

EXERCISE NO. 10**-:10.1:-**

State the order of each of the following matrices:

i)

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 2 \end{bmatrix}$$

ii)

$$B = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 5 & 4 \\ 3 & 6 & 3 \end{bmatrix}$$

iii)

$$C = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 6 & 7 & 8 \\ 3 & 5 & 2 & 1 \end{bmatrix}$$

iv)

$$D = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 3 & 3 \\ 6 & 5 & 4 \\ 8 & 7 & 5 \end{bmatrix}$$

SOLUTION

- i) The order of the matrix is 2×3
- ii) The order of the matrix is 3×3
- iii) The order of the matrix is 3×4
- iv) The order of the matrix is 4×3

-:10.2:-

Write the general 2×2 matrix, using double subscript notation.

SOLUTION

The general 2×2 matrix is given below:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2 \times 2}$$

-:10.3:-

Write the general 4×4 matrix using double subscript notation.

SOLUTION

The general 4×4 matrix is given below:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}_{4 \times 4}$$

-:10.4:-

The dimension of A is 3×4 , the dimension of B is 4×1 , the dimension of C is 1×3 , the dimension of D is 4×3 and the dimension of E is 3×3 . Determine the dimension of

- (a) AB (b) BA (c) CA (d) AD
 (e) DA (f) BCE (g) CEA (h) ABCD
 (i) DABCE

SOLUTION

- a) $AB = (3 \times 4)(4 \times 1) = 3 \times 1$
 b) $BA = (4 \times 1)(3 \times 4) = \text{Does not exist}$
 c) $CA = (1 \times 3)(3 \times 4) = 1 \times 4$
 d) $AD = (3 \times 4)(4 \times 3) = 3 \times 3$
 e) $DA = (4 \times 3)(3 \times 4) = 4 \times 4$
 f) $BCE = (4 \times 1)(1 \times 3)(3 \times 3) = 4 \times 3$
 g) $CEA = (4 \times 1)(1 \times 3)(3 \times 3) = 4 \times 3$
 h) $ABCD = (3 \times 4)(4 \times 1)(1 \times 3)(4 \times 3) = \text{Does not exist}$
 i) $DABCE = (4 \times 3)(3 \times 4)(4 \times 1)(1 \times 3)(3 \times 3) = 4 \times 3$

-:10.5:-

If

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix}$$

Find $A+B$, $B+A$, $A-B$ and $B-A$

SOLUTION

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+6 & 4+5 \\ 3+2 & 2+6 \\ 2+1 & 5+1 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 5 & 8 \\ 3 & 6 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6+1 & 5+4 \\ 2+3 & 6+2 \\ 1+2 & 1+5 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 5 & 8 \\ 3 & 6 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1-6 & 4-5 \\ 3-2 & 2-6 \\ 2-1 & 5-1 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 1 & -4 \\ 1 & 4 \end{bmatrix}$$

$$B - A = \begin{bmatrix} 6 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 6-1 & 5-4 \\ 2-3 & 6-2 \\ 1-2 & 1-5 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -1 & 4 \\ -1 & -4 \end{bmatrix}$$

∴ 10.6:-

$$\text{If } A = \begin{bmatrix} x-2y & y & x+z \\ x & x-y & 4x+y \\ 4x+3y & -x+y & 3y \end{bmatrix}, B = \begin{bmatrix} x+2y & x-y & x-z \\ -x+3y & x+y & 3x-y \\ -4x & x-y & 2x-3y \end{bmatrix}$$

Then find $A + B$ and $A - B$

SOLUTION

$$\begin{aligned} A + B &= \begin{bmatrix} x-2y & y & x+z \\ x & x-y & 4x+y \\ 4x+3y & -x+y & 3y \end{bmatrix} + \begin{bmatrix} x+2y & x-y & x-z \\ -x+3y & x+y & 3x-y \\ -4x & x-y & 2x-3y \end{bmatrix} \\ &= \begin{bmatrix} x-2y+x+2y & y+x-y & x+z+x-z \\ x-x+3y & x-y+x+y & 4x+y+3x-y \\ 4x+3y-4x & -x+y+x-y & 3y+2x-3y \end{bmatrix} \\ &= \begin{bmatrix} 2x & x & 2x \\ 3y & 2x & 7x \\ 3y & 0 & 2x \end{bmatrix} \end{aligned}$$

Now

$$A - B = \begin{bmatrix} x-2y & y & x+z \\ x & x-y & 4x+y \\ 4x+3y & -x+y & 3y \end{bmatrix} - \begin{bmatrix} x+2y & x-y & x-z \\ -x+3y & x+y & 3x-y \\ -4x & x-y & 2x-3y \end{bmatrix}$$

$$\begin{aligned}
 A - B &= \begin{bmatrix} x-2y-x-2y & y-x+y & x+z-x+z \\ x+x-3y & x-y-x-y & 4x+y-3x+y \\ 4x+3y+4x & x+y-x+y & 3y-2x+3y \end{bmatrix} \\
 &= \begin{bmatrix} -4x & -x+2y & 2z \\ 2x-3y & -2y & x+2y \\ 8x+3y & -2x+2y & -2x+6y \end{bmatrix}
 \end{aligned}$$

∴10.7:-

$$\text{If } A = \begin{bmatrix} a & a+b \\ b-c & -c \\ a+c & b+c \end{bmatrix}, B = \begin{bmatrix} -a+b & -b \\ c & b+c \\ -a & -c \end{bmatrix}$$

Then find $A + B$ and $A - B$

SOLUTION

$$\begin{aligned}
 A + B &= \begin{bmatrix} a & a+b \\ b-c & -c \\ a+c & b+c \end{bmatrix} + \begin{bmatrix} -a+b & -b \\ c & b+c \\ -a & -c \end{bmatrix} \\
 &= \begin{bmatrix} a-a+b & a+b-b \\ b-c+c & -c+b+c \\ a+c-a & b+c-c \end{bmatrix} = \begin{bmatrix} b & a \\ b & b \\ c & b \end{bmatrix}
 \end{aligned}$$

Now

$$\begin{aligned}
 A - B &= \begin{bmatrix} a & a+b \\ b-c & -c \\ a+c & b+c \end{bmatrix} - \begin{bmatrix} -a+b & -b \\ c & b+c \\ -a & -c \end{bmatrix} \\
 &= \begin{bmatrix} a+a-b & a+b+b \\ b-c-c & -c-b-c \\ a+c+a & b+c+c \end{bmatrix} = \begin{bmatrix} 2a-b & a+2b \\ b-2c & -b-2c \\ 2a+c & b+2c \end{bmatrix}
 \end{aligned}$$

∴10.8:-

Find A if $2A + 3B = C$ where

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 4 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 3 & -1 \\ 2 & 0 \\ -1 & -1 \end{bmatrix}$$

Hint: Since $2A + 3B = C$, it follows that $2A = C - 2B$ and

$$A = \frac{1}{2}(C - 3B)$$

SOLUTION

We know that $2A + 3B = C$

$$2A = C - 3B \text{ and } A = \frac{1}{2}(C - 3B)$$

$$3B = 3 \begin{bmatrix} 1 & 2 \\ 0 & 4 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 0 & 12 \\ 12 & 0 \end{bmatrix}$$

$$(C - 3B) = \begin{bmatrix} 3 & -1 \\ 2 & 0 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & 12 \\ 12 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3 & -1-6 \\ 2-0 & 0-12 \\ -1-12 & -1-0 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 2 & -12 \\ -13 & -1 \end{bmatrix}$$

So

$$A = \frac{1}{2}(C - 3B) = \frac{1}{2} \begin{bmatrix} 0 & -7 \\ 2 & -12 \\ -13 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{7}{2} \\ 1 & -6 \\ -\frac{13}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & -3.5 \\ 1 & -6 \\ -6.5 & -0.5 \end{bmatrix}$$

-:10.9:-

Find B if $-A + 2B = 6C$, where

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -1 & 7 \\ 3 & 0 & 0 \\ 4 & 1 & -5 \end{bmatrix}$$

Hint: Since $-A + 2B = 6C$, it follows that $2B = A + 6C$ and

$$B = \frac{1}{2}(A + 3C)$$

SOLUTION

Since $-A + 2B = 6C$, it follows $2B = A + 6C$ and $B = \frac{1}{2}(A + 6C)$

$$6C = 6 \begin{bmatrix} 2 & -1 & 7 \\ 3 & 0 & 0 \\ 4 & 1 & -5 \end{bmatrix} = \begin{bmatrix} 12 & -6 & 42 \\ 18 & 0 & 0 \\ 24 & 6 & -30 \end{bmatrix}$$

$$A + 6C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 12 & -6 & 42 \\ 18 & 0 & 0 \\ 24 & 6 & -30 \end{bmatrix} = \begin{bmatrix} 13 & -4 & 41 \\ 21 & 0 & 1 \\ 25 & 7 & -29 \end{bmatrix}$$

$$B = \frac{1}{2}(A + 6C) = \frac{1}{2} \begin{bmatrix} 13 & -4 & 41 \\ 21 & 0 & 1 \\ 25 & 7 & -29 \end{bmatrix} = \begin{bmatrix} \frac{13}{2} & -2 & \frac{41}{2} \\ \frac{21}{2} & 0 & \frac{1}{2} \\ \frac{25}{2} & \frac{7}{2} & -\frac{29}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 7.5 & -1 & 20.5 \\ 10.5 & 0 & 0.5 \\ 12.5 & 3.5 & -14.5 \end{bmatrix}$$

:-10.10:-

Write the transpose of the following matrices.

$$(i) A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 4 & 2 \\ 6 & 0 & 7 \end{bmatrix} \quad (ii) A = \begin{bmatrix} 3 & 2 \\ 4 & 6 \\ 7 & 2 \end{bmatrix}$$

SOLUTION

$$(i) A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 4 & 2 \\ 6 & 0 & 7 \end{bmatrix}$$

Interchange the rows and columns

$$A^t = \begin{bmatrix} 3 & 1 & 6 \\ 2 & 4 & 0 \\ 6 & 2 & 7 \end{bmatrix}$$

$$(ii) \quad A = \begin{bmatrix} 3 & 2 \\ 4 & 6 \\ 7 & 2 \end{bmatrix}$$

Interchange the rows and columns

$$A^t = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 6 & 2 \end{bmatrix}$$

-:10.11:-

If A is matrix, what is $[A^t]^t$

SOLUTION

If A is the matrix then its transpose is A^t . The transpose of A^t is again the original matrix i.e. $[A^t]^t = A$

-:10.12:-

Find x, y, z and w from the following matrices.

$$a) \quad \begin{bmatrix} x & 1 & 0 \\ 0 & y & z \\ w & 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix} \quad b) \quad \begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & w \end{bmatrix}$$

SOLUTION

(a)

$$\begin{bmatrix} x & 1 & 0 \\ 0 & y & z \\ w & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$

The two matrices are equal. In the equality of matrices the corresponding elements are same. Hence from the above matrices we find $x=3$, $y=2$, $z=3$ and $w=4$

(b)

$$\begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & w \end{bmatrix}$$

Here $x=4$, $y=1$, $z=1$ and $w=2$

-:10.13:-

Solve for x, y and z if

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

SOLUTION

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

We first add the two matrices of left hand side

$$\begin{bmatrix} x & y \\ y & z \end{bmatrix} + \begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 2+2x & y-y \\ y+3y & z-4z \end{bmatrix} = \begin{bmatrix} 3x & 0 \\ 4y & -3z \end{bmatrix}$$

Hence

$$\begin{bmatrix} 3x & 0 \\ 4y & -3z \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 8 & 9 \end{bmatrix}$$

From the two equal matrices, we have

$$\begin{array}{rcl} 3x = 6 & 4y = 8 & -3z = 9 \\ x = 2 & y = 2 & z = -3 \end{array}$$

-:10.14 :-

If

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

Find the $3A+4B=2C$ **SOLUTION**

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix}$$

$$3A + 4B - 2C$$

$$3A = 3 \begin{bmatrix} 2 & -5 & 1 \\ 3 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 6 & -15 & 3 \\ 9 & 0 & -12 \end{bmatrix}$$

$$4B = 4 \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -8 & -12 \\ 0 & -4 & 20 \end{bmatrix}$$

$$2C = 2 \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 & -4 \\ 2 & -2 & -2 \end{bmatrix}$$

$$\begin{aligned}
 3A + 4B &= \begin{bmatrix} 6 & -15 & 3 \\ 9 & 0 & -12 \end{bmatrix} + \begin{bmatrix} 4 & -8 & -12 \\ 0 & -4 & 20 \end{bmatrix} \\
 &= \begin{bmatrix} 6+4 & -15-8 & 3-12 \\ 9+0 & 0-4 & -12+20 \end{bmatrix} = \begin{bmatrix} 10 & -23 & -9 \\ 9 & -4 & 8 \end{bmatrix} \\
 3A + 4B - 2C &= \begin{bmatrix} 10 & -23 & -9 \\ 9 & -4 & 8 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 2 & -2 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 10+0 & -23-2 & -9-4 \\ 9-2 & -4+2 & 8+2 \end{bmatrix} = \begin{bmatrix} 10 & -25 & -13 \\ 7 & -2 & 10 \end{bmatrix}
 \end{aligned}$$

-:10.15:-

Find XZ , ZX and XY , if possible for

$$X = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, Y = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, Z = \begin{bmatrix} 2 & 1 & 4 \end{bmatrix}$$

SOLUTION

$$XZ = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2(2) & 2(1) & 2(4) \\ 1(2) & 1(1) & 1(4) \\ 3(2) & 3(1) & 3(4) \end{bmatrix} = \begin{bmatrix} 4 & 2 & 8 \\ 2 & 1 & 4 \\ 6 & 3 & 12 \end{bmatrix}$$

$$ZX = \begin{bmatrix} 2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2(2) + 1(1) + 4(3) \end{bmatrix} = \begin{bmatrix} 4 + 1 + 12 \end{bmatrix} = 17$$

$$XY = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \text{Does not exist}$$

-:10.16:-

Verify that $AB = AC$, but that $B \neq C$, for

$$A = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 5 & 6 \\ -1 & -3 \end{bmatrix}$$

SOLUTION

$$\begin{aligned}
 AB &= \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} 2(1) + 4(1) & 2(2) + 4(-1) \\ 1(1) + 2(1) & 1(2) + 2(-1) \end{bmatrix} = \begin{bmatrix} 2+4 & 4-4 \\ 1+2 & 2-2 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 3 & 0 \end{bmatrix}
 \end{aligned}$$

and

$$\begin{aligned}
 AC &= \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ -1 & -3 \end{bmatrix} \\
 &= \begin{bmatrix} 2(5) + 4(-1) & 2(6) + 4(-3) \\ 1(5) + 2(-1) & 1(6) + 2(-3) \end{bmatrix} = \begin{bmatrix} 10-4 & 12-12 \\ 5-2 & 6-6 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 3 & 0 \end{bmatrix}
 \end{aligned}$$

Hence $AB = AC$ where $B \neq C$

-:10.17:-

If

$$A = \begin{bmatrix} 3 & 1 & 6 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 1 & 3 \\ 5 & 2 & 1 \end{bmatrix}$$

Find AB and BA .**SOLUTION**

$$\begin{aligned}
 AB &= \begin{bmatrix} 3 & 1 & 6 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 & 1 \\ 2 & 1 & 3 \\ 5 & 2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 3(4) + 1(2) + 6(5) & 3(1) + 1(1) + 6(2) & 3(1) + 1(3) + 6(1) \\ 2(4) + 1(2) + 0(5) & 2(1) + 1(1) + 0(2) & 2(1) + 1(3) + 0(1) \\ 1(4) + 2(2) + 3(5) & 1(1) + 2(1) + 3(2) & 1(1) + 2(3) + 3(1) \end{bmatrix} \\
 &= \begin{bmatrix} 12+2+30 & 3+1+12 & 3+3+6 \\ 8+2+0 & 2+1+0 & 2+3+0 \\ 4+4+15 & 1+2+6 & 1+6+3 \end{bmatrix} = \begin{bmatrix} 44 & 16 & 12 \\ 10 & 3 & 5 \\ 23 & 9 & 10 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 BA &= \begin{bmatrix} 4 & 1 & 1 \\ 2 & 1 & 3 \\ 5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 6 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 4(3)+1(2)+1(1) & 4(1)+1(1)+1(2) & 4(6)+1(0)+1(3) \\ 2(3)+1(2)+3(1) & 2(1)+1(1)+3(2) & 2(6)+1(0)+3(3) \\ 5(3)+2(2)+1(1) & 5(1)+2(1)+1(2) & 5(6)+2(0)+1(3) \end{bmatrix} \\
 &= \begin{bmatrix} 12+2+1 & 4+1+2 & 24+0+3 \\ 6+2+3 & 2+1+6 & 12+0+9 \\ 15+4+1 & 5+2+2 & 30+0+3 \end{bmatrix} = \begin{bmatrix} 15 & 7 & 27 \\ 11 & 9 & 21 \\ 20 & 9 & 33 \end{bmatrix}
 \end{aligned}$$

∴10.18:-

If

$$A = \begin{bmatrix} 3 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 & 6 & 8 \\ 2 & 1 & 7 & 1 \\ 3 & 2 & 2 & 2 \end{bmatrix}$$

Find AB. Does the product BA exist?

SOLUTION

$$\begin{aligned}
 AB &= \begin{bmatrix} 3 & 1 & 3 \\ 2 & 1 & 6 \end{bmatrix} \begin{bmatrix} 4 & 1 & 6 & 8 \\ 2 & 1 & 7 & 1 \\ 3 & 2 & 2 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 3(4)+1(2)+3(3) & 3(1)+1(1)+3(2) & 3(6)+1(7)+3(2) & 3(8)+1(1)+3(2) \\ 2(4)+1(2)+6(3) & 2(1)+1(1)+6(2) & 2(6)+1(7)+6(2) & 2(8)+1(1)+6(2) \end{bmatrix} \\
 &= \begin{bmatrix} 12+2+9 & 3+1+6 & 18+7+6 & 14+1+6 \\ 8+2+18 & 2+1+12 & 12+7+12 & 16+1+12 \end{bmatrix} = \begin{bmatrix} 23 & 10 & 31 & 31 \\ 28 & 15 & 31 & 29 \end{bmatrix}
 \end{aligned}$$

Multiplication of two matrices is only be possible if the number of columns of first matrix is equal to the number of rows in the second matrix. Here the number of columns of B and number of rows of A are not equal. The product BA does not exist.

∴10.19:-

$$\text{If } A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ 6 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 3 \\ 2 & 1 \end{bmatrix}$$

Show that $A(B+C) = AB+AC$

SOLUTION

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ 6 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 3 \\ 2 & 1 \end{bmatrix}$$

$$B+C = \begin{bmatrix} 2 & 1 \\ 6 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2+3 & 1+3 \\ 6+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 8 & 3 \end{bmatrix}$$

$$\begin{aligned} A(B+C) &= \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} 3(5)+1(8) & 3(4)+1(3) \\ 2(5)+2(8) & 2(4)+2(3) \end{bmatrix} \\ &= \begin{bmatrix} 15+8 & 12+3 \\ 10+16 & 8+6 \end{bmatrix} = \begin{bmatrix} 23 & 15 \\ 26 & 14 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 3(2)+1(6) & 3(1)+1(2) \\ 2(2)+2(6) & 2(1)+2(2) \end{bmatrix} \\ &= \begin{bmatrix} 6+6 & 3+2 \\ 4+12 & 2+4 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ 16 & 6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} AC &= \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 9+1 \\ 6+4 & 6+2 \end{bmatrix} \\ &= \begin{bmatrix} 11 & 10 \\ 10 & 8 \end{bmatrix} \end{aligned}$$

$$AB+AC = \begin{bmatrix} 12 & 5 \\ 16 & 6 \end{bmatrix} + \begin{bmatrix} 11 & 10 \\ 10 & 8 \end{bmatrix} = \begin{bmatrix} 12+11 & 5+10 \\ 16+10 & 6+8 \end{bmatrix} = \begin{bmatrix} 23 & 15 \\ 26 & 14 \end{bmatrix}$$

Hence $A(B+C) = AB+AC$

-:10.20:-

If

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$$

Then find BA

SOLUTION

$$A = \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\begin{aligned}
 BA &= \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 4(3) + (-1)(2) & 4(1) + (-1)(0) \\ 2(3) + 3(2) & 3(1) + 3(0) \end{bmatrix} \\
 &= \begin{bmatrix} 12-3 & 4+0 \\ 6+6 & 2+0 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 12 & 2 \end{bmatrix}
 \end{aligned}$$

:-10.21:-

If

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} a & 2 \\ 7 & b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

Then find a and b

SOLUTION

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} a & -2 \\ 7 & b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

$$\begin{aligned}
 &\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} a & -2 \\ 7 & b \end{bmatrix} \\
 &= \begin{bmatrix} 2(a) + 3(7) & 2(-2) + 3(b) \\ 4(a) + 5(7) & 4(-2) + 5(b) \end{bmatrix} = \begin{bmatrix} 2a + 21 & 4 + 3b \\ 4a + 35 & 8 + 5b \end{bmatrix}
 \end{aligned}$$

We know that

$$= \begin{bmatrix} 2(a) + 21 & 4 + 3b \\ 4a + 35 & 8 + 5b \end{bmatrix} = \begin{bmatrix} 31 & 1 \\ 55 & 3 \end{bmatrix}$$

We can put the corresponding values as

$$2a + 21 = 31$$

or

$$4a + 35 = 55$$

$$2a = 31 - 21 = 10$$

$$4a = 55 - 35 = 20$$

$$a = \frac{10}{2} = 5$$

$$a = \frac{20}{4} = 5$$

Similarly

$$4 + 3b = 1$$

or

$$8 + 5b = 3$$

$$3b = 1 - 4 = -3$$

$$5b = 3 - 8 = -5$$

$$b = -1$$

$$b = -1$$

Hence $a = 5$, $b = -1$

-:10.22:-

$$\text{If } A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 8 \\ 1 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 4 & 8 \\ 6 & 1 & 4 \end{bmatrix}$$

Then find (i) $A+B$ (ii) $A-B$ (iii) AB **SOLUTION**

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 8 \\ 1 & 1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 & 2 \\ 0 & 4 & 8 \\ 6 & 1 & 4 \end{bmatrix}$$

(i)

$$A+B = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 8 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 2 \\ 0 & 4 & 8 \\ 6 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1+2 & 4+1 & 3+2 \\ 2+0 & 1+4 & 8+8 \\ 1+6 & 1+1 & 2+4 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 5 \\ 2 & 5 & 16 \\ 7 & 2 & 6 \end{bmatrix}$$

(ii)

$$\begin{aligned} A-B &= \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 8 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 2 \\ 0 & 4 & 8 \\ 6 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 8 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & -1 & -2 \\ 0 & -4 & -8 \\ -6 & -1 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 1-2 & 4-1 & 3-2 \\ 2+0 & 1-4 & 8-8 \\ 1-6 & 1-1 & 2-4 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 1 \\ 2 & -3 & 0 \\ -5 & 0 & -2 \end{bmatrix} \end{aligned}$$

(iii)

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 8 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 0 & 4 & 8 \\ 6 & 1 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1(2)+4(0)+3(6) & 1(1)+4(4)+3(1) & 1(2)+4(8)+3(4) \\ 2(2)+1(0)+8(6) & 2(1)+1(4)+8(1) & 2(2)+1(8)+9(4) \\ 1(2)+1(0)+2(6) & 1(1)+1(4)+2(1) & 1(2)+1(8)+2(4) \end{bmatrix} \\ &= \begin{bmatrix} 2+0+18 & 1+16+3 & 2+32+12 \\ 4+0+48 & 2+4+8 & 4+8+32 \\ 2+0+12 & 1+4+12 & 2+8+8 \end{bmatrix} = \begin{bmatrix} 20 & 20 & 46 \\ 52 & 14 & 44 \\ 14 & 17 & 18 \end{bmatrix} \end{aligned}$$

-:10.23:-

If

$$A = \begin{bmatrix} 5 & 4 & 3 \\ 6 & 3 & 1 \\ 8 & 9 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 5 \\ 3 & 1 & 6 \end{bmatrix}$$

Then find (i) $2A+4B$ (ii) AB **SOLUTION**(i) $2A + 4B$

$$2A = 2 \begin{bmatrix} 5 & 4 & 3 \\ 6 & 3 & 1 \\ 8 & 9 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 8 & 6 \\ 12 & 6 & 2 \\ 16 & 18 & 4 \end{bmatrix}$$

$$4B = 4 \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 5 \\ 3 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 12 & 16 \\ 8 & 16 & 20 \\ 12 & 4 & 24 \end{bmatrix}$$

$$2A + 4B = \begin{bmatrix} 10 & 8 & 6 \\ 12 & 6 & 2 \\ 16 & 18 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 12 & 16 \\ 8 & 16 & 20 \\ 12 & 4 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 10+4 & 8+12 & 6+16 \\ 12+8 & 6+16 & 2+20 \\ 16+12 & 18+4 & 4+24 \end{bmatrix} = \begin{bmatrix} 14 & 20 & 22 \\ 20 & 22 & 22 \\ 28 & 22 & 28 \end{bmatrix}$$

(ii) AB

$$AB = \begin{bmatrix} 5 & 4 & 3 \\ 6 & 3 & 1 \\ 8 & 9 & 2 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 2 & 4 & 5 \\ 3 & 1 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5(1)+4(2)+3(3) & 5(3)+4(4)+3(1) & 5(4)+4(5)+3(6) \\ 6(1)+3(2)+1(3) & 6(3)+3(4)+1(1) & 6(4)+3(5)+1(6) \\ 8(1)+9(2)+2(3) & 8(3)+9(4)+2(1) & 8(4)+9(5)+2(6) \end{bmatrix}$$

$$= \begin{bmatrix} 5+8+9 & 15+16+3 & 20+20+18 \\ 6+6+3 & 18+12+1 & 24+15+6 \\ 8+18+6 & 24+36+2 & 32+45+12 \end{bmatrix} = \begin{bmatrix} 22 & 34 & 58 \\ 15 & 31 & 45 \\ 32 & 62 & 89 \end{bmatrix}$$

-:10.24:-

Find (a) AB (b) BA, given

$$A = \begin{bmatrix} 7 & 7 \\ 6 & 2 \\ 1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 9 & 1 \\ 2 & 12 & 7 \end{bmatrix}$$

SOLUTION

(a) AB

$$AB = \begin{bmatrix} 7 & 7 \\ 6 & 2 \\ 1 & 8 \end{bmatrix} \begin{bmatrix} -3 & 9 & 1 \\ 2 & 12 & 7 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 7 & 7 \\ 6 & 2 \\ 1 & 8 \end{bmatrix} \begin{bmatrix} -3 & 9 & 1 \\ 2 & 12 & 7 \end{bmatrix} = \begin{bmatrix} 7(-3) + 7(2) & 7(9) + 7(12) & 7(1) + 7(7) \\ 6(-3) + 2(2) & 6(9) + 2(12) & 6(1) + 2(7) \\ 1(-3) + 8(2) & 1(9) + 8(12) & 1(1) + 8(7) \end{bmatrix} \\ &= \begin{bmatrix} -21 + 14 & 63 + 84 & 7 + 49 \\ -18 + 4 & 54 + 24 & 6 + 14 \\ -3 + 16 & 9 + 96 & 1 + 56 \end{bmatrix} = \begin{bmatrix} -7 & 147 & 56 \\ -14 & 78 & 20 \\ 13 & 105 & 57 \end{bmatrix} \end{aligned}$$

(b) BA

$$\begin{aligned} BA &= \begin{bmatrix} -3 & 9 & 1 \\ 2 & 12 & 7 \end{bmatrix} \begin{bmatrix} 7 & 7 \\ 6 & 2 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} -3(7) + 9(6) + 1(1) & -3(7) + 9(2) + 1(8) \\ 2(7) + 12(6) + 7(1) & 2(7) + 12(2) + 7(8) \end{bmatrix} \\ &= \begin{bmatrix} -21 + 54 + 1 & -21 + 18 + 8 \\ 14 + 72 + 7 & 14 + 24 + 56 \end{bmatrix} = \begin{bmatrix} 34 & 5 \\ 93 & 94 \end{bmatrix} \end{aligned}$$

-:10.25:-

Find (a) AB (b) BA, given

$$A = \begin{bmatrix} 4 & 9 & 8 \\ 7 & 6 & 2 \\ 1 & 5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$$

SOLUTION

(a) AB

$$\begin{aligned}
 AB &= \begin{bmatrix} 4 & 9 & 8 \\ 7 & 6 & 2 \\ 1 & 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 4(1)+9(5)+8(0) & 4(2)+9(3)+8(2) & 4(0)+9(1)+8(4) \\ 7(1)+6(5)+2(0) & 7(2)+6(3)+2(2) & 7(0)+5(1)+2(4) \\ 1(1)+5(5)+3(0) & 1(2)+5(3)+3(2) & 1(0)+5(1)+3(4) \end{bmatrix} \\
 &= \begin{bmatrix} 4+45+0 & 8+27+16 & 0+9+32 \\ 7+30+0 & 14+18+4 & 0+6+8 \\ 1+25+0 & 2+15+6 & 0+5+12 \end{bmatrix} = \begin{bmatrix} 49 & 51 & 31 \\ 37 & 36 & 14 \\ 26 & 23 & 17 \end{bmatrix}
 \end{aligned}$$

(b) BA

$$\begin{aligned}
 BA &= \begin{bmatrix} 1 & 2 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & 9 & 8 \\ 7 & 6 & 2 \\ 1 & 5 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 1(4)+2(7)+0(1) & 1(9)+2(6)+0(5) & 1(8)+2(2)+0(3) \\ 5(4)+3(7)+1(1) & 5(9)+3(6)+1(5) & 5(8)+3(2)+1(3) \\ 0(4)+2(7)+4(1) & 0(9)+2(6)+4(5) & 0(8)+2(2)+4(3) \end{bmatrix} \\
 &= \begin{bmatrix} 4+14+0 & 9+12+0 & 8+4+0 \\ 20+21+1 & 45+18+5 & 40+6+3 \\ 0+14+4 & 0+12+20 & 0+4+12 \end{bmatrix} = \begin{bmatrix} 18 & 21 & 12 \\ 42 & 68 & 49 \\ 18 & 32 & 16 \end{bmatrix}
 \end{aligned}$$

-:10.26:-

Find (a) AB (b) BA, for a case where B is an identity matrix, given.

$$A = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

SOLUTION

(a) AB

$$AB = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned}
 AB &= \begin{bmatrix} 23(1) + 6(0) + 14(0) & 23(0) + 6(1) + 14(0) & 23(0) + 6(0) + 14(1) \\ 18(1) + 12(0) + 9(0) & 18(0) + 12(1) + 9(0) & 18(0) + 12(0) + 9(1) \\ 24(1) + 2(0) + 6(0) & 24(0) + 2(1) + 6(0) & 24(0) + 2(0) + 6(1) \end{bmatrix} \\
 &= \begin{bmatrix} 23+0+0 & 0+6+0 & 0+0+14 \\ 18+0+0 & 0+12+0 & 0+0+9 \\ 24+0+0 & 0+2+0 & 0+0+6 \end{bmatrix} = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix}
 \end{aligned}$$

(b) BA

$$\begin{aligned}
 AB &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 1(23) + 0(18) + 0(24) & 1(6) + 0(12) + 0(2) & 1(14) + 0(9) + 0(6) \\ 0(23) + 1(18) + 0(24) & 0(6) + 1(12) + 0(2) & 0(14) + 1(9) + 0(6) \\ 0(23) + 0(18) + 1(24) & 0(6) + 0(12) + 1(2) & 0(14) + 0(9) + 1(6) \end{bmatrix} \\
 &= \begin{bmatrix} 23+0+0 & 6+0+0 & 14+0+0 \\ 0+18+0 & 0+12+0 & 0+9+0 \\ 0+0+24 & 0+0+2 & 0+0+6 \end{bmatrix} = \begin{bmatrix} 23 & 6 & 14 \\ 18 & 12 & 9 \\ 24 & 2 & 6 \end{bmatrix}
 \end{aligned}$$

-:10.27:-

Find the determinants of the following matrices.

i) $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$

ii) $B = \begin{bmatrix} c & m \\ n & p \end{bmatrix}$

iii) $C = \begin{bmatrix} 6 & 2 \\ 4 & 1 \end{bmatrix}$

iv) $D = \begin{bmatrix} 15 & 8 \\ 3 & 4 \end{bmatrix}$

SOLUTION

(i)

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}, \text{ then } |A| = \begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix} = 12 - 20 = -8$$

(ii)

$$B = \begin{bmatrix} c & m \\ n & p \end{bmatrix}, \text{ then } |B| = \begin{vmatrix} c & m \\ n & p \end{vmatrix} = cp - mn$$

(iii)

$$C = \begin{bmatrix} 6 & 2 \\ 4 & 1 \end{bmatrix}, \text{ then } |C| = \begin{vmatrix} 6 & 2 \\ 4 & 1 \end{vmatrix} = 6 - 8 = -2$$

(iv)

$$D = \begin{bmatrix} 15 & 8 \\ 3 & 4 \end{bmatrix}, \text{ then } |D| = \begin{vmatrix} 15 & 8 \\ 3 & 4 \end{vmatrix} = 60 - 24 = 36$$

-:10.28:-

Find the value of x when

$$\text{i) } \begin{bmatrix} 8 & x \\ 2 & 4 \end{bmatrix} \text{ is a singular matrix} \quad \text{ii) } \begin{bmatrix} 2 & 1 \\ 3 & x \end{bmatrix} \text{ is a singular}$$

SOLUTION

$$\begin{bmatrix} 8 & x \\ 2 & 4 \end{bmatrix} \text{ is a singular}$$

$$\text{then } \begin{vmatrix} 8 & x \\ 2 & 4 \end{vmatrix} = 0$$

$$32 - 2x = 0$$

$$-2x = -32$$

$$2x = 32$$

$$x = 16$$

-:10.29:-

Find the inverse of the following matrices.

$$\text{i) } A = \begin{bmatrix} 4 & 6 \\ 10 & 8 \end{bmatrix}$$

$$\text{ii) } B = \begin{bmatrix} -3 & -27 \\ -6 & -18 \end{bmatrix}$$

$$\text{iii) } C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{iv) } D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

SOLUTION

(i)

$$A = \begin{bmatrix} 4 & 6 \\ 10 & 8 \end{bmatrix}, |A| = \begin{vmatrix} 4 & 6 \\ 10 & 8 \end{vmatrix} = 32 - 60 = -28$$

$$A^{-1} = -\frac{1}{28} \begin{bmatrix} 4 & 6 \\ 10 & 8 \end{bmatrix} = \begin{bmatrix} \frac{-8}{28} & \frac{6}{28} \\ \frac{10}{28} & \frac{4}{28} \end{bmatrix} = \begin{bmatrix} \frac{-4}{14} & \frac{3}{14} \\ \frac{5}{7} & \frac{1}{7} \end{bmatrix}$$

(ii)

$$B = \begin{bmatrix} -3 & -27 \\ -6 & -18 \end{bmatrix}, \quad |B| = \begin{vmatrix} -3 & -27 \\ -6 & -18 \end{vmatrix} = 54 - 162 = -108$$

$$B^{-1} = -\frac{1}{108} \begin{bmatrix} -18 & 27 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} \frac{18}{108} & \frac{-27}{108} \\ \frac{-6}{108} & \frac{3}{108} \end{bmatrix} = \begin{bmatrix} \frac{6}{36} & \frac{-9}{36} \\ \frac{-2}{36} & \frac{1}{36} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{-1}{4} \\ \frac{-1}{18} & \frac{1}{36} \end{bmatrix}$$

(iii)

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |C| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$C^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(iv)

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \quad |D| = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = 3 - 0 = 3$$

$$D^{-1} = \frac{1}{3} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$$

-:10.30:-

$$\text{If } A = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Prove that

(i) $AA^{-1} = A^{-1}A = I$

(ii) $BB^{-1} = B^{-1}B = I$

(iii) $B^2 = I$

SOLUTION

(i)

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix}, \quad |A| = \begin{vmatrix} 2 & 1 \\ 0 & 8 \end{vmatrix} = 16 - 0 = 16$$

$$A^{-1} = \frac{1}{16} \begin{bmatrix} 8 & -1 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix} = \begin{bmatrix} \frac{16}{16} + 0 & \frac{-2}{16} + \frac{2}{16} \\ 0 + 0 & 0 + \frac{16}{16} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$A^{-1}A = \begin{bmatrix} \frac{8}{16} & -\frac{1}{16} \\ 0 & \frac{2}{16} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} \frac{16}{16} + 0 & \frac{8}{16} - \frac{8}{16} \\ 0 + 0 & 0 + \frac{16}{16} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence $AA^{-1} = A^{-1}A$

(ii)

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad |B| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$B^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BB^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$B^{-1}B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(iii)

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

-:10.31:-

Solve the following sets of equations with the help of matrices.

a) $x + y = 8$

$2x - y = 7$

c) $3x + 2y = 1$

$5x - 3y = 27$

b) $2x + 5y = 19$

$x + 3y = 11$

d) $3x + 2y = 12$

$x + 5y = 17$

$$\begin{aligned} \text{e) } 2x - 3y &= 1 \\ x + 4y &= 6 \end{aligned}$$

$$\begin{aligned} \text{f) } 7x - 3y &= 3 \\ 2x + y &= 2 \end{aligned}$$

SOLUTION

$$\begin{aligned} \text{(a) } x + y &= 8 \\ 2x - y &= 7 \end{aligned}$$

Here

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

So

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3 \neq 0$$

$$A^{-1} = \frac{-1}{-3} \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 8 \\ 7 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} + \frac{7}{3} \\ \frac{16}{3} - \frac{7}{3} \end{bmatrix} = \begin{bmatrix} \frac{15}{3} \\ \frac{9}{3} \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Therefore $x = 5, y = 3$

(b)

$$2x + 5y = 19$$

$$x + 3y = 11$$

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1 \neq 0$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 19 \\ 11 \end{bmatrix} = \begin{bmatrix} 57 - 55 \\ -19 + 22 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore, $x = 2, y = 3$

(c)

$$3x + 2y = 1$$

$$5x - 3y = 27$$

$$A = \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 27 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 27 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 5 & -3 \end{vmatrix} = -9 - 10 = -19 \neq 0$$

$$A^{-1} = \frac{-1}{19} \begin{bmatrix} -3 & -2 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{19} & \frac{2}{19} \\ \frac{5}{19} & \frac{-3}{19} \end{bmatrix} \begin{bmatrix} 1 \\ 27 \end{bmatrix} = \begin{bmatrix} \frac{3}{19} + \frac{54}{19} \\ \frac{5}{19} - \frac{81}{19} \end{bmatrix} = \begin{bmatrix} \frac{57}{19} \\ \frac{-76}{19} \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

Therefore, $x = 3, y = -4$

(d)

$$3x + 2y = 12$$

$$x + 5y = 17$$

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 17 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$A^{-1} = \frac{1}{15 - 2} \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{13} \begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & -\frac{2}{13} \\ -\frac{1}{13} & \frac{3}{13} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & -\frac{2}{13} \\ -\frac{1}{13} & \frac{3}{13} \end{bmatrix} \begin{bmatrix} 12 \\ 17 \end{bmatrix} = \begin{bmatrix} \frac{60}{13} - \frac{34}{13} \\ -\frac{12}{13} + \frac{51}{13} \end{bmatrix} = \begin{bmatrix} \frac{26}{13} \\ \frac{39}{13} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore, $x = 2, y = 3$

(e)

$$2x - 3y = 1$$

$$x + 4y = 6$$

$$A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -3 \\ 1 & 4 \end{vmatrix} = 8 + 3 = 11$$

$$A^{-1} = \frac{1}{11} \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{2}{11} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{2}{11} \end{bmatrix} \begin{bmatrix} 1 \\ 6 \end{bmatrix} = \begin{bmatrix} \frac{4}{11} + \frac{18}{11} \\ -\frac{1}{11} + \frac{12}{11} \end{bmatrix} = \begin{bmatrix} \frac{22}{11} \\ \frac{11}{11} \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Therefore, $x = 2, y = 1$

(f)

$$7x - 3y = 3$$

$$2x + y = 2$$

Here

$$A = \begin{bmatrix} 7 & -3 \\ 2 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$\begin{bmatrix} 7 & -3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 7 & -3 \\ 2 & 1 \end{vmatrix} = 7 \cdot 1 - 6 = 13$$

$$A^{-1} = \frac{1}{13} \begin{bmatrix} 1 & 3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{-2}{13} & \frac{7}{13} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{13} \\ \frac{-2}{13} & \frac{7}{13} \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{13} + \frac{6}{13} \\ \frac{-6}{13} + \frac{14}{13} \end{bmatrix} = \begin{bmatrix} \frac{9}{13} \\ \frac{8}{13} \end{bmatrix}$$

Therefore, $x = \frac{9}{13}$, $y = \frac{8}{13}$

-:10.32:-

Solve the following sets of equations with the help of matrices.

$$\begin{array}{ll} \text{a)} & \text{(i) } \begin{array}{l} 2x - 6y = -12 \\ 3x - 2y = -4 \end{array} \quad \text{(ii) } \begin{array}{l} \frac{3}{2}x + y = \frac{3}{4} \\ x - 2y = 2 \end{array} \end{array}$$

SOLUTION

(a)

$$\begin{array}{l} \text{(i)} \quad 2x - 6y = 12 \\ \quad \quad 3x - 2y = -4 \end{array}$$

Here

$$A = \begin{bmatrix} 2 & -6 \\ 3 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} -12 \\ -4 \end{bmatrix}$$

So

$$\begin{bmatrix} 2 & -6 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -12 \\ -4 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$A = \begin{vmatrix} 2 & -6 \\ 3 & -2 \end{vmatrix} = -4 + 18 = 14 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{14} \begin{bmatrix} -2 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{-2}{14} & \frac{6}{14} \\ \frac{3}{14} & \frac{2}{14} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-2}{14} & \frac{6}{14} \\ \frac{3}{14} & \frac{2}{14} \end{bmatrix} \begin{bmatrix} -12 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{24}{14} - \frac{24}{14} \\ \frac{36}{14} - \frac{8}{14} \end{bmatrix} = \begin{bmatrix} \frac{0}{14} \\ \frac{28}{14} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Therefore $x = 0, y = 2$

(ii)

$$\frac{3}{2}x + y = \frac{3}{4}$$

$$x - 2y = 2$$

Here $A = \begin{bmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix}$

So $\begin{bmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix}$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} \frac{3}{2} & 1 \\ 1 & -2 \end{vmatrix} = -3 - 1 = -4 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = -\frac{1}{4} \begin{bmatrix} 2 & -1 \\ -1 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix} \begin{bmatrix} \frac{3}{4} \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} + \frac{1}{2} \\ \frac{3}{16} - \frac{3}{4} \end{bmatrix} = \begin{bmatrix} \frac{3+4}{8} \\ \frac{3-12}{16} \end{bmatrix} = \begin{bmatrix} \frac{7}{8} \\ -\frac{9}{16} \end{bmatrix}$$

Therefore $x = 7/8, y = -9/16$

(b)

$$2x + 3y = 10$$

$$4x + 8y = 24$$

Here

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 10 \\ 24 \end{bmatrix}$$

So

$$\begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 24 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 8 \end{vmatrix} = 16 - 12 = 4 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 8 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{8}{4} & \frac{-3}{4} \\ \frac{-4}{4} & \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} 2 & \frac{-3}{4} \\ -1 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 10 \\ 24 \end{bmatrix} = \begin{bmatrix} 20 - 18 \\ -10 + 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Therefore $x = 2, y = 2$

(c)

$$3x + 2y = 12$$

$$4x + 5y = 23$$

Here

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

So

$$\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B$$

$$|A| = \begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 15 - 8 = 7 \neq 0$$

The matrix A is non-singular, A^{-1} exists.

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} \frac{5}{7} & -\frac{2}{7} \\ -\frac{4}{7} & \frac{3}{7} \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B = \begin{bmatrix} \frac{5}{7} & -\frac{2}{7} \\ -\frac{4}{7} & \frac{3}{7} \end{bmatrix} \begin{bmatrix} 12 \\ 23 \end{bmatrix} = \begin{bmatrix} \frac{60}{7} - \frac{46}{7} \\ -\frac{48}{7} + \frac{69}{7} \end{bmatrix} = \begin{bmatrix} \frac{14}{7} \\ \frac{21}{7} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Therefore $x = 2, y = 3$

-:10.33:-

Solve the following sets of equations by Cramer's Rule

a) $2x + 4y = 5$

b) $x + 3y = 3$

$6x - y = 1$

$2x + 5y = 7$

c) $x - 2y = 1$

d) $5x + 2y = 4$

$2x - 5y = 1$

$3x - 2y = 12$

SOLUTION

(a)

$$2x + 4y = 5$$

$$6x - y = 1$$

$$\begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\text{Here } \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

By Cramer's rule the value of x and y are

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 5 & 4 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 \\ 6 & -1 \end{vmatrix}} = \frac{-5 - 4}{-2 - 24} = \frac{-9}{-26} = \frac{9}{26}$$

and

$$y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 2 & 5 \\ 6 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 4 \\ 6 & -1 \end{vmatrix}} = \frac{2 - 30}{-2 - 24} = \frac{-28}{-26} = \frac{14}{13}$$

Hence $x = 9/26$ and $y = 14/13$

(b)

$$x + 3y = 3$$

$$2x + 5y = 7$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

$$\text{Here } \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

By Cramer's rule the value of x and y are

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 3 & 3 \\ 7 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}} = \frac{15 - 21}{5 - 6} = \frac{-6}{-1} = 6$$

and

$$y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix}} = \frac{7 - 6}{5 - 6} = \frac{-1}{-1} = -1$$

Hence $x = 6$ and $y = -1$

(c)

$$x - 2y = 1$$

$$2x - 5y = 1$$

$$\begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{Here } \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 2 & -5 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

By Cramer's rule the value of x and y are

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 1 & -2 \\ 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & -5 \end{vmatrix}} = \frac{-5+2}{-5+4} = \frac{-3}{-1} = 3$$

and

$$y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -2 \\ 2 & -5 \end{vmatrix}} = \frac{1-2}{-5+4} = \frac{-1}{-1} = 1$$

Hence $x = 3$ and $y = 1$

(d)

$$5x + 2y = 4$$

$$3x - 2y = 12$$

$$\begin{bmatrix} 5 & 2 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$$

$$\text{Here } \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 3 & -2 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$$

By Cramer's rule the value of x and y are

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 4 & 2 \\ 12 & -2 \end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{-8-24}{-10-6} = \frac{-32}{-16} = 2$$

and

$$y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\begin{vmatrix} 5 & 4 \\ 3 & 12 \end{vmatrix}}{\begin{vmatrix} 5 & 2 \\ 3 & -2 \end{vmatrix}} = \frac{60-12}{-10-6} = \frac{48}{-16} = -3$$

Hence $x = 2$ and $y = -3$