mx + c$$

$$y = -1x - 4 \Rightarrow y = -x - 4$$

-:5.22:-

Find the slope and y-intercept of $3x + 2y - 7 = 0$

SOLUTION

$$3x + 2y - 7 = 0$$

$$2y = -3x + 7$$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

$$y = mx + c$$

Hence Slope = $m = -3/2$ and Intercept = $c = 7/2$

-:5.23:-

Find the slope and y-intercept of $3x + 2y + 7 = 0$

SOLUTION

$$3x + 2y + 7 = 0$$

$$2y = -3x - 7$$

$$y = -\frac{3}{2}x - \frac{7}{2}$$

$$y = mx + c$$

Hence Slope = $m = -3/2$ and Intercept = $c = -7/2$

-:5.24:-

Find the slope and y-intercept of $5x - 4y + 8 = 0$

SOLUTION

$$5x - 4y + 8 = 0$$

$$-4y = -5x - 8 \Rightarrow 4y = 5x + 8$$

$$y = \frac{5}{4}x + 2 \Rightarrow y = mx + c$$

Hence Slope = $m = 5/4$ and Intercept = $c = 2$

:-5.25:-

Find the slope and y-intercept of $3x - 6y + 4 = 0$

SOLUTION

$$3x - 6y + 4 = 0$$

$$-6y = -3x - 4$$

$$6y = 3x + 4$$

$$y = \frac{3}{6}x + \frac{4}{6}$$

$$y = \frac{1}{2}x + \frac{2}{3}$$

$$y = mx + c$$

Hence Slope = $m = 1/2$ and Intercept = $c = 2/3$

:-5.26:-

Find the slope of straight lines which passes through (3, 5) and (5, 3).

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = 3$, $y_1 = 5$, $x_2 = 5$ and $y_2 = 3$

$$m = \frac{3-5}{5-3} = \frac{-2}{2} = -1$$

:-5.27:-

Find the slope of straight lines which passes through (1, 4) and (-1, 4).

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $x_1 = 1$, $y_1 = 4$, $x_2 = -1$ and $y_2 = 4$

$$m = \frac{4-4}{-1-1} = \frac{0}{-2} = 0$$

-:5.28:-

Find the slope of straight lines which passes through

$$\left(\frac{2}{3}, \frac{3}{2}\right) \text{ and } \left(\frac{6}{7}, \frac{7}{6}\right).$$

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Here } x_1 = \frac{2}{3}, y_1 = \frac{3}{2}, x_2 = \frac{6}{7}, \text{ and } y_2 = \frac{7}{6}$$

$$m = \frac{\frac{7}{6} - \frac{3}{2}}{\frac{6}{7} - \frac{2}{3}} = \frac{\frac{7-9}{6}}{\frac{18-14}{21}}$$

$$\frac{-2}{6} = \frac{-2}{6} \times \frac{21}{4} = \frac{-42}{24} = \frac{-7}{4}$$

-:5.29:-

Find the slope of straight lines which passes through

$$\left(\frac{1}{4}, \frac{4}{7}\right) \text{ and } \left(\frac{4}{7}, \frac{1}{4}\right).$$

SOLUTION

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Here } x_1 = \frac{1}{4}, y_1 = \frac{4}{7}, x_2 = \frac{4}{7}, \text{ and } y_2 = \frac{1}{4}$$

$$m = \frac{\frac{1}{4} - \frac{4}{7}}{\frac{4}{7} - \frac{1}{4}} = \frac{\frac{7-16}{28}}{\frac{16-7}{28}} = \frac{-9}{9} = -1$$

-:5.30:-

Graph the following linear function, $y = 6x - 2$

SOLUTIONPutting $x = -2, -1, 0, 1, 2, 3$

$$y = 6(-2) - 2 = -12 - 2 = -14$$

$$y = 6(1) - 2 = 6 - 2 = 4$$

$$y = 6(-1) - 2 = -6 - 2 = -8$$

$$y = 6(2) - 2 = 12 - 2 = 10$$

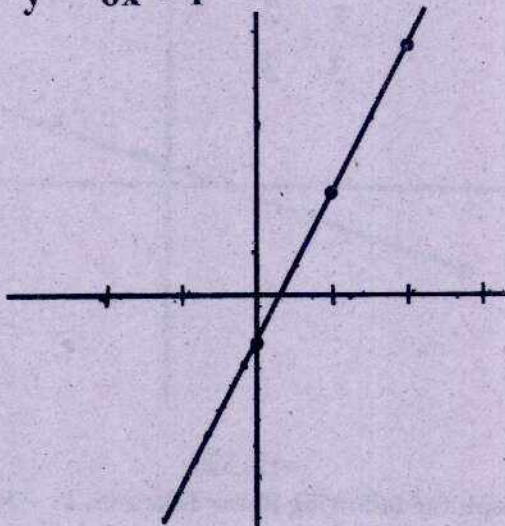
$$y = 6(0) - 2 = 0 - 2 = -2$$

$$y = 6(3) - 2 = 18 - 2 = 16$$

X	-2	-1	0	1	2	3
Y	-14	-8	-2	4	10	16

Graph

$$y = 6x - 1$$



-:5.31:-

Graph the following linear function, $y = \frac{x}{3} + \frac{2}{3}$ **SOLUTION**

$$y = \frac{x}{3} + \frac{2}{3}$$

Putting $x = -11, -5, 1, 4, 7$

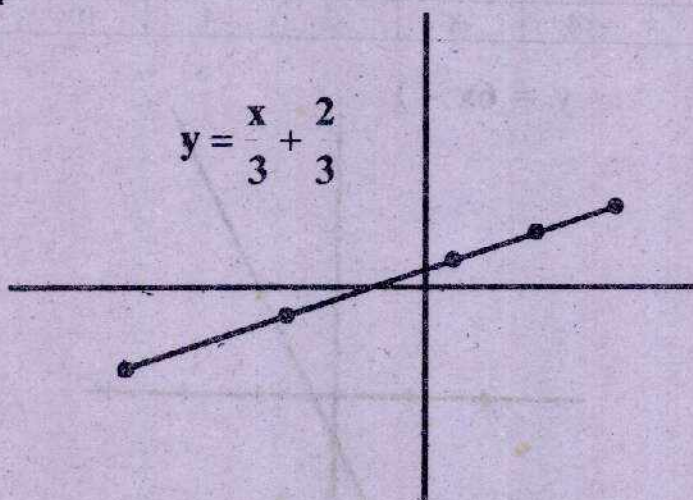
$$y = \frac{-11}{3} + \frac{2}{3} = \frac{-11+2}{3} = \frac{-9}{3} = -3, \quad y = \frac{-5}{3} + \frac{2}{3} = \frac{-5+2}{3} = \frac{-3}{3} = -1$$

$$y = \frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = \frac{3}{3} = 1, \quad y = \frac{4}{3} + \frac{2}{3} = \frac{4+2}{3} = \frac{6}{3} = 2$$

$$y = \frac{7}{3} + \frac{2}{3} = \frac{7+2}{3} = \frac{9}{3} = 3$$

x	-11	-5	1	4	7
y	-3	-1	1	2	3

Graph



-:5.32:-

Graph the following linear function, $2x - 3y = 12$ **SOLUTION**

$$2x - 3y = 12$$

$$-3y = 12 - 2x$$

$$y = \frac{2x - 12}{3}$$

Putting $x = 0, 3, 6, 9, 12$

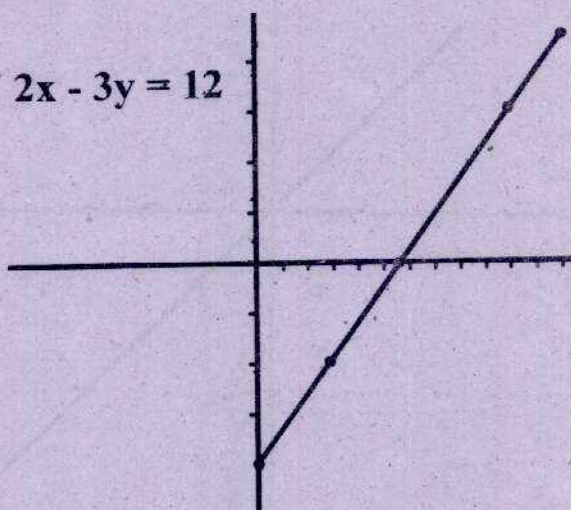
$$y = \frac{2(0) - 12}{3} = \frac{-12}{3} = -4, \quad y = \frac{2(3) - 12}{3} = \frac{6 - 12}{3} = \frac{-6}{3} = -2$$

$$y = \frac{2(6) - 12}{3} = \frac{12 - 12}{3} = 0, \quad y = \frac{2(9) - 12}{3} = \frac{18 - 12}{3} = 2$$

$$y = \frac{2(12) - 12}{3} = \frac{24 - 12}{3} = 4$$

X	0	3	6	9	12
Y	-4	-2	0	2	4

Graph



-:5.33:-

Graph the following linear function, $3x + 2y = 0$ **SOLUTION**

$$3x + 2y = 0$$

$$2y = -3x$$

$$y = \frac{-3x}{2}$$

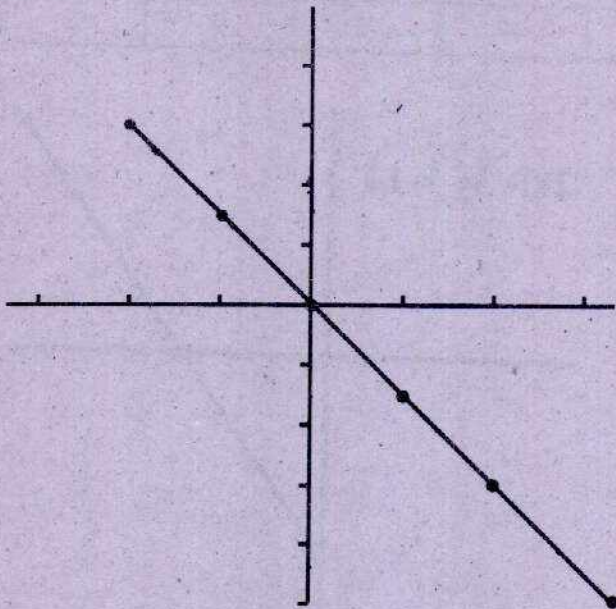
Putting $x = -2, -1, 0, 1, 2, 3$

$$y = \frac{-3(-2)}{2} = \frac{6}{2} = 3, \quad y = \frac{-3(-1)}{2} = \frac{3}{2} = 1.5, \quad y = \frac{-3(0)}{2} = \frac{0}{2} = 0,$$

$$y = \frac{-3(1)}{2} = \frac{-3}{2} = -1.5, \quad y = \frac{-3(2)}{2} = \frac{-6}{2} = -3, \quad y = \frac{-3(3)}{2} = \frac{-9}{2} = -4.5$$

X	-2	-1	0	1	2	3
Y	3	1.5	0	-1.5	-3	-4.5

Graph



-:5.34:-

Graph the following linear function, $6x + 5y = 9$ **SOLUTION**

$$6x + 5y = 9$$

$$5y = 9 - 6x$$

$$y = \frac{9 - 6x}{5}$$

Putting $x = -1, 0, 1, 2, 3, 4$

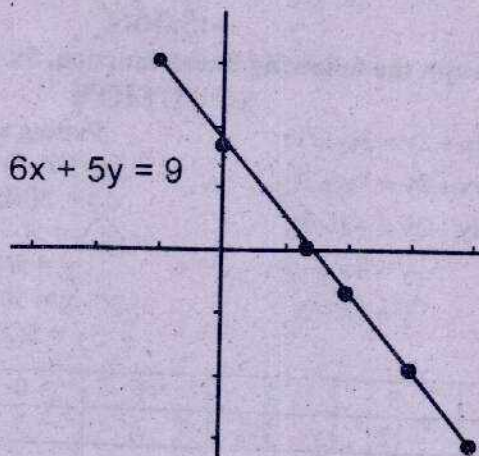
$$y = \frac{9 - 6(-1)}{5} = \frac{9 + 6}{5} = \frac{15}{5} = 3, \quad y = \frac{9 - 6(0)}{5} = \frac{9 - 0}{5} = 1.8$$

$$y = \frac{9 - 6(1)}{5} = \frac{9 - 6}{5} = \frac{3}{5} = 0.6, \quad y = \frac{9 - 6(2)}{5} = \frac{9 - 12}{5} = \frac{-3}{5} = -0.6$$

$$y = \frac{9 - 6(3)}{5} = \frac{9 - 18}{5} = \frac{-9}{5} = -1.8, \quad y = \frac{9 - 6(4)}{5} = \frac{9 - 24}{5} = \frac{-15}{5} = -3$$

X	-1	0	1	2	3	4
Y	3	1.8	0.6	-0.6	-1.8	-3

Graph



-:5.35:-

Graph the following linear function, $\frac{x}{2} + \frac{y}{4} = 1$ **SOLUTION**

$$\frac{x}{2} + \frac{y}{4} = 1$$

$$\frac{2x + y}{4} = 1$$

$$2x + y = 4$$

$$y = 4 - 2(-2) = 4 + 4 = 8$$

$$y = 4 - 2(-1) = 4 + 2 = 6$$

$$y = 4 - 2(0) = 4 + 0 = 4$$

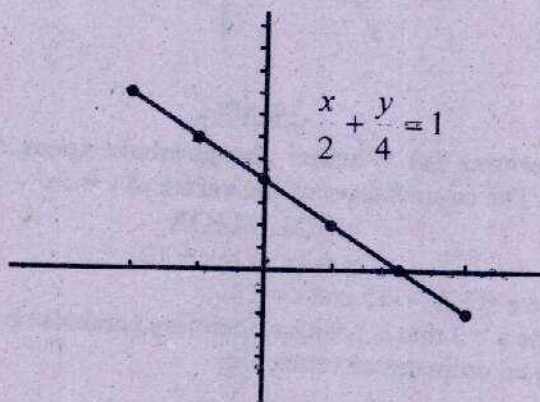
$$y = 4 - 2(1) = 4 - 2 = 2$$

$$y = 4 - 2(2) = 4 - 4 = 0$$

$$y = 4 - 2(3) = 4 - 6 = -2$$

X	-2	-1	0	1	2	3
Y	8	6	4	2	0	-2

Graph



-:5.36:-

Graph the following linear function, $5x + 2y = 3(y - 1)$ **SOLUTION**

$$5x + 2y = 3(y - 1)$$

$$5x + 2y = 3y - 3$$

$$2y - 3y = -3 - 5x$$

$$-y = -5x - 3$$

$$y = 5x + 3$$

$$\text{Putting } x = -3, -2, -1, 0, 1, 2$$

$$y = 5(-3) + 3 = -15 + 3 = -12$$

$$y = 5(-2) + 3 = -10 + 3 = -7$$

$$y = 5(-1) + 3 = -5 + 3 = -2$$

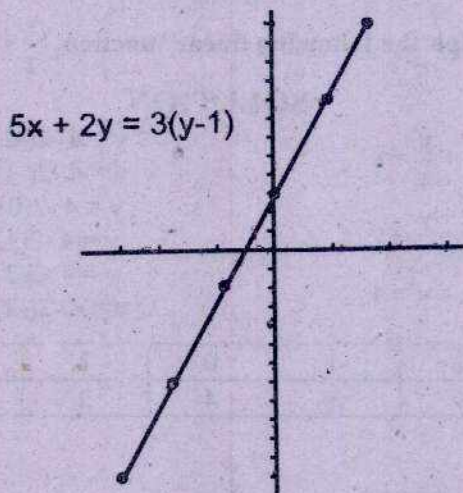
$$y = 5(0) + 3 = 0 + 3 = 3$$

$$y = 5(1) + 3 = 5 + 3 = 8$$

$$y = 5(2) + 3 = 10 + 3 = 13$$

X	-3	-2	-1	0	1	2
Y	-12	-7	-2	3	8	13

Graph



-:5.37:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 3x^2 - 15x + 12$

SOLUTION

$$y = 3x^2 - 15x + 12$$

Here $a = 3$, $b = -15$ and $c = 12$

- (a) Since $a > 0$ that is positive, therefore parabola opens upward
 (b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-15)}{2(3)} = \frac{15}{6} = \frac{5}{2}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(3)(12) - (15)^2}{4(3)}$$

$$= \frac{144 - 225}{12} = -\frac{81}{12} = -\frac{27}{4}$$

Hence Co-ordinate of vertex are $x = 5/2$ and $y = -27/4$ **-.5.38:-**

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 12x^2 - 2x - 4$

SOLUTION

$$y = 12x^2 - 2x - 4$$

Here $a = 12$, $b = -2$ and $c = -4$ (a) $a = 12$ Since $a > 0$ i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(12)} = \frac{2}{24} = \frac{1}{12}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(12)(-4) - (-2)^2}{4(12)}$$

$$= \frac{-192 - 4}{24} = -\frac{196}{24} = -\frac{49}{6}$$

Hence Co-ordinate of vertex are $x = 1/12$ and $y = -49/6$ **-.5.39:-**

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = -2x^2 + 11x - 12$

SOLUTION

$$y = -2x^2 + 11x - 12$$

Here $a = -2$, $b = 11$ and $c = -12$ (a) $a = -2$ Since $a < 0$ i.e. a is negative, therefore parabola opens downward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-11)}{2(-2)} = \frac{-11}{-4} = \frac{11}{4}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(-2)(-12) - (-11)^2}{4(-2)}$$

$$= \frac{96 - 121}{-8} = \frac{-25}{-8} = \frac{25}{8}$$

Hence Co-ordinate of vertex are $x = 11/4$ and $y = 25/8$

-:5.40:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = x^2 - 9x + 18$

SOLUTION

$$y = x^2 - 9x + 18$$

Here $a = 1$, $b = -9$ and $c = 18$

(a) $a = 1$

Since $a > 0$ i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-9)}{2(1)} = \frac{9}{2}$$

and

$$y = \frac{4ac - b^2}{4a} = \frac{4(1)(18) - (-9)^2}{4(1)}$$

$$= \frac{72 - 81}{4} = -\frac{9}{4}$$

Hence Co-ordinate of vertex are $x = 9/2$ and $y = -9/4$

-:5.41:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = x^2 + 2x - 8$

SOLUTION

$$y = x^2 + 2x - 8$$

Here $a = 1$, $b = 2$ and $c = -8$

(a) $a = 1$

Since $a > 0$ i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(2)}{2(1)} = \frac{-2}{2} = -1$$

and

$$\begin{aligned} y &= \frac{4ac - b^2}{4a} = \frac{4(1)(-8) - (2)^2}{4(1)} \\ &= \frac{-32 - 4}{4} = \frac{-36}{4} = -9 \end{aligned}$$

Hence Co-ordinate of vertex are $x = -1$ and $y = -9$

-:5.42:-

Determine (a) Whether the parabola opens downwards or upward (b) The co-ordinates of the vertex of $y = 2x^2 - 12x + 18$

SOLUTION

$$y = 2x^2 - 12x + 18$$

Here $a = 2$, $b = -12$ and $c = 18$

(a) $a = 2$

Since $a > 0$ i.e. a is positive, therefore parabola opens upward

(b) The co-ordinates of vertex are:

$$x = \frac{-b}{2a} = \frac{-(-12)}{2(2)} = \frac{12}{4} = 3$$

and

$$\begin{aligned} y &= \frac{4ac - b^2}{4a} = \frac{4(2)(18) - (-12)^2}{4(2)} \\ &= \frac{144 - 144}{8} = \frac{0}{8} = 0 \end{aligned}$$

Hence Co-ordinate of vertex are $x = 3$ and $y = 0$

-:5.43:-

Find the vertex and graph the following quadratic function:

$$y = 4x^2 - 3x + 1$$

SOLUTION

$$y = 4x^2 - 3x + 1$$

Vertex is

$$\frac{-b}{2a} = \frac{-(-3)}{2(4)} = \frac{3}{8}, \quad f\left(\frac{-b}{2a}\right) = f\left(\frac{3}{8}\right)$$

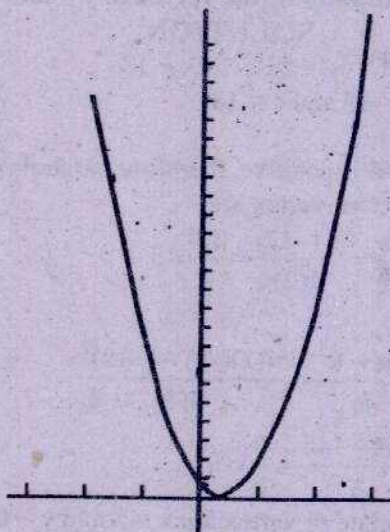
$$= 4\left(\frac{3}{8}\right)^2 - 3\left(\frac{3}{8}\right) + 1 = 4\left(\frac{9}{64}\right) - \frac{9}{8} + 1$$

$$= \frac{9}{16} - \frac{9}{8} + 1 = \frac{9 - 18 + 16}{16} = \frac{7}{16}$$

So the vertex is $\left(\frac{3}{8}, \frac{7}{16}\right)$

x	-2	-1	0	$\frac{3}{8}$	1	2	3
$4x^2$	16	4	0	$\frac{9}{16}$	4	16	36
$-3x$	6	3	0	$-\frac{9}{8}$	-3	-6	-9
$+1$	1	1	1	1	1	1	1
$y = 4x^2 - 3x + 1$	23	8	1	$\frac{7}{16}$	2	11	28

Graph



:-5.44:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - 2x$$

SOLUTION

$$y = x^2 - 2x$$

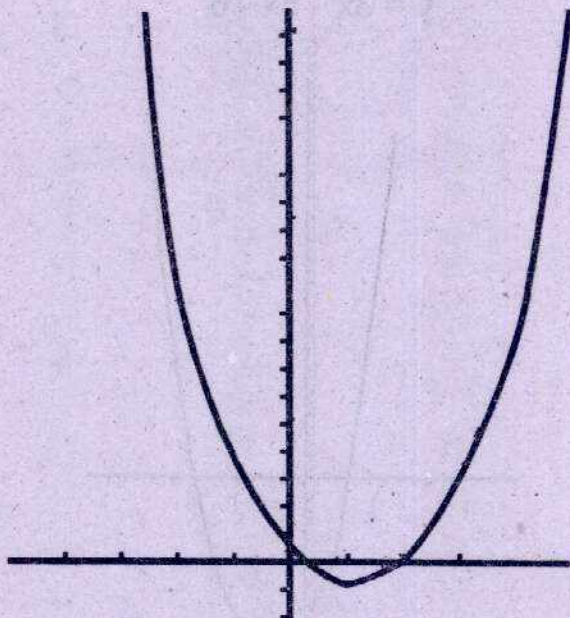
$$\frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1, \quad f\left(\frac{-b}{2a}\right) = f(1)$$

$$= (1)^2 - 2(1) = 1 - 2 = -1$$

Hence vertex is (1, -1)

x	-3	-2	-1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
$-2x$	6	4	2	0	-2	-4	-6	-8	-10
$y = x^2 - 2x$	15	8	3	0	-1	0	3	8	15

Graph



-:5.45:-

Find the vertex and graph the following quadratic function:

$$y = 3x^2 - 5x - 6$$

SOLUTION

$$y = 3x^2 - 5x - 6$$

$$\frac{-b}{2a} = \frac{-(-5)}{2(3)} = \frac{5}{6}, \quad f\left(\frac{-b}{2a}\right) = f\left(\frac{5}{6}\right)$$

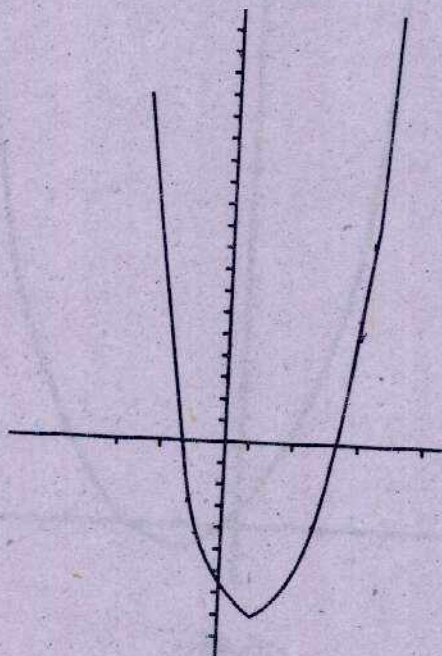
$$= 3\left(\frac{5}{6}\right)^2 - 5\left(\frac{5}{6}\right) - 6 = \frac{25}{12} - \frac{25}{6} - 6$$

$$= \frac{25 - 50 - 72}{12} = -\frac{97}{12} = -8\frac{1}{12}$$

x	-2	-1	0	$\frac{5}{6}$	1	2	3	4
$3x^2$	12	3	0	$\frac{25}{12}$	3	12	27	48
$-5x$	10	5	0	$-\frac{25}{6}$	-5	-10	-15	-20
-6	-6	-6	-6	-6	-6	-6	-6	-6
$y = 3x^2 - 5x - 6$	16	2	-6	$-\frac{97}{12}$	-8	-4	6	22

Graph

$$y = 3x^2 - 5x - 6$$



-:5.46:-

Find the vertex and graph the following quadratic function:

$$y = 4x^2 - 4x + 1$$

SOLUTION

$$y = 4x^2 - 4x + 1$$

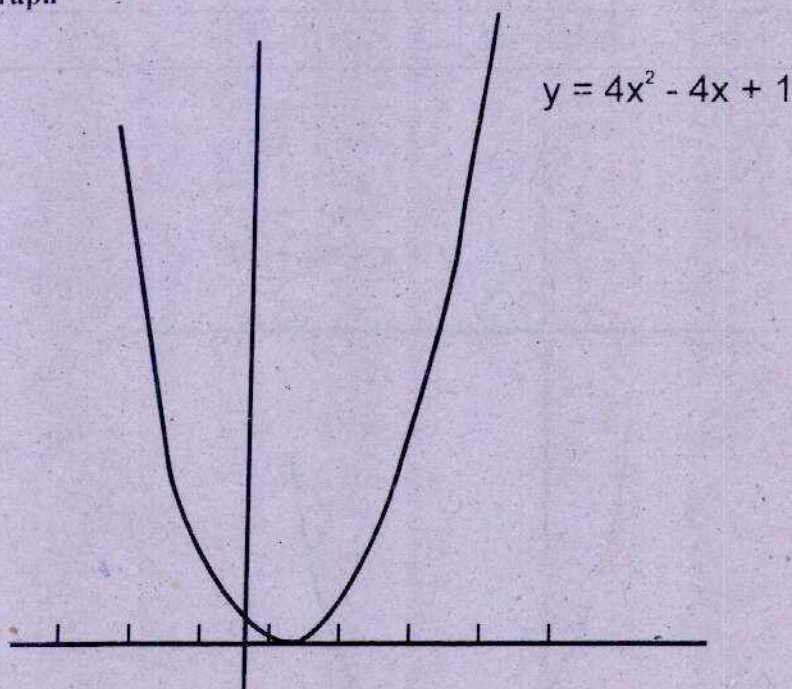
$$\frac{-b}{2a} = \frac{-(-4)}{2(4)} = \frac{4}{8} = \frac{1}{2}$$

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{1}{2}\right) \Rightarrow 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = 1 - 2 + 1 = 0$$

Hence vertex is $(\frac{1}{2}, 0)$

x	-2	-1	0	$\frac{1}{2}$	1	2	3	4
$4x^2$	16	4	0	1	4	16	36	64
$-4x$	8	4	0	-2	-4	-8	-12	-16
$+1$	1	1	1	1	1	1	1	1
$y = 4x^2 - 4x + 1$	25	9	1	0	1	9	25	49

Graph



-:5.47:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - 4x - 17$$

SOLUTION

$$y = x^2 - 4x - 17$$

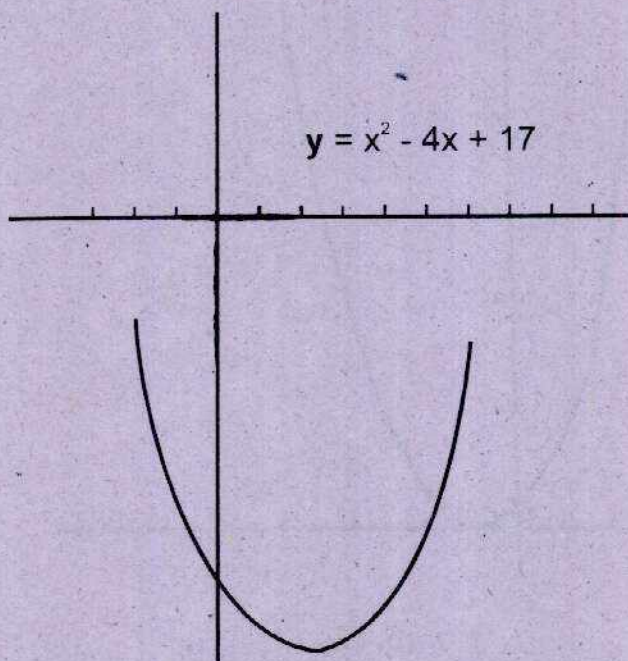
$$\frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$$

$$f\left(\frac{-b}{2a}\right) = f(2) = (2)^2 - 4(2) - 17 = 4 - 8 - 17 = -21$$

Hence vertex is (2, -21)

x	-2	-1	0	1	2	3	4	5	6
x^2	4	1	0	1	4	9	16	25	36
$-4x$	8	4	0	-4	-8	-12	-16	-20	-24
-17	-17	-17	-17	-17	-17	-17	-17	-17	-17
$y = x^2 - 4x - 17$	-5	-12	-17	-20	-21	-20	-17	-12	-5

Graph



:-5.48:-

Find the vertex and graph the following quadratic function:

$$y = x^2 - x + 6$$

SOLUTION

$$y = x^2 - x + 6$$

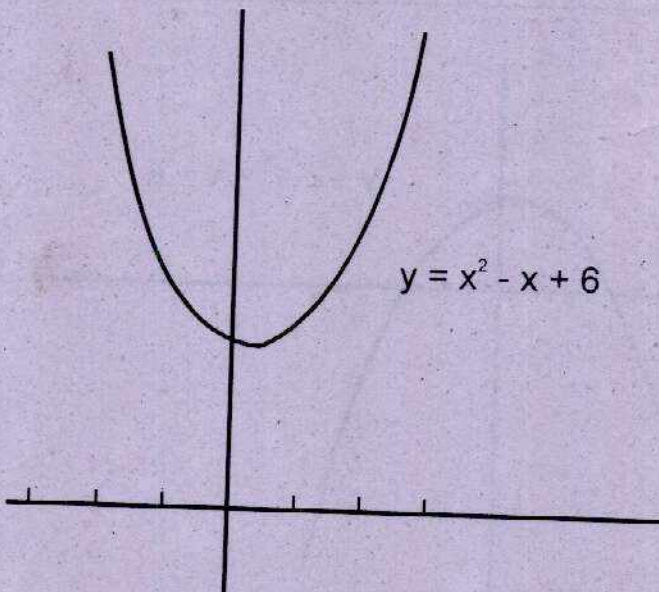
$$\frac{-b}{2a} = \frac{-(-1)}{2(1)} = \frac{1}{2}$$

$$\begin{aligned} f\left(\frac{-b}{2a}\right) &= f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 6 \\ &= \frac{1}{4} - \frac{1}{2} + 6 = \frac{23}{4} = 5\frac{3}{4} \end{aligned}$$

$$\text{Vertex is } \left(\frac{1}{2}, \frac{23}{4}\right)$$

x	-2	-1	0	$\frac{1}{2}$	1	2	3
x^2	4	1	0	$\frac{1}{4}$	1	4	9
-x	2	1	0	$-\frac{1}{2}$	-1	-2	-3
6	6	6	6	6	6	6	6
$y = x^2 - x + 6$	12	8	6	$\frac{23}{4}$	6	8	12

Graph



-:5.49:-

Find the vertex and graph the following quadratic function:

$$y = -2x^2 - x + 4$$

SOLUTION

$$y = -2x^2 - x + 4$$

$$\frac{-b}{2a} = \frac{-(-1)}{2(2)} = \frac{1}{4} = -\frac{1}{4}$$

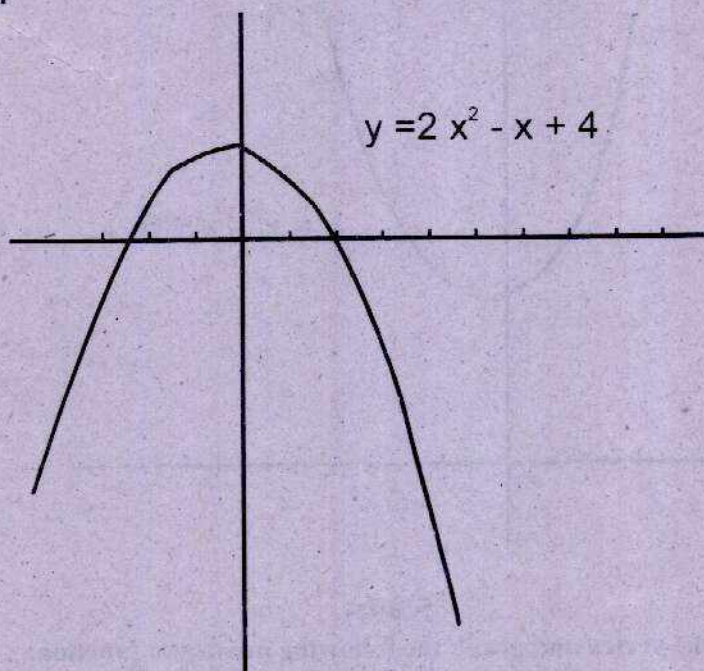
$$f\left(\frac{-b}{2a}\right) = f\left(-\frac{1}{4}\right) = -2\left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right) + 4$$

$$= -\frac{1}{8} + \frac{1}{4} + 4 = \frac{1+2+32}{8} = \frac{33}{8} = 4\frac{1}{8}$$

Hence vertex is $\left(-\frac{1}{4}, \frac{33}{8}\right)$

x	-3	-2	-1	$-\frac{1}{4}$	0	1	2	3
$-2x^2$	-18	-8	-2	$-\frac{1}{8}$	0	-2	-8	-18
$-x$	3	2	1	$\frac{1}{4}$	0	-1	-2	-3
4	4	4	4	4	4	4	4	4
$y = -2x^2 - x + 4$	-11	-2	3	$\frac{33}{8}$	4	1	-6	-17

Graph



-:5.50:-

Represent each of the following functions by a table and by a graph, letting x take on values within the limits indicated.

- a) $y = 2x - 3$ x between 0 and 8
 b) $3y = 6 - x$ x between 1 and 5
 c) $y = x^2 - 4x - 1$ x between -2 and 5
 d) $y = 16 + 10 - x^2$ x between a and b

SOLUTION(a) $y = 2x - 3$ Putting x between 0 and 8

$$y = 2(0) - 3 = 0 - 3 = -3$$

$$y = 2(1) - 3 = 2 - 3 = -1$$

$$y = 2(2) - 3 = 4 - 3 = 1$$

$$y = 2(3) - 3 = 6 - 3 = 3$$

$$y = 2(4) - 3 = 8 - 3 = 5$$

$$y = 2(5) - 3 = 10 - 3 = 7$$

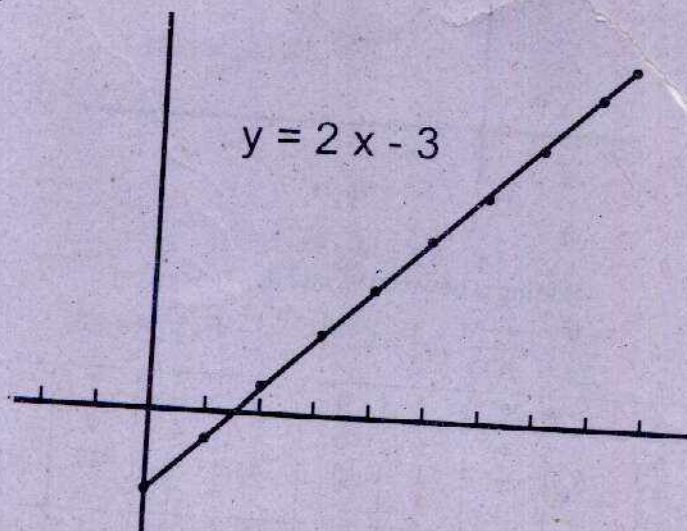
$$y = 2(6) - 3 = 12 - 3 = 9$$

$$y = 2(7) - 3 = 14 - 3 = 11$$

$$y = 2(8) - 3 = 16 - 3 = 13$$

x	0	1	2	3	4	5	6	7	8
y	-3	-1	1	3	5	7	9	11	13

Graph



(b)

$$3y = 6 - x$$

$$y = \frac{6 - x}{3}$$

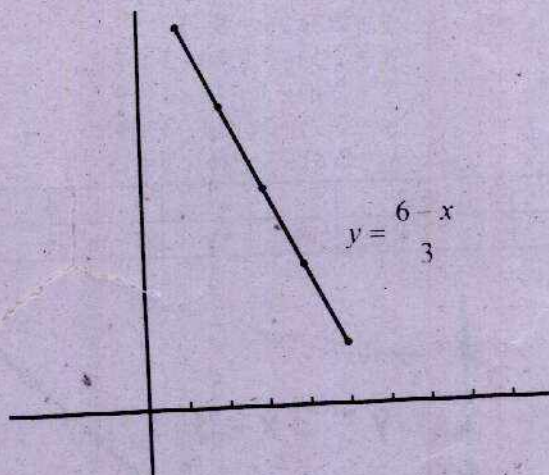
Putting x between 1 and 5

$$y = \frac{6-1}{3} = \frac{5}{3}, \quad y = \frac{6-2}{3} = \frac{4}{3}, \quad y = \frac{6-3}{3} = \frac{3}{3} = 1,$$

$$y = \frac{6-4}{3} = \frac{2}{3}, \quad y = \frac{6-5}{3} = \frac{1}{3}$$

X	1	2	3	4	5
y	$\frac{5}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$	$\frac{1}{3}$

Graph



(c)

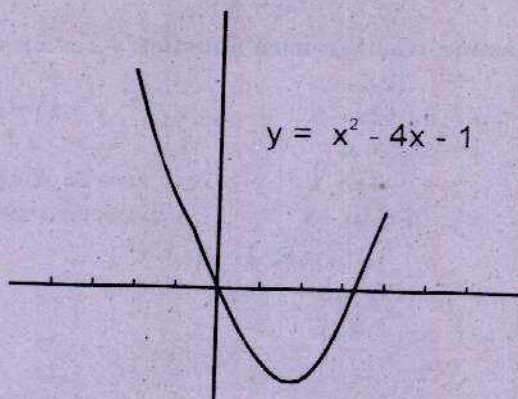
$$y = x^2 - 4x - 1$$

Putting x between -2 and 5

$$\frac{-b}{2a} = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2, \quad f\left(\frac{-b}{2a}\right) = 4 - 8 - 1 = -5$$

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-4x$	8	4	0	-4	-8	-12	-16	-20
-1	-1	-1	-1	-1	-1	-1	-1	-1
$y = x^2 - 4x - 1$	11	4	-1	-4	-5	-4	-1	4

Graph



(d)

$$y = 16 + 10x - x^2$$

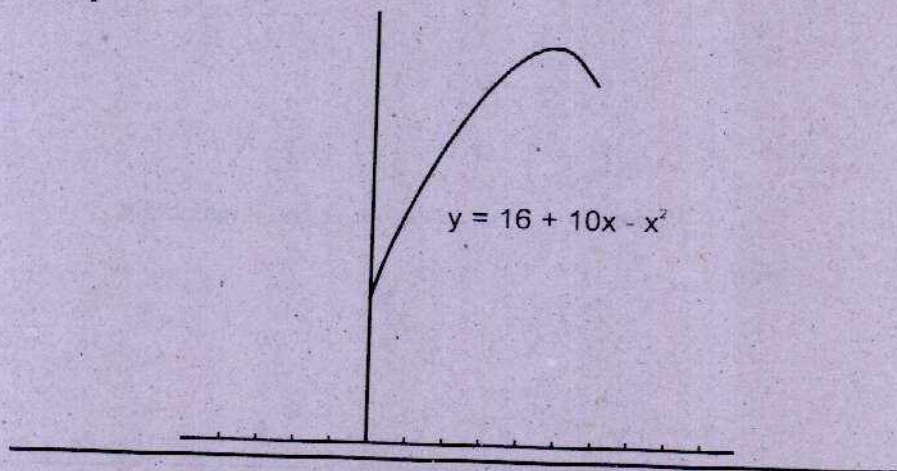
Putting x between -2 and 5

$$\frac{-b}{2a} = \frac{-(-10)}{2(-1)} = -\frac{10}{2} = -5$$

$$f\left(\frac{-b}{2a}\right) = f(-5) = 16 + 10(-5) - (-5)^2 = 16 - 50 - 25 = -59$$

x	0	1	2	3	4	5	6
16	16	16	16	16	16	16	16
$10x$	0	10	20	30	40	50	60
$-x^2$	0	-1	-4	-9	-16	-25	-36
$y = 16 + 10x - x^2$	16	25	32	37	40	41	40

Graph



-:5.51:-

Determine whether each function's vertex is minimum or maximum.

i) $y = 8x + 2x - x^2$

ii) $y = x^2 + 2x$

iii) $y = x^2/2 + x$

iv) $y = -4 + 7x - 4x^2$

c) $y = x^2 - 4x - 1$

x between -2 and 5

d) $y = 16 + 10 - x^2$

x between a and b

SOLUTION

(i) $y = 8 + 2x - x^2$

$$\frac{-b}{2a} = \frac{-(2)}{2(-1)} = \frac{-2}{-2} = 1$$

$$f\left(\frac{-b}{2a}\right) = f(1) = 8 + 2(1) - (1)^2 = 8 + 2 - 1 = 9$$

Vertex is (1, 9), as $a < 0$ vertex is maximum

(ii) $y = x^2 + 2x$

$$\frac{-b}{2a} = \frac{-(2)}{2(1)} = -1$$

$$f\left(\frac{-b}{2a}\right) = f(-1) = 1 + 2(-1)^2 = 1 - 2 = -1$$

Vertex is (-1, -1), as $a > 0$ vertex is maximum

(iii)

$$y = \frac{x^2}{2} + x$$

$$\frac{-b}{2a} = \frac{-1}{2(\frac{1}{2})} = \frac{-1}{1} = -1$$

$$f\left(\frac{-b}{2a}\right) = f(-1) = \frac{1}{2} - 1 = -\frac{1}{2}$$

Vertex is (-1, -1/2), as $a > 0$, vertex is maximum

(iv)

$$y = -4 + 7x - 4x^2$$

$$\frac{-b}{2a} = \frac{-(7)}{2(-4)} = \frac{-7}{-8} = \frac{7}{8}$$

$$f\left(\frac{-b}{2a}\right) = f\left(\frac{7}{8}\right) = -4 + 7\left(\frac{7}{8}\right) - 4\left(\frac{7}{8}\right)^2$$

$$f\left(\frac{-b}{2a}\right) = -4 + \frac{49}{8} - 4 \cdot \frac{49}{64} = -4 + \frac{49}{8} - \frac{49}{16}$$

$$= \frac{-64 + 98 - 49}{16} = -\frac{15}{16}$$

Hence vertex is $\left(\frac{7}{8}, -\frac{15}{16}\right)$, as $a < 0$ vertex is maximum

SET - B

-:5.1:-

The population size y of a certain city at time t is given by

$$y = f(t) = 4t^2 + 2t$$

(a) What is $f(1)$, (b) What is $f(2)$ and (c) What is $f(3)$

SOLUTION

$$y = f(t) = 4t^2 + 2t$$

- (a) $f(1) = 4(1)^2 + 2(1) = 4 + 2 = 6$
 (b) $f(2) = 4(2)^2 + 2(2) = 16 + 4 = 20$
 (c) $f(3) = 4(3)^2 + 2(3) = 36 + 6 = 42$

-:5.2:-

A train traveled 60 miles the first hour and 50 miles each hour thereafter. Find the function which gives the distance covered in t hours. Also find the distance covered in one hour, 2 hour, 3 hours and 4 hours.

SOLUTION

Here total time = t hours

Speed of first hour = 60 miles

Speed of remaining $(t - 1)$ hours = 50 miles each hour

So Function = $f(t) = 50(t - 1) + 60$

- $f(1) = 50(1 - 1) + 60 = 50(0) + 60 = 60$ miles
 $f(2) = 50(2 - 1) + 60 = 50 + 60 = 110$ miles
 $f(3) = 50(3 - 1) + 60 = 100 + 60 = 160$ miles
 $f(4) = 50(4 - 1) + 60 = 150 + 60 = 210$ miles

-:5.3:-

Find the quantity demanded from the following function when price (P) is 5, 10, 15, 20, 25. If $q_d = 5 + 2P$

SOLUTION

Here quantity demand function is

$$q_d = 5 + 2P$$

When $P = 5$

$$q_d = 5 + 2(5) = 5 + 10 = 15$$

When $P = 10$

$$q_d = 5 + 2(10) = 5 + 20 = 25$$

When $P = 15$

$$q_d = 5 + 2(15) = 5 + 30 = 35$$

When $P = 20$

$$q_d = 5 + 2(20) = 5 + 40 = 45$$

When $P = 25$

$$q_d = 5 + 2(25) = 5 + 50 = 55$$

-:5.4:-

Find the quantity demanded from the following function when price (P) is 20, 40, 60, 80, 100. If $q_d = 300 - 3P$

SOLUTION

Here quantity demand function is

$$q_d = 300 - 3P$$

When $P = 20$

$$q_d = 300 - 3(20) = 300 - 60 = 240$$

When $P = 40$

$$q_d = 300 - 3(40) = 300 - 120 = 180$$

When $P = 60$

$$q_d = 300 - 3(60) = 300 - 180 = 120$$

When $P = 80$

$$q_d = 300 - 3(80) = 300 - 240 = 60$$

When $P = 100$

$$q_d = 300 - 3(100) = 300 - 300 = 0$$

-:5.5:-

If the demand function and supply function for a specified per meter cloth are: $P + 2q = 100$ and $45P - 20q = 350$ respectively, compare the quantity demanded and quantity supplied when $P = 14$. Are there surplus cloth or not enough to meet demand.

SOLUTION

Here quantity demand function is

$$P + 2q = 100$$

Price = $P = 14$

The quantity demanded when $P = 14$ is

$$14 - 2q = 100$$

$$2q = 100 - 14$$

$$2q = 86$$

$$q = 43$$

The supply function is $45P - 20q = 350$

The quantity supplied when price = $P = 14$ is

$$45(14) - 20q = 350$$

$$630 - 20q = 350$$

$$-20q = 350 - 630$$

$$-20q = -280$$

$$q = 14$$

Cloth is not enough to meet the demand.

-:5.6:-

A factory determines that the overheads for producing a quantity of a certain item is Rs. 300 and the cost of each item is Rs. 20. Express the total expenses as a function of the number of items produced and compute the expenses for producing 12, 25, 50, 75 and 100 items.

SOLUTION

Let n represent the number of items produced. Then $20n + 300$ represent the total expenses. Let us use E to represent the expenses function, so that we have

$$E(n) = 20n + 300, \text{ where } n \text{ is a whole number}$$

We obtain:

$$E(12) = 20(12) + 300 = 240 + 300 = 540$$

$$E(25) = 20(25) + 300 = 500 + 300 = 800$$

$$E(50) = 20(50) + 300 = 1000 + 300 = 1300$$

$$E(75) = 20(75) + 300 = 1500 + 300 = 1800$$

$$E(100) = 20(100) + 300 = 2000 + 300 = 2300$$

-:5.7:-

Suppose that the cost function for producing certain items is given by $C(n) = 3n + 5$, where n represents the number of items produced. Compute $C(150)$, $C(500)$, $C(750)$ and $C(1500)$

SOLUTION

Cost function is

$$C(n) = 3n + 5$$

When

$$n = 150 \quad C(150) = 3(150) + 5 = 450 + 5 = \text{Rs. } 455$$

$$n = 500 \quad C(500) = 3(500) + 5 = 1500 + 5 = \text{Rs. } 1505$$

$$n = 750 \quad C(750) = 3(750) + 5 = 2250 + 5 = \text{Rs. } 2255$$

$$n = 1500 \quad C(1500) = 3(1500) + 5 = 4500 + 5 = \text{Rs. } 4505$$

-:5.8:-

The profit function for selecting items is given by

$$P(n) = -n^2 + 500n + 61500$$

Compute $P(200)$, $P(230)$, $P(250)$ and $P(260)$.

SOLUTION

Here

$$\begin{aligned} P(n) &= -n^2 + 500n + 61500 \\ P(200) &= -(200)^2 + 500(200) + 61500 \\ &= -40000 + 100000 + 61500 \\ &= -40000 + 161500 = 121500 \\ P(230) &= -(230)^2 + 500(230) + 61500 \\ &= -52900 + 115000 + 61500 \\ &= -52900 + 176500 = 123600 \\ P(250) &= -(250)^2 + 500(250) + 61500 \\ &= -62500 + 125000 + 61500 \\ &= -62500 + 186500 = 124000 \\ P(260) &= -(260)^2 + 500(260) + 61500 \\ &= -67600 + 130000 + 61500 \\ &= -67600 + 191500 = 123900 \end{aligned}$$

-:5.9:-

The height of a projectile fired vertically into the air at an initial velocity of 64 feet per second is a function of the time (t) and is given by $h(t) = 64t - 16t^2$

Compute $h(1)$, $h(2)$, $h(3)$ and $h(4)$

SOLUTION

Here function of height $h(t) = 64t - 16t^2$

When

$$h(1) = 64(1) - 16(1)^2 = 64 - 16 = 48 \text{ feet}$$

$$h(2) = 64(2) - 16(2)^2 = 128 - 16(4) = 128 - 64 = 64 \text{ feet}$$

$$h(3) = 64(3) - 16(3)^2 = 192 - 16(9) = 192 - 144 = 48 \text{ feet}$$

$$h(4) = 64(4) - 16(4)^2 = 256 - 16(16) = 256 - 256 = 0 \text{ feet}$$

-:5.10:-

A car rental agency charges Rs. 500 per day plus Rs. 1.25 a mile. Therefore, the daily charge for renting a car is a function of the number of miles travel (m) and can be expressed as:

$$C(m) = 500 + 1.25m$$

Compute $C(75)$, $C(100)$, $C(150)$ and $C(500)$

SOLUTION

Here $C(m) = 500 + 1.25m$

When $C(75) = 500 + 1.25(75) = 500 + 93.75 = \text{Rs. } 593.75$

$$C(100) = 500 + 1.25(100) = 500 + 125 = \text{Rs. } 625$$

$$C(150) = 500 + 1.25(150) = 500 + 187.50 = \text{Rs. } 687.50$$

$$C(500) = 500 + 1.25(500) = 500 + 625 = \text{Rs. } 1125$$

-:5.11:-

A producer knows that he can sell as many items at Rs. 0.25 each as he can produce in a day. If the cost function is

$$C(x) = \text{Rs. } 0.20x + \text{Rs. } 70$$

Find the break even point.

SOLUTION

The amount received function for this problem would be $R(x) = \text{Rs. } 0.25x$. At break-even point the amount received must equal to the cost. Setting these two quantities equal gives:

$$R(x) = C(x)$$

$$0.25x = \text{Rs. } 0.20x + \text{Rs. } 70$$

$$0.25x - 0.20x = 70 \Rightarrow 0.05x = 70$$

$$x = 1400$$

Substituting this value into $R(x) = 0.25x$

$$R(1400) = 0.25(1400) = 350$$

Hence break-even point is 1400, 350.

-:5.12:-

A firm knows that it can sell as many items at Rs. 1.25 each as it can produce in a day. If the cost function is $C(x) = 0.90x + \text{Rs. } 105$. Find the break-even point.

SOLUTION

The amount received function for this problem is $R(x) = \text{Rs. } 1.25x$. At the break-even point, the amount received must equal to cost

$$R(x) = C(x)$$

$$\text{Rs. } 1.25x = \text{Rs. } 0.90x + \text{Rs. } 105$$

$$1.25x - 0.90x = 105$$

$$0.35x = 105$$

$$x = \frac{105}{0.35} = \frac{10500}{35}$$

$$x = 300$$

Substituting this value into $R(x) = 1.25x$

$$R(300) = 1.25(300) = 375$$

Hence break-even point is 300, 375

-:5.13:-

The cost function for storing a particular item at XYZ corporation was found to be $f(x) = 0.005x + 0.80$ where x is the cost of the item. What is the cost of storing 84 items.

SOLUTION

Here

$$f(x) = 0.005x + 0.80$$

$$f(84) = 0.005(84) + 0.80 = 0.42 + 0.80 = 1.22$$

-:5.14:-

Suppose a calculator has the total cost function $C(x) = 17x + 3400$ and the revenue function $R(x) = 34x$

- What is the equation of the profit function for the calculator?
- What is the profit on 300 units?

SOLUTION

$$(a) \quad P(x) = R(x) - C(x)$$

$$\begin{aligned} \text{Profit Function} &= \text{Revenue Function} - \text{Cost Function} \\ &= 34x - (17x + 3400) \\ &= 34x - 17x - 3400 \\ &= 17x - 3400 \end{aligned}$$

$$(b) \quad \text{Profit on 300 units}$$

$$\begin{aligned} P(300) &= 17(300) - 3400 \\ &= 5100 - 3400 = \text{Rs. } 1700 \end{aligned}$$