

**Unit
2**

Vectors and Equilibrium

How are mobile cranes able to lift heavy load without toppling over?

**LEARNING
OUTCOMES**

After studying this unit the students will be able to

- ⊗ describe the Cartesian coordinate system.
- ⊗ determine the sum of vectors using head-to-tail rule.
- ⊗ represent a vector into two perpendicular components.
- ⊗ determine the sum of vectors using perpendicular components.
- ⊗ describe scalar product of two vectors in term of angle between them.
- ⊗ describe vector product of two vectors in term of angle between them.
- ⊗ state the method to determine the direction of vector product of two vectors.
- ⊗ define the torque as vector product $\mathbf{r} \times \mathbf{F}$.
- ⊗ list applications of torque or moment due to a force.
- ⊗ state first condition of equilibrium.
- ⊗ state second condition of equilibrium.
- ⊗ solve two dimensional problems involving forces (statics) using 1st and 2nd conditions of equilibrium.

Vectors were developed in the late nineteenth century as mathematical tools for studying physics. In the following century, vectors became an essential tool for anyone using mathematics or physics. In order to navigate, pilots need to know what effect a crosswind will have on the direction in which they intend to fly. In order to build bridges, engineers need to know what load a particular design will support. There and many other technological examples have made vectors an integral part to study physics.

2.1 VECTORS

A vector is a mathematical quantity having both magnitude and direction. Some quantities (such as weight, velocity, or friction) require both a magnitude (or size) and a direction for a complete description and are called vectors. Vectors, have both magnitude and direction and obey the rules of vector algebra. Unlike scalars they cannot be added, subtracted and multiplied by using ordinary algebra.

Representation of vectors: Vectors can be represented in two ways.

A. Symbolic representation: Symbolically a vector is represented by a bold face letter either capital or small. (e.g $\mathbf{F}$, $\mathbf{f}$). Vector can also be represented by a simple face letter with an arrow over or below it. In this book we will represent vectors by letter with an arrow over it.

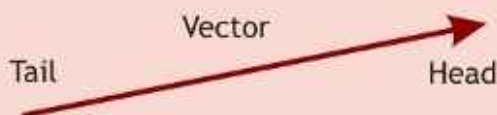
For example $\vec{A}$, $\vec{B}$, $\vec{C}$, or $\vec{D}$

To indicate only the magnitude of vector, the quantity may be printed in italics or the mathematicians' absolute-value sign may be used.

For example $|\vec{A}|$, $|\vec{B}|$, $|\vec{C}|$ or $|\vec{D}|$

B. Graphical Representation: Graphically a vector is represented by an arrow, the length of the arrow gives the magnitude (under certain scale) and the arrow head points the direction of the vector.

The starting point of the vector is called tail of the vector and the ending point is called head of the vector as shown in the figure.



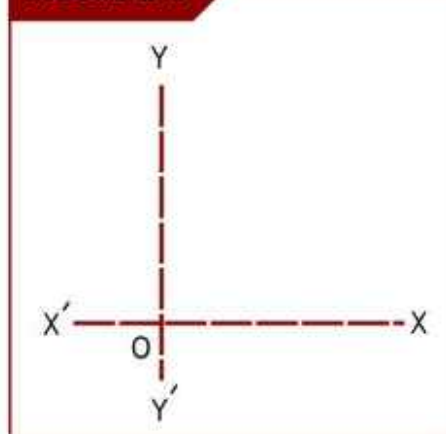
Geometric vectors are those that are considered without reference to coordinate axes (any sets of values that indicate the position of a point in a given reference system.). The ability to use vectors in applications usually requires us to place them on a **coordinate plane**. These are referred to as **algebraic vectors**.

Mathematicians started using coordinates to analyze physical situations in about the fourteenth century. However, a great deal of the credit for developing the methods used with coordinate systems should be given to the *French mathematician Rene Descartes (1596- 1650)*.

2.2 CARTESIAN COORDINATE SYSTEM

The Cartesian coordinate system (sometimes called the number plane) consists of a horizontal line called the x-axis (XOX') and a vertical line called the y-axis (YOY') intersecting at a right angle (90°) at a point 'O' called the origin (as shown in Figure 2.1). The x and y-axis are usually drawn in the plane of the page. The z-axis (ZOZ') comes directly out of the page at the origin which is not shown here for simplicity.

FIGURE 2.1



STEPS TO REPRESENT A VECTOR IN CARTESIAN COORDINATE SYSTEM

The following method is used to represent a vector

1. Draw a Cartesian Coordinate System.
2. Select a suitable scale.
3. Draw a line in the specified direction. Cut the line equal to the magnitude of the vector according to the selected scale.
4. Put an arrow in the direction of the vector.

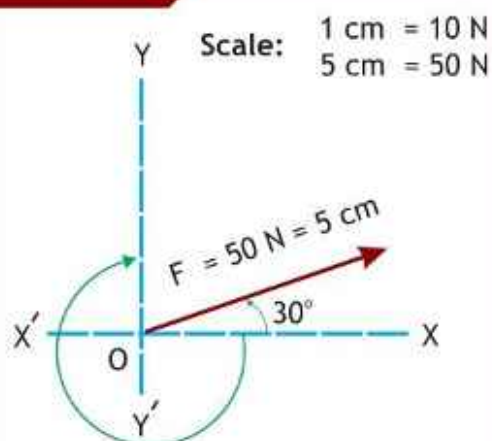
2.2.1 Representing a vector in Cartesian Coordinate System

The most commonly used method to represent a vector is with Cartesian coordinates.

2.2.1 Representing a vector in plane

To represent a vector in plane (two dimensional) two mutually perpendicular lines are drawn for Cartesian coordinate system to represent a vector. For example a force vector of 50 N making an angle of 30° with the horizontal is drawn as shown in Figure 2.2. The scale is selected such that 5 cm equals to 50 N, and angle is drawn in degrees. The angle by convention is measured from the positive x-axis in anti clockwise direction.

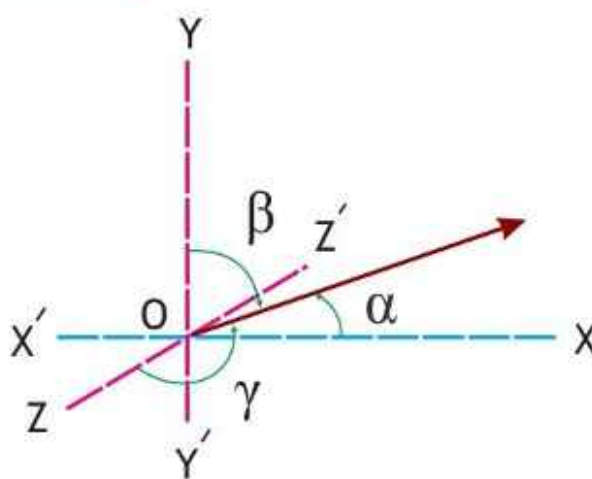
FIGURE 2.2



2.2.2 Representing a vector in space

To represent a vector in space (three dimensional) three mutually perpendicular lines XOX' , YOY' and ZOZ' are drawn in Cartesian coordinate system and vector is represented as shown in the Figure 2.3. The direction of vector in space is specified by three angle symbols α for x-axis, β for y-axis and γ for z - axis respectively.

FIGURE 2.3



UNIT VECTORS

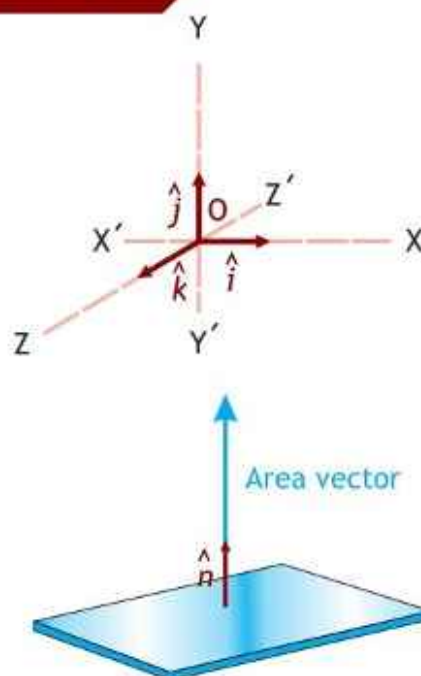
The dimensionless vector having magnitude 1 and used to represent the direction of a vector is called unit vector. The unit vector is generally represented by letter with a cap or hat (^) over it. For example the unit vector for vector $\vec{A}$ is $\hat{A}$. The unit vector is obtained by dividing vector by its own magnitude, e.g. A vector $\vec{A}$ can be written as

$$\vec{A} = |\vec{A}| \hat{A}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

Some Important Unit Vectors: For Cartesian coordinate system the unit vectors perfectly along x-axis is represented as $\hat{i}$. The unit vector along y-axis is represented by $\hat{j}$. And the unit vector along z-axis is represented by $\hat{k}$. The three unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ do not change the magnitude or the dimensions of anything; they only indicate directions. Similarly the unit vector $\hat{n}$ is normal, or perpendicular to a surface at a given point. For example area is sometime taken as a vector quantity in physics and therefore this description is helpful.

FIGURE 2.4



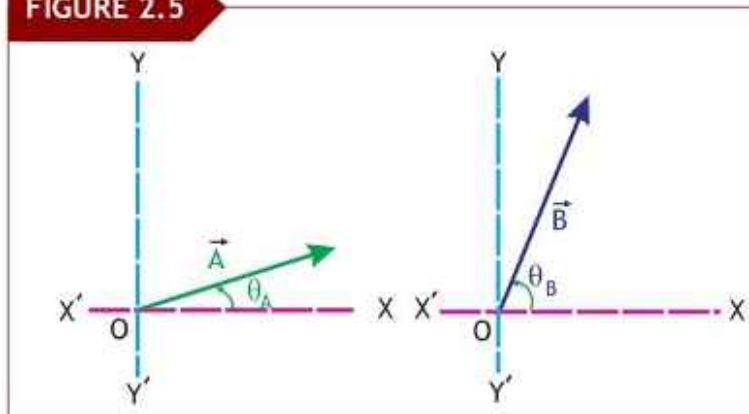
2.3 ADDITION OF VECTORS

Vectors may be added geometrically by drawing them to a common scale and placing them head to tail. Joining the tail of the first vector with the head of the last will give another vector which is the sum of these vectors called resultant vector.

2.3.1 Addition of two Vectors:

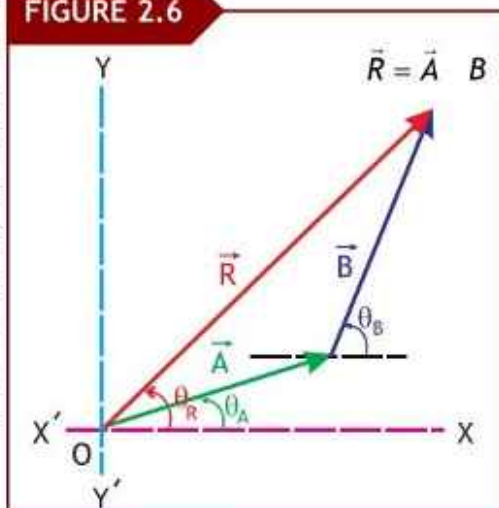
As an example consider two vectors $\vec{A}$ and $\vec{B}$, drawn to same scale making certain angles θ_A and θ_B with the x-axis respectively as shown in the Figure 2.5.

FIGURE 2.5



To add these vectors we redraw them to a common scale and place them head to tail as in the steps mentioned above. Such that the tail of vector $\vec{B}$ is on head of vector $\vec{A}$. Joining the tail of the first vector with the head of the last will give another vector which is the sum of these vectors called resultant vector $\vec{R}$ as shown in Figure 2.6. The resultant will have the same effect as the combined effect of both vectors.

FIGURE 2.6



STEPS TO ADD TWO VECTORS BY HEAD TO TAIL RULE

The following steps are followed to add two vectors by head to tail rule.

1. Sketch one vector according to selected scale in a given direction.
2. Now put the tail of the second vector on the head of the first vector according to selected scale in the given direction.
3. By joining the tail of the first vector to the head of the second vector and putting an arrow on the line pointing away from the origin gives the resultant vector.
4. To determine the resultant measure the length of R and convert it back according to given scale, it will give the magnitude (or size) of the resultant vector. To determine the direction measure the angle of resultant with x-axis.

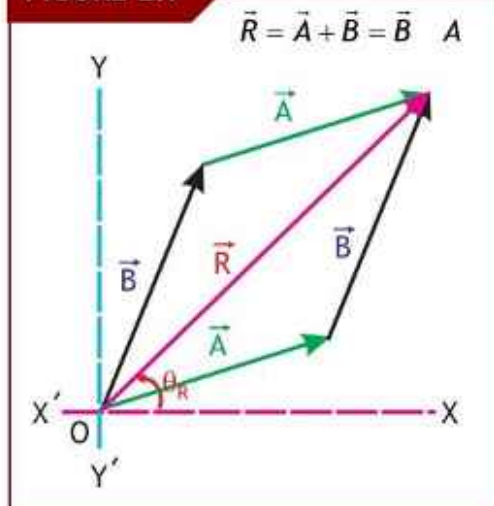
2.3.2 Vector Addition is

Commutative: Vector addition obeys the commutative property, it means that the order in which the vectors are added has no physical significance.

$$\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$

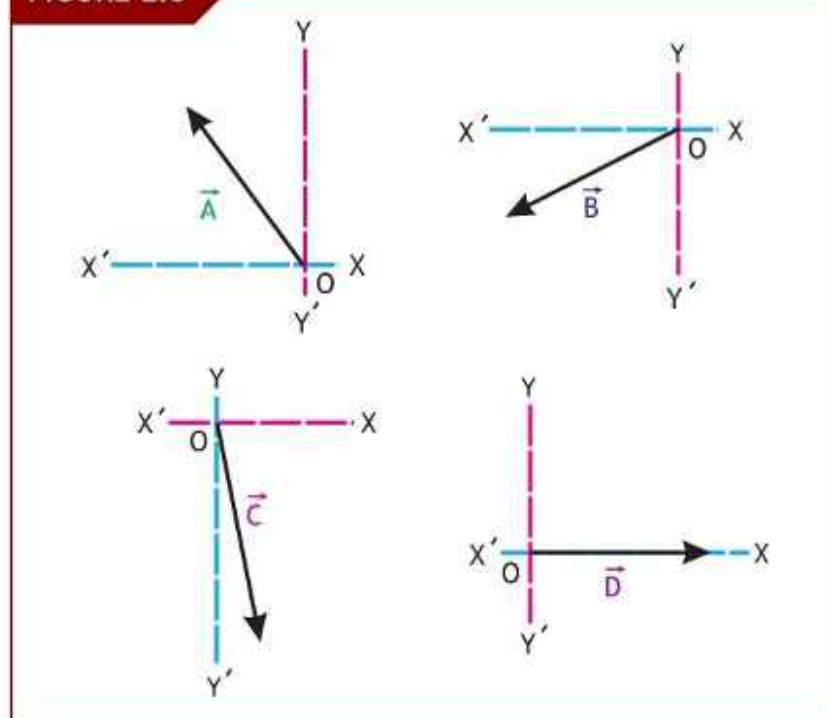
When two or more vectors are added together, they must all have the same units and they all must be the same type of quantity.

FIGURE 2.7



2.3.3 Addition of More than two Vectors: The rules for vector addition can be extended to any number of vectors. For example consider four vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ in xy - plane as shown in the Figure 2.8.

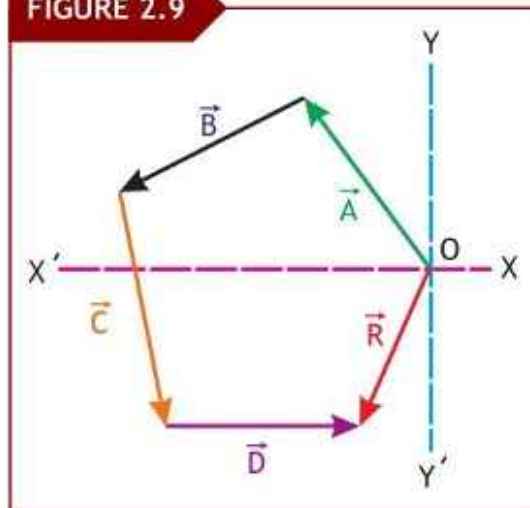
FIGURE 2.8



To add these vectors draw them to a common scale and placing them head to tail. Such that the tail of each vector is on head of the previous vector.

To determine the resultant measure the length of R and convert it back according to given scale, it will give the magnitude (or size) of the resultant vector. To determine the direction measure the angle of resultant with x -axis.

FIGURE 2.9



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

When more than two vectors are added, still the sum is independent of the order of the addition. Commutative property holds good in this case as well. It does not matter in which order you put these vectors the result will always be the same. For instance

$$\vec{R} = \vec{B} + \vec{D} + \vec{A} + \vec{C} = \vec{B} + \vec{A} + \vec{C} + \vec{D} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

ACTIVITY

Try drawing these vectors in different orders and confirm that the magnitude and direction of resultant R of these vectors is same.

NULL VECTORS

The vector having ZERO magnitude and have some arbitrary direction is called a NULL vector, (usually denoted as $\vec{O}$). Such vectors are obtained by addition, subtraction and cross ($\times$) multiplication of vectors. For example when a vector A is subtracted from itself then:

$$\vec{A} + (-\vec{A}) = \vec{O}$$

Example 2.1**SHIP DISPLACEMENT**

A ship leaves port and travels 200 km at 30° north of east. Then it changes its direction and travels 350 km in a direction 140° north of east to reach destination. Calculate straight line distance covered by ship?

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Procedure.

1. selecting scale:

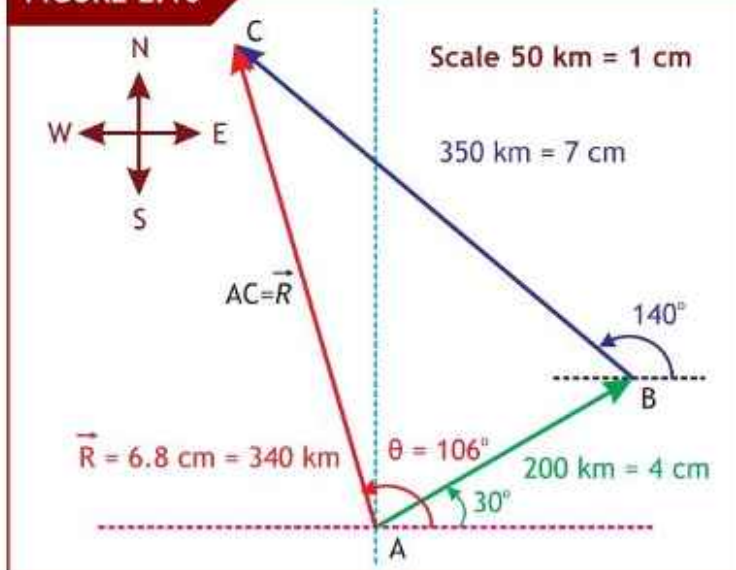
Let $5 \text{ km} = 1 \text{ cm}$

$200 \text{ km} = 4 \text{ cm}$, $\theta = 30^\circ$
with the east.

$350 \text{ km} = 7 \text{ cm}$, $\theta = 140^\circ$
with the east.

REQUIRED

Resultant $\vec{R} = ?$

FIGURE 2.10**SOLUTION**

2. Finding the resultant: Using head to tail rule to get the resultant $\vec{R}$.

We measure the length of vector $\vec{R}$ (with scale) which was about 6.8 cm ($6.8 \times 50 = 340 \text{ km}$), and with the protector we also calculate the value of angle, which is $\theta = 106^\circ$ with east.

34 km, 106° with east

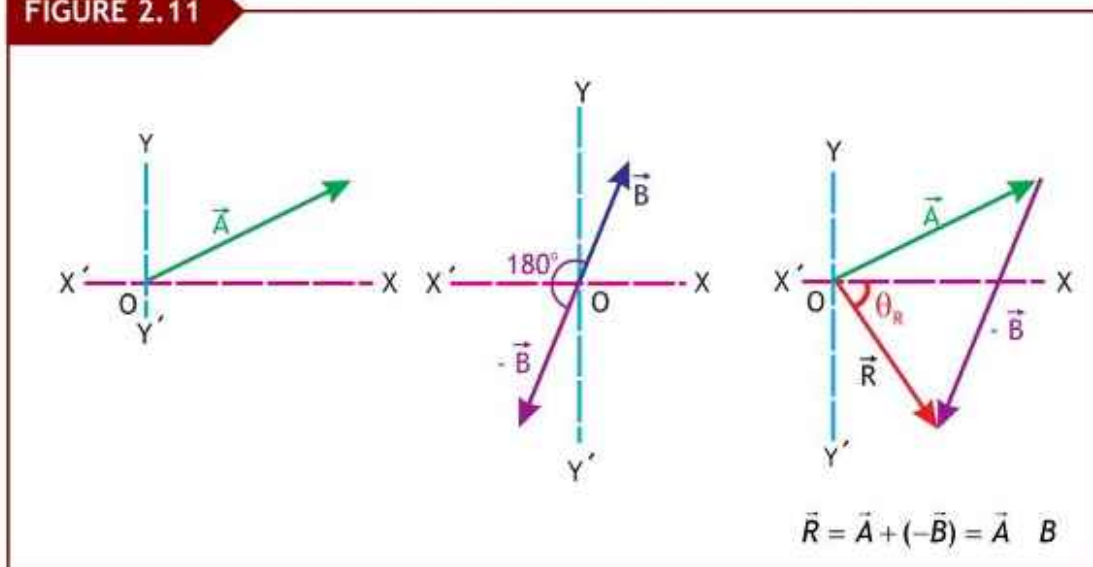
Answer

Assignment 2.1**AIRPLANE THROUGH WIND**

An airplane is moving at 120 m/s at an angle of 10° with x-axis, through a 30 m/s cross wind, blowing at angle of 260° with x-axis. Determine the resultant velocity of the airplane. (20 m)

2.3.4 Subtraction of vectors: There is no direct method for the subtraction of vectors. Subtraction of one vector from another vector means addition of the negative of the vector with the first.

FIGURE 2.11



If vector $\vec{B}$ is to be subtracted from vector $\vec{A}$. Then we first find the negative of vector $\vec{B}$ (which is $-\vec{B}$). Then we follow the rules of vector addition to get the resultant $\vec{R}$ as shown in Figure 2.11. Such that

$$\vec{R} = \vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$

2.4 MULTIPLICATION OF A VECTOR BY A SCALAR

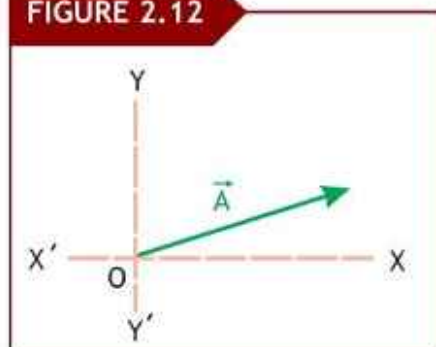
If k is a scalar and $\vec{A}$ is a vector, then the scalar multiple is the vector whose length is k times the length of $\vec{A}$ and whose direction depending upon the value k .

If $k = 0$ then $k\vec{A} = \vec{0}$.

If $k > 0$ then $+k \cdot \vec{A} = k\vec{A}$.

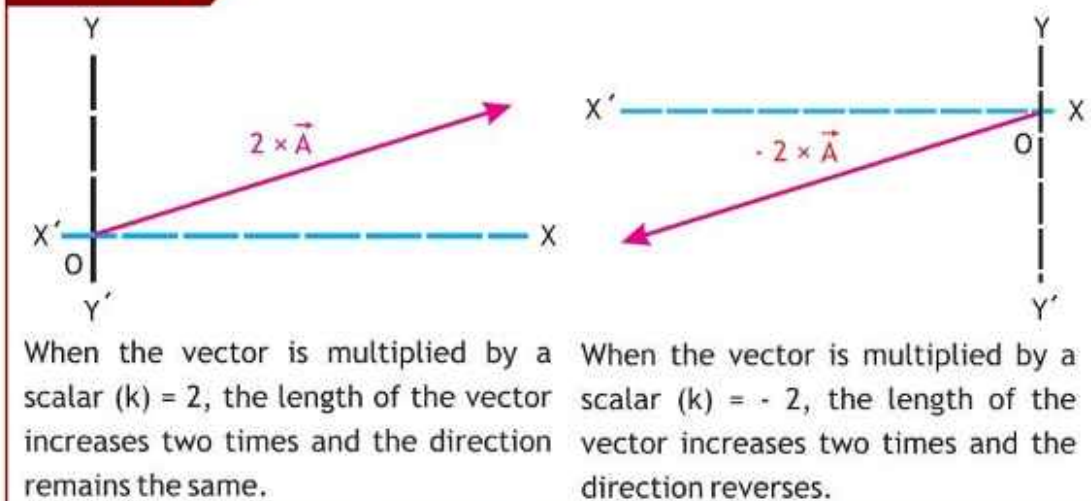
If $k < 0$ then $-k \cdot \vec{A} = -k\vec{A}$.

FIGURE 2.12



As an example consider a vector $\vec{A}$ in the xy plane, making certain angle with the x -axis as shown in Figure 2.12.

FIGURE 2.13



2.5 RESOLUTION OF VECTORS

The process of splitting a vector into two or more vectors is called **resolution of a vector**. The vectors so obtained are called **components of the vectors**. If these components in which a vector is split are perpendicular to each other then such components are called **rectangular components of vector**.

Consider a vector $\vec{A}$ in the Cartesian coordinate System, represented by the line OP, making an angle θ as shown in the figure 2.14.

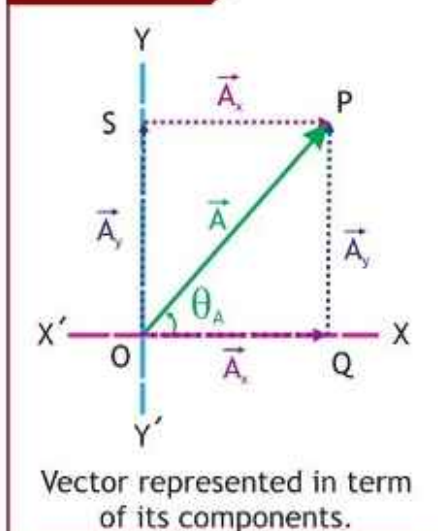
Draw perpendiculars from point P on x-axis and y-axis which meets the axis at points Q and S respectively. Put arrow head from the direction of O towards Q and S such that they represent vectors as $\vec{A}_x$ (OQ) and $\vec{A}_y$ (OS), called the rectangular components of vector $\vec{A}$.

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

TIP

The effective value of a vector in a particular direction is called component of a vector.

FIGURE 2.14



$$\text{or } A = A_x \hat{i} + A_y \hat{j} \quad \text{--- (1)}$$

2.5.1 Components represented in terms of vector: From Figure 2.14 consider a triangle OPQ, without considering the sides as vectors, as shown in Figure 2.15. This forms a right angle triangle OPQ for which we have.

$$\cos \theta = \frac{\text{Base}}{\text{hyp}} = \frac{A_x}{A} \quad \text{and} \quad \sin \theta = \frac{\text{Perp}}{\text{hyp}} = \frac{A_y}{A}$$

$$A_x = A \cos \theta \quad \text{--- (2)} \quad \text{and} \quad A_y = A \sin \theta \quad \text{--- (3)}$$

Equation (2) and equation (3) are used to represent the components in terms of its vector. Putting equation 2 and equation 3 in equation 1 we get

$$A = A \cos \theta \hat{i} + A \sin \theta \hat{j} \quad \text{--- 2.1}$$

From a right angle triangle ΔOPQ , using Pythagoras theorem

$$(\text{hyp})^2 = (\text{base})^2 + (\text{perp})^2$$

$$\sqrt{(\text{hyp})^2} = \sqrt{(\text{base})^2 + (\text{perp})^2}$$

$$\text{or } \text{hyp} = \sqrt{(\text{base})^2 + (\text{perp})^2}$$

$$\text{therefore } |A| = \sqrt{A_x^2 + A_y^2} \quad \text{--- 2.2}$$

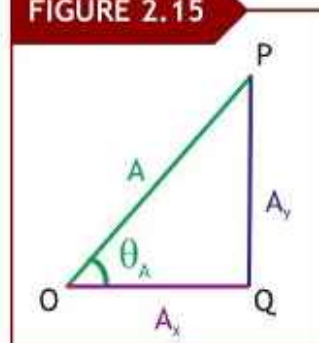
The magnitude of vector can now be determined if the values of the magnitudes of components are known. Also to determine the direction in right angle triangle ΔOPQ , we have

$$\tan \theta = \frac{\text{perp}}{\text{base}} = \frac{A_y}{A_x} \quad \text{and} \quad \theta = \tan^{-1} \frac{A_y}{A_x} \quad \text{--- 2.3}$$

In three dimensional components vector A can be written as

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \quad \text{--- 2.4}$$

FIGURE 2.15



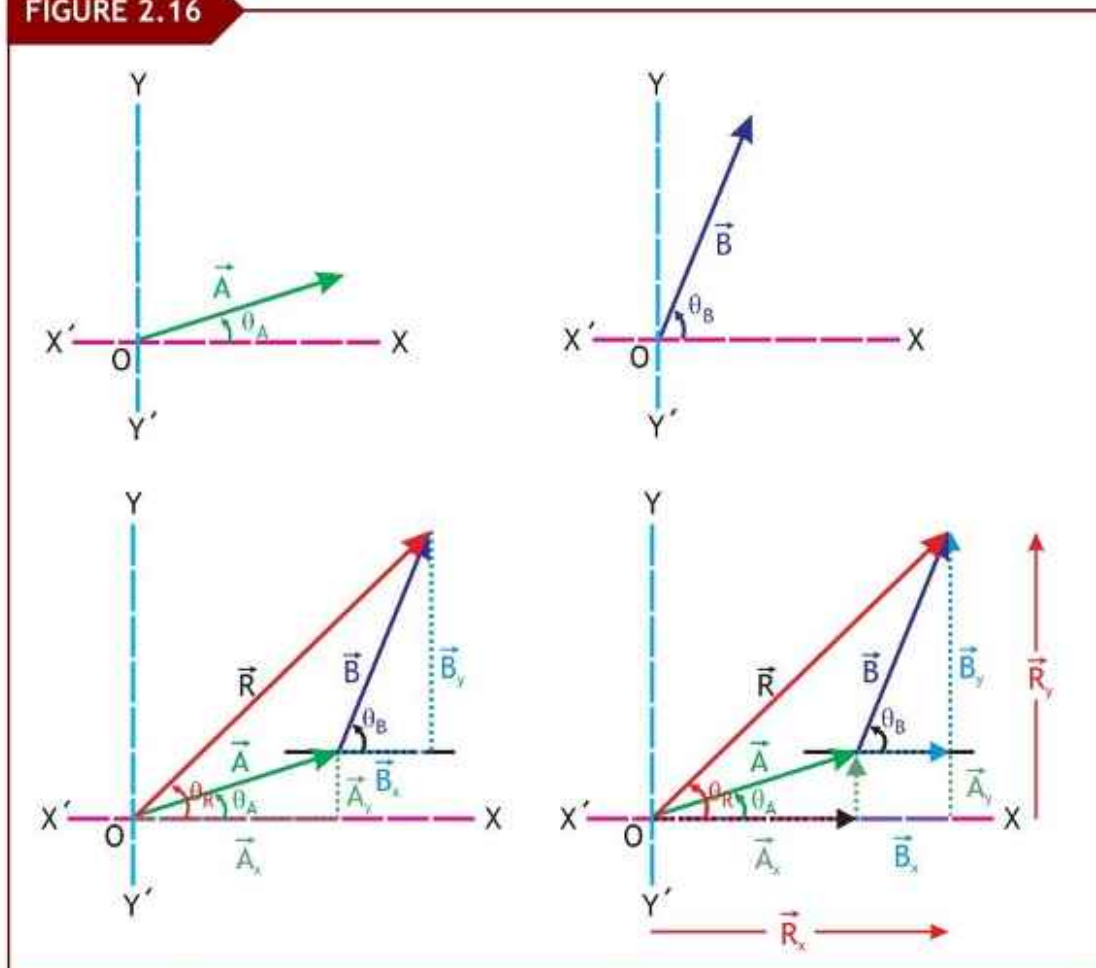
2.6 ADDITION OF VECTORS BY RECTANGULAR COMPONENTS

The analytical method for addition of vectors is called *addition of vectors by rectangular components*. This method is more mathematical in nature rather than geometrical, therefore it is regarded as more precise and accurate.

Consider two vectors $\vec{A}$ and $\vec{B}$ making angles θ_A and θ_B respectively with x-axis. Now when these vectors are added by head to tail rule we get a resultant vector $\vec{R}$ making an angle θ_R with the x-axis as shown in figure 2.16. Such that $\vec{R}$ is

$$\vec{R} = \vec{A} + \vec{B} \quad \text{--- (1)}$$

FIGURE 2.16



From the figure we see that $\vec{R}_x = \vec{A}_x + \vec{B}_x$ or $R_x \hat{i} = A_x \hat{i} + B_x \hat{i}$

$$\text{therefore } |R_x| = |A_x + B_x| \text{ ————— (2)}$$

similarly $\vec{R}_y = \vec{A}_y + \vec{B}_y$ or $R_y \hat{j} = A_y \hat{j} + B_y \hat{j}$

$$\text{therefore } |R_y| = |A_y + B_y| \text{ ————— (3)}$$

Also by rectangular components $\vec{R} = R_x \hat{i} + R_y \hat{j}$ ————— (4)

Putting equation 2 and equation 3 in equation 4 we get

$$\vec{R} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} \text{ ————— 2.5}$$

By rectangular components the magnitude is

$$|R| = \sqrt{R_x^2 + R_y^2} \text{ ————— (5)}$$

Putting equation 2 and equation 3 in equation 5 we get

$$R = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \text{ ————— 2.6}$$

By rectangular components the direction is

$$\theta_R = \tan^{-1} \frac{R_y}{R_x} \text{ ————— (6)}$$

Putting equation 2 and equation 3 in equation 6 we get

$$\theta_R = \tan^{-1} \frac{A_y + B_y}{A_x + B_x} \text{ ————— 2.7}$$

Equations (2.5), (2.6) and (2.7) are obtained for two vectors added by head to tail rule the same procedure applies for any number of vectors $\vec{A}, \vec{B}, \vec{C} \dots$ to get a resultant vector $\vec{R}$. Generally

$$\vec{R} = (A_x + B_x + C_x + \dots) \hat{i} + (A_y + B_y + C_y + \dots) \hat{j} \text{ ————— 2.8}$$

$$\text{and } |R| = \sqrt{(A_x + B_x + C_x + \dots)^2 + (A_y + B_y + C_y + \dots)^2} \text{ ————— 2.9}$$

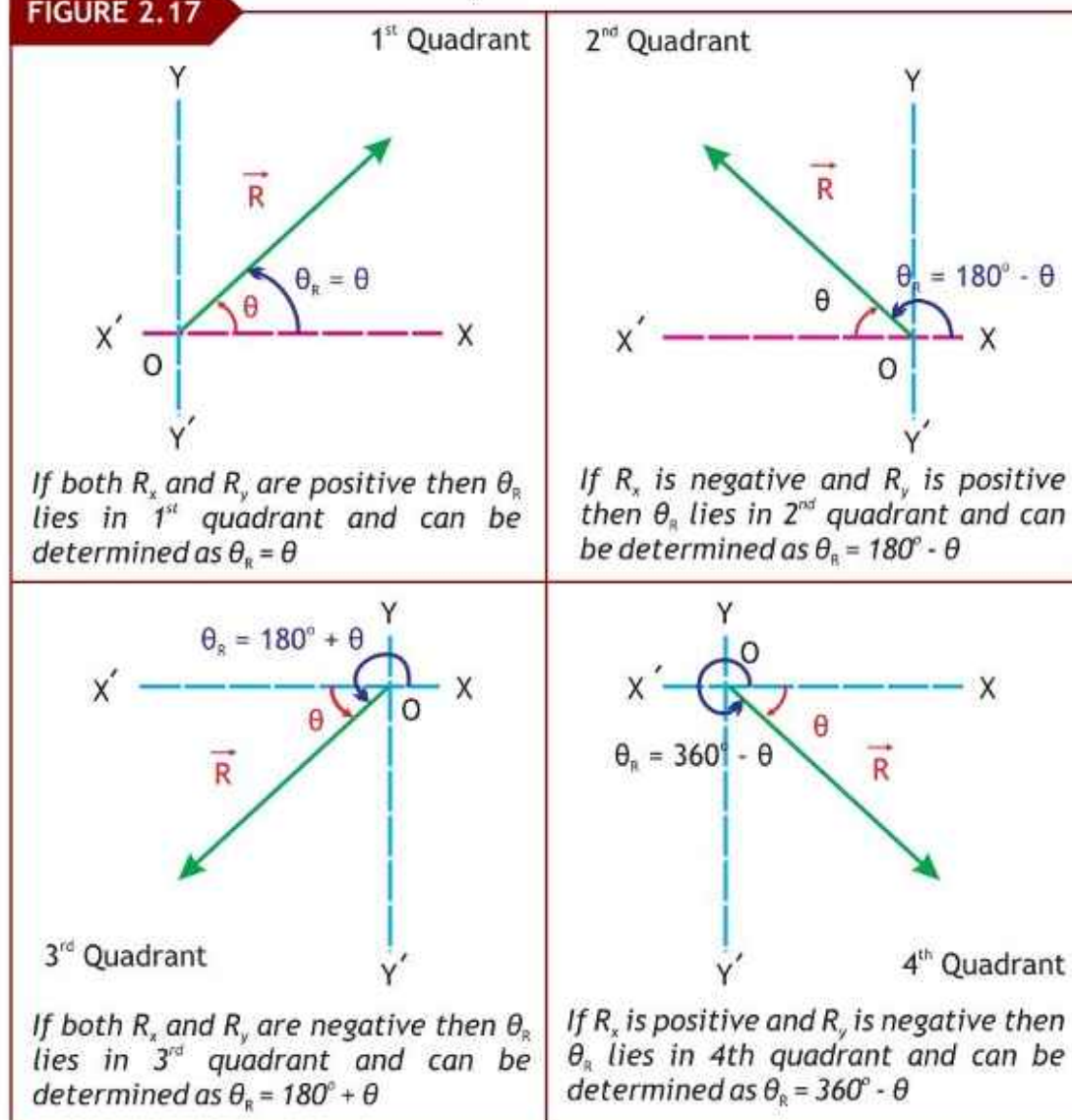
and $\theta_R = \tan^{-1} \frac{A_y + B_y + C_y + \dots}{A_x + B_x + C_x + \dots}$

2.10

2.6.1 Determination of Angle: To find the direction θ of $\vec{R}$ along x-axis in counter clockwise sense, we determine angle ' θ ' irrespective of the positive and negative signs of R_x and R_y as

$$\theta = \tan^{-1} \frac{R_y}{R_x} \text{ and then follow the steps as in figure 2.17}$$

FIGURE 2.17



Example 2.2

VECTOR ADDITION

Two forces $\vec{F}_1 = 15 \text{ N}$ making an angle $\theta_1 = 70^\circ$ with positive x-axis and Force $\vec{F}_2 = 25 \text{ N}$ making an angle $\theta_2 = 220^\circ$ with positive x-axis, act at a point, calculate the resultant force $\vec{F}_R$.

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Force $F_1 = 15 \text{ N}$, angle $\theta_1 = 70^\circ$

Force $F_2 = 25 \text{ N}$, angle $\theta_2 = 220^\circ$

REQUIRED

Resultant Force $\vec{F}_R = ?$

SOLUTION

By rectangular components

$$\vec{F}_R = \vec{F}_{Rx} \hat{i} + F_{Ry} \hat{j}$$

$$\text{or } F_R = F_{Rx} \hat{i} + F_{Ry} \hat{j}$$

$$\text{or } F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad \text{--- (1)}$$

By addition of vectors by rectangular components

$$F_{Rx} = F_{1x} + F_{2x} \quad \text{--- (2)} \quad \text{and} \quad F_{Ry} = F_{1y} + F_{2y} \quad \text{--- (3)}$$

Now from the given data we can easily determine the rectangular components of each vector as shown in Figure 2.18

For vector F_1

$$F_{1x} = F_1 \cos \theta_1$$

$$\text{or } F_{1x} = 15 \text{ N} \cos 70^\circ$$

$$\text{Hence } F_{1x} = 5.13 \text{ N} \quad \text{--- (4)}$$

$$F_{1y} = F_1 \sin \theta_1$$

$$\text{or } F_{1y} = 15 \text{ N} \sin 70^\circ$$

$$\text{Hence } F_{1y} = 14.09 \text{ N} \quad \text{--- (5)}$$

For vector F_2

$$F_{2x} = F_2 \cos \theta_2$$

$$\text{or } F_{2x} = 25 \text{ N} \cos 220^\circ$$

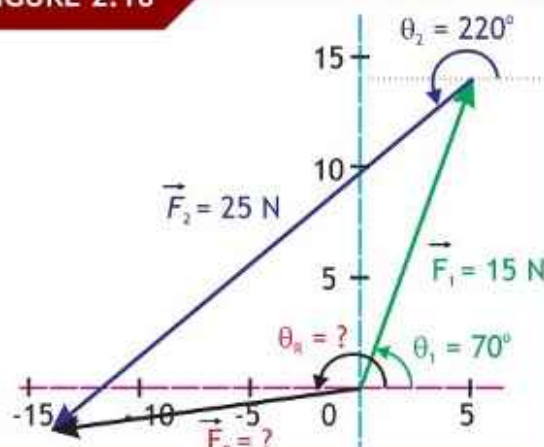
$$\text{Hence } F_{2x} = -19.15 \text{ N} \quad \text{--- (6)}$$

$$F_{2y} = F_2 \sin \theta_2$$

$$\text{or } F_{2y} = 25 \text{ N} \sin 220^\circ$$

$$\text{Hence } F_{2y} = -16.07 \text{ N} \quad \text{--- (7)}$$

FIGURE 2.18



putting values of F_{1x} and F_{2x} from equation 4 and 6 in equation 2, we get F_{Rx} as

$$F_{Rx} = 5.13\text{ N} - 19.15\text{ N} \quad \text{Therefore} \quad F_{Rx} = -14.02\text{ N} \quad \text{--- (8)}$$

putting values of F_{1y} and F_{2y} from equation 5 and 7 in equation 3, we get F_{Ry} as

$$F_{Ry} = 14.09\text{ N} - 16.07\text{ N} \quad \text{Therefore} \quad F_{Ry} = -1.98\text{ N} \quad \text{--- (9)}$$

putting values of F_{Rx} and F_{Ry} from equation 8 and 9 in equation 1, we get F_R as

$$F_R = \sqrt{(-14.02\text{ N})^2 + (-1.98\text{ N})^2} \quad \text{or} \quad F_R = 14.16\text{ N} \quad \text{--- Answer}$$

Now to determine the angle, we use $\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}}$

$$\theta = \tan^{-1} \frac{1.98\text{ N}}{14.02\text{ N}} \quad \text{Therefore} \quad \theta = 7.97^\circ$$

As F_{Rx} and F_{Ry} are both negative therefore the resultant lies in the third quadrant where

$\theta_R = 180^\circ + \theta$, therefore

$$\theta_R = 180^\circ + \theta = 180^\circ + 7.97^\circ$$

Hence $\theta_R = 187.97^\circ$ --- Answer

EXTENSION EXERCISE

In example 2.2 also calculate the resultant force for

- $\vec{F}_R = \vec{F}_1 + \vec{F}_2$ and
- $\vec{F}_R = \vec{F}_2 - \vec{F}_1$.

Assignment 2.2

VECTOR ADDITION

A Force $F_1 = 20\text{ N}$ making an angle $\theta_1 = 30^\circ$ with positive x-axis and Force $F_2 = 30\text{ N}$ making an angle $\theta_2 = 150^\circ$ with positive x-axis, acts at a point, calculate the resultant force.
($F_R = 26.46\text{ N}$, $\theta_R = 109.11^\circ$)

Example 2.3

ANGLE BETWEEN FORCES

Find the angle between two forces of equal magnitude such that the magnitude of their resultant is also equal to either of them.

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Force ' $|F_1|$ ' = $|F|$

Force ' $|F_2|$ ' = $|F|$

Resultant ' $|F_R|$ ' = $|F|$

$$|F_R| = |\vec{F}_1| = |\vec{F}_2| = |F|$$

REQUIRED

Angle ' θ ' = ?

SOLUTION

Resolving forces into their rectangular components as shown in Figure 2.19, we get

For force $F_1 = F$ and $\theta_1 = 0^\circ$

$$F_{1x} = F_1 \cos \theta_1 = F \cos 0^\circ = F$$

$$\text{and } F_{1y} = F_1 \sin \theta_1 = F \sin 0^\circ = 0$$

For force $F_2 = F$ and $\theta_2 = \theta$

$$F_{2x} = F_2 \cos \theta_2 = F \cos \theta$$

$$\text{and } F_{2y} = F_2 \sin \theta_2 = F \sin \theta$$

By addition of vectors by rectangular components

$$F_{Rx} = F_{1x} + F_{2x} = F + F \cos \theta \quad \text{--- (1)}$$

$$\text{and } F_{Ry} = F_{1y} + F_{2y} = 0 + F \sin \theta = F \sin \theta \quad \text{--- (2)}$$

By addition of vectors by rectangular components the resultant force F_R is

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} \quad \text{--- (3)}$$

Putting equation 1 and equation 2 in equation 3 we get

$$F_R = \sqrt{(F + F \cos \theta)^2 + (F \sin \theta)^2} \quad \text{since } (a + b)^2 = a^2 + b^2 + 2ab$$

$$\text{hence } F_R = \sqrt{(F^2 + F^2 \cos^2 \theta + 2FF \cos \theta) + (F^2 \sin^2 \theta)}$$

$$\text{or } F_R = \sqrt{F^2 + F^2 (\cos^2 \theta + \sin^2 \theta) + 2F^2 \cos \theta} \quad \text{as } \cos^2 \theta + \sin^2 \theta = 1$$

$$\text{hence } F_R = \sqrt{F^2 + F^2 + 2F^2 \cos \theta} \quad \text{or } F = \sqrt{2 + 2 \cos \theta}$$

$$\text{squaring } 1 = 2 + 2 \cos \theta \quad \text{or } \theta = \cos^{-1} \left(-\frac{1}{2} \right)$$

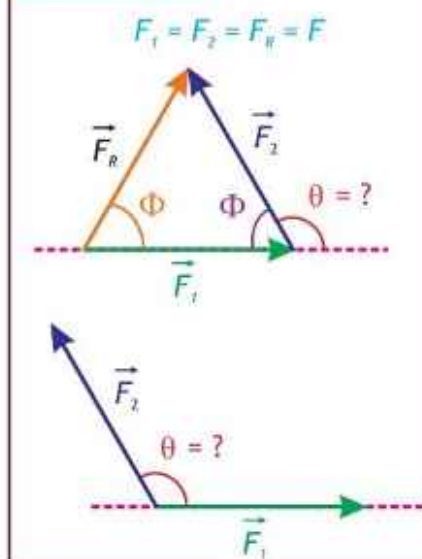
therefore

$$\theta = 120^\circ$$

Answer

Hence the angle between the two forces must be 120° .

FIGURE 2.19



Assignment 2.3**RESULTANT VECTOR**

Two forces of 20 N and 10 N are making an angle of 120° with each other. Find a single pull that would (a) replace the given forces system (b) balance the given forces system.
 ((a) $F_R = 17.32 \text{ N}$, $\theta_R = 30^\circ$, (b) $F_R = 17.32 \text{ N}$, $\theta_R = 210^\circ$)

2.7 PRODUCT OF VECTORS

Depending upon the physical nature of the given vectors, there are two ways to multiply a vector by a vector: one way produces a scalar (called the scalar product), and the other produces a new vector (called the vector product).

2.7.1 Scalar Product: When a vector is multiplied by a vector and the resultant obtained is a scalar quantity, such type of vector multiplication is called *scalar product*.

$$\text{vector} \cdot \text{vector} = \text{scalar}$$

For example if two vectors $\vec{A}$ and $\vec{B}$ are multiplied such that their result is a scalar C , then such type of multiplication is called as scalar product. Usually a DOT ($\cdot$) is placed between the two vectors to represent therefore it is also called as *dot product*.

$$\vec{A} \cdot \vec{B} = C$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

2.11

Where θ is the smaller of the angle between the two vectors $\vec{A}$ and $\vec{B}$.

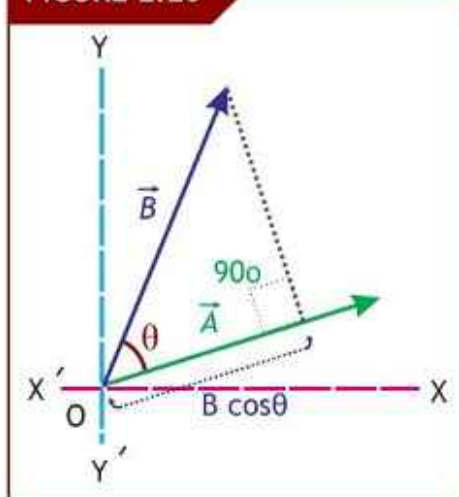
Consider the Figure 2.20, in which two vectors $\vec{A}$ and $\vec{B}$ are shown

$$\vec{A} \cdot \vec{B} = \begin{matrix} \text{(magnitude} \\ \text{of } \vec{A}) \end{matrix} \begin{matrix} \text{(Component of } \vec{B} \\ \text{parallel to } \vec{A}) \end{matrix}$$

From the figure $\vec{A} \cdot \vec{B} = A (B \cos \theta)$

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \text{--- (1)}$$

Scalar product is commutative: Scalar product of two vectors obeys the commutative law

FIGURE 2.20

Consider the Figure 2.21, Now if we want to multiply vector $\vec{B}$ with vector $\vec{A}$

$$\vec{B} \cdot \vec{A} = (\text{magnitude of } \vec{B}) (\text{Component of } \vec{A} \text{ parallel to } \vec{B})$$

From the figure $\vec{B} \cdot \vec{A} = B (A \cos \theta)$

$$\vec{B} \cdot \vec{A} = BA \cos \theta$$

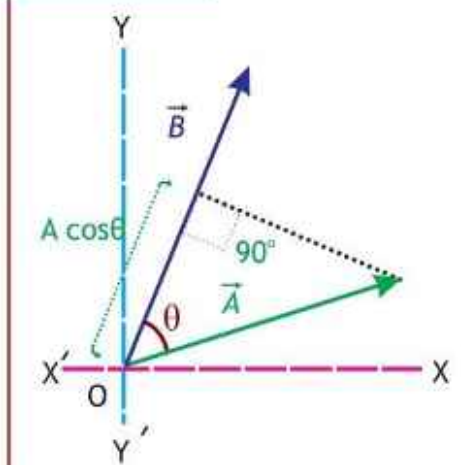
Since in magnitude $AB = BA$, therefore

$$\vec{B} \cdot \vec{A} = AB \cos \theta \quad \text{--- (2)}$$

Comparing equation 1 and equation 2

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

FIGURE 2.21



This shows that scalar product is commutative. Examples of scalar product are work, power etc.

2.7.2. Vector Product: When a vector is multiplied by a vector and the resultant obtained is a vector quantity, such type of vector multiplication is called **vector product**.

$$\text{vector} \times \text{vector} = \text{vector}$$

For example if two vectors $\vec{A}$ and $\vec{B}$ are multiplied such that their result is also a vector $\vec{C}$, then such type of multiplication is called as vector product. Usually a CROSS ($\times$) is placed between the two vectors to represent, therefore it is also called as **cross product**.

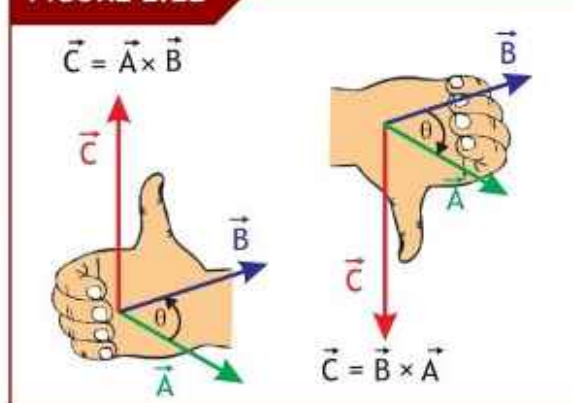
$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

2.12

Where θ is the smaller of the angle between the two vectors. Where $\hat{n}$ denote the unit vector determined by **RIGHT HAND RULE**, as shown in Figure 2.22.

FIGURE 2.22



The rule is to rotate the fingers of your right hand from the first vector to the second the stretched **THUMB** will point in the direction of the resultant vector.

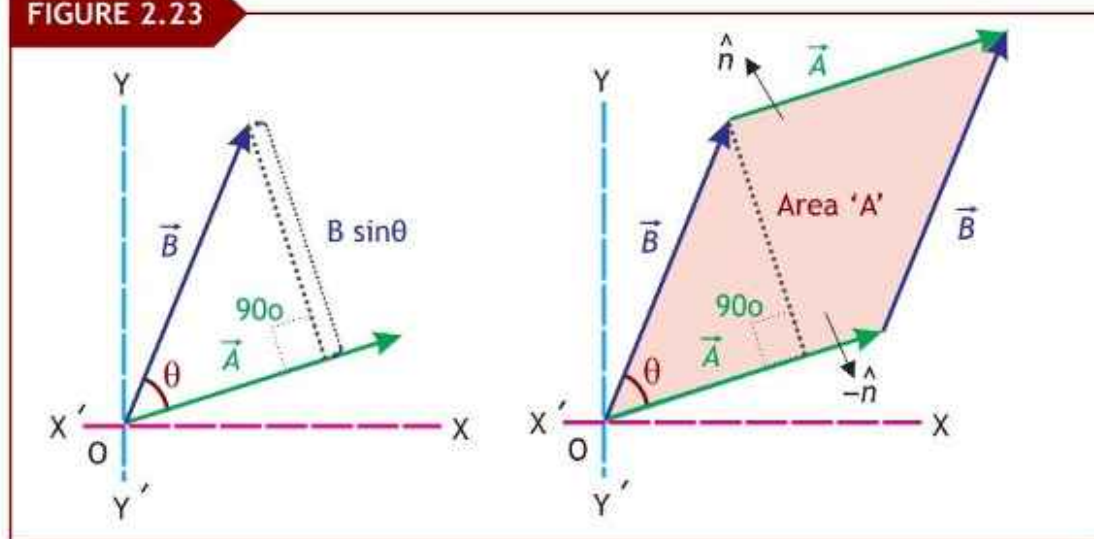
Consider the Figure 2.23, in which two vectors $\vec{A}$ and $\vec{B}$ are shown

$$\vec{A} \times \vec{B} = (\text{magnitude of } \vec{A}) \times (\text{Component of } \vec{B} \text{ perpendicular to } \vec{A})$$

$$\vec{A} \times \vec{B} = A(B \sin \theta) \hat{n}$$

$$\text{Or simply } \vec{A} \times \vec{B} = AB \sin \theta \hat{n} \quad \text{--- (1)}$$

FIGURE 2.23



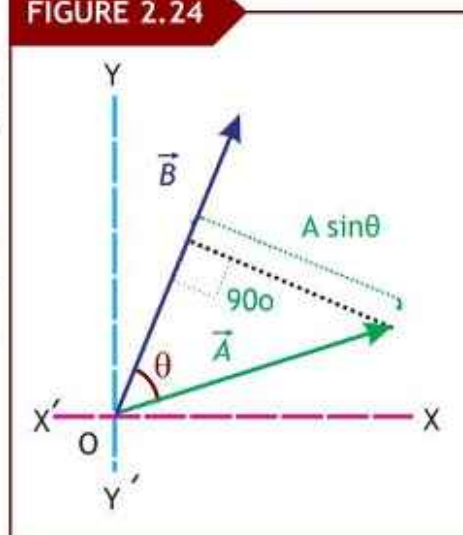
The magnitude $AB \sin \theta$, gives the area of the plane determined by the two vector $\vec{A}$ and $\vec{B}$, and the unit vector $\hat{n}$ gives the direction of the area of the plane. Here in this case it is out of the page as determined by right hand rule.

Vector product is anti commutative:

Consider the Figure 2.24, where we want to multiply vector $\vec{B}$ with vector $\vec{A}$. By definition of cross product

$$\vec{B} \times \vec{A} = (\text{magnitude of } \vec{B}) \times (\text{Component of } \vec{A} \text{ perpendicular to } \vec{B})$$

FIGURE 2.24



$$\vec{B} \times \vec{A} = B(A \sin \theta) (-\hat{n})$$

$$\text{Or simply } \vec{A} \times \vec{B} = -AB \sin \theta \hat{n} \quad \text{--- (2)}$$

Comparing equation 1 and equation 2

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

The unit vector $\hat{n}$ (negative) gives the direction of the area of the plane opposite to what it was for $\vec{A} \times \vec{B}$. Here in this case it is into the page. Hence vector product is **NOT** commutative, it is rather **ANTI COMMUTATIVE**.

Examples of vector product are torque, Angular momentum etc.

Example 2.4

PRODUCT OF VECTORS

Vector $\vec{A}$ having magnitude 3.2 makes 50° with x-axis and vector $\vec{B}$ with magnitude 5.2 makes 110° with x-axis. What is the magnitude of their dot and cross products?

GIVEN

Vector $|\vec{A}| = 3.2$, angle $\theta_1 = 50^\circ$ with x-axis

Vector $|\vec{B}| = 5.1$, angle $\theta_2 = 110^\circ$ with x-axis

REQUIRED

$$|\vec{A} \cdot \vec{B}| = ?$$

$$|\vec{A} \times \vec{B}| = ?$$

SOLUTION

The angle θ is the smaller of the angle between two vectors as shown in Figure 2.25, we have

$$\theta = \theta_2 - \theta_1 = 110^\circ - 50^\circ = 60^\circ$$

The magnitude of dot product is

$$|\vec{A} \cdot \vec{B}| = AB \cos \theta$$

putting values

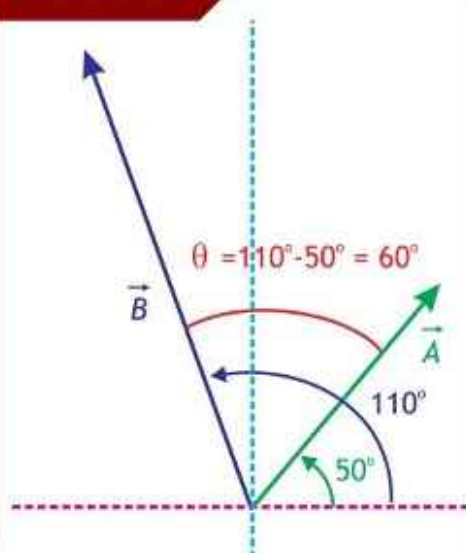
$$|\vec{A} \cdot \vec{B}| = (3.2)(5.1)(\cos 60^\circ)$$

hence

$$|\vec{A} \cdot \vec{B}| = 8.2$$

Answer

FIGURE 2.25



The magnitude of cross product is

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

putting values $|\vec{A} \times \vec{B}| = (3.2)(5.1)(\sin 60^\circ)$

therefore

$$|\vec{A} \times \vec{B}| = 14.1$$

Answer

Assignment 2.4

VECTOR MULTIPLICATION

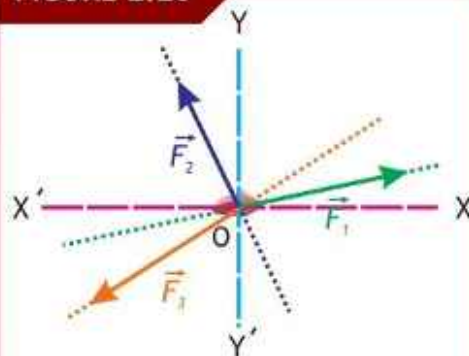
Show that $(\vec{A} \cdot \vec{B})^2 + (\vec{A} \times \vec{B})^2 = A^2 B^2$

2.8 CONCURRENT FORCES

When two or more forces are acting on a body and the line of action of these forces pass through a common point, the forces are said to be concurrent forces.

For example let three dogs are pulling a piece of meat with forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ as shown in the Figure 2.26, the forces are concurrent as their line of action passes through a common point.

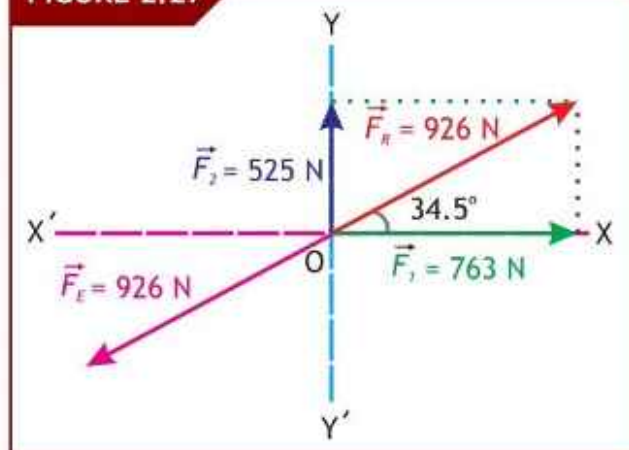
FIGURE 2.26



Two or more concurrent forces can be balanced by a single force called **equilibrant force**.

For example consider two concurrent forces $\vec{F}_1 = 763 \text{ N}$ and $\vec{F}_2 = 525 \text{ N}$ acting at right angle to each other with their resultant $\vec{F}_R = 926 \text{ N}$ at 34.5° with x-axis. The equilibrant force is equal in magnitude to that of the resultant force but it acts in the opposite direction as shown in Figure 2.27.

FIGURE 2.27



In this case, the equilibrant is 926 N at 214.5° ($180^\circ + 34.5^\circ = 214.5^\circ$).

2.9 TORQUE OR MOMENT OF FORCE

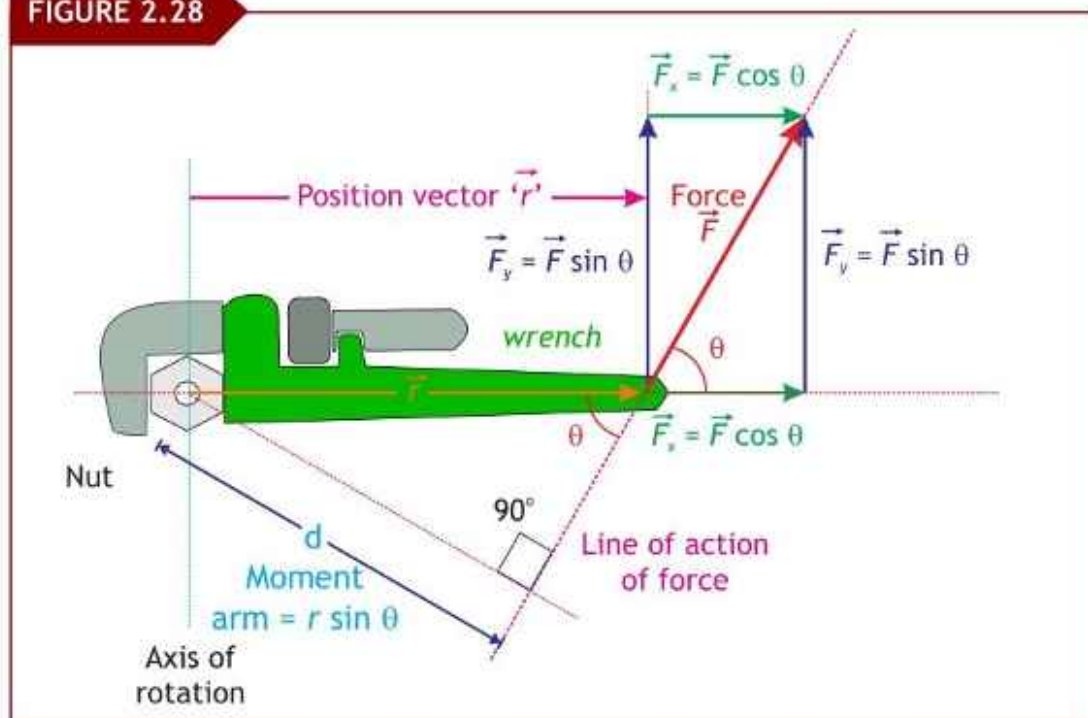
Turning effect produced in a body about a fixed point due to applied force is called torque or moment of force.

The applied force $\vec{F}$ generally can act at an angle θ with respect to the position vector $\vec{r}$ locating the point of application of the force. We define the torque $\vec{\tau}$ resulting from the force $\vec{F}$ with the expression

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{or} \quad \tau = rF \sin \theta \hat{n} \quad \text{2.13}$$

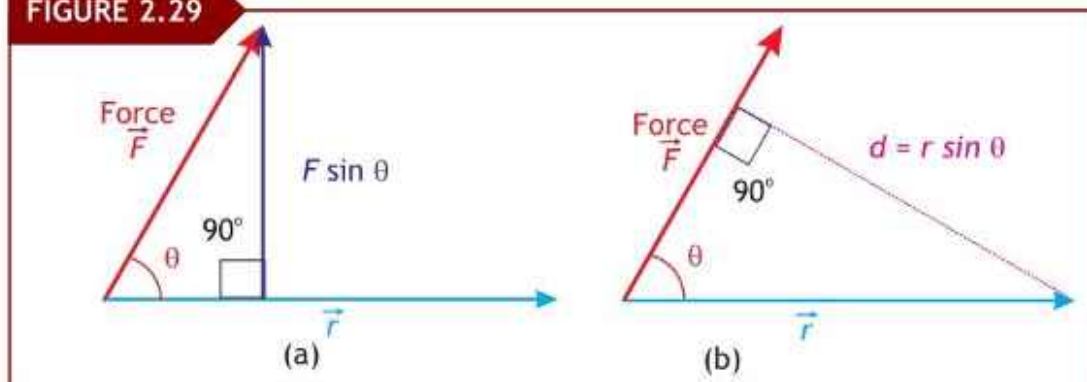
Torque is a vector quantity. Torque has units of newton metres (N m) in the SI.

FIGURE 2.28



It is very important to recognize that torque is defined only when a reference axis is specified, from which the position vector $\vec{r}$ is determined. Looking at the force components in Figure 2.28, we see that the component $F \cos \theta$ parallel to $\vec{r}$ will not cause a rotation of the wrench around the pivot point because its line of action passes right through the pivot point. So the effective component of force that produces the rotation is $F \sin \theta$.

FIGURE 2.29



Therefore the torque is the product of the magnitude of distance to the point of application of force (position vector $\vec{r}$) and the perpendicular component of the force ($F \sin \theta$), as shown in Figure 2.29 (a).

$$\tau = (r)(F \sin \theta) \hat{n} \quad \text{or} \quad \vec{\tau} = \vec{r} \times \vec{F}$$

The second way to interpret Equation is to associate the sine function with this distance as $r \sin \theta$ and multiply it with magnitude of force $\vec{F}$ as shown in Figure 2.29 (b), so that we can write

$$\tau = (F)(r \sin \theta) \hat{n} \quad \text{or} \quad \vec{\tau} = \vec{F} \times \vec{r}$$

since $d = r \sin \theta$ therefore

$$|\tau| = Fd$$

2.14

The quantity d , called the moment arm (lever arm) of the force $\vec{F}$ represents the perpendicular distance from the rotation axis to the line of action of $\vec{F}$. In some problems, this approach to the calculation of the torque is easier than that of resolving the force into components.

Factors on which torque depends: By the definition of torque we can see that, torque depends up on the following three factors

- A. Magnitude of applied force $\vec{F}$
- B. Magnitude of position vector $\vec{r}$
- C. Angle between applied force and position vector θ

Torque will be maximum when the angle between $\vec{r}$ and $\vec{F}$ is 90° (or 270°)

Torque will be minimum when the angle between $\vec{r}$ and $\vec{F}$ is 0° (or 180°)

DO YOU KNOW**Mobile Crane Counterweights - an Important Safety Factor:**

Mobile cranes are vehicles designed to lift, lower and transport heavy loads. A mobile crane moves the heavy loads.

Cranes operate on the principle of lever. The shorter end of the beam is applied by a force and the longer end (called boom) can rotate and move the load radially inward or outward, to position the object at the correct location.

Since boom is having large length even a small load lifted at its end will produce large torque and there is a danger for the crane to topple over.

To overcome such a situation, cranes have a counter weight at the other side that moves in an opposite direction from the object that is lifted. The counter weight exerts a torque on the crane in equal and opposite direction to the torque from the load. Mathematically

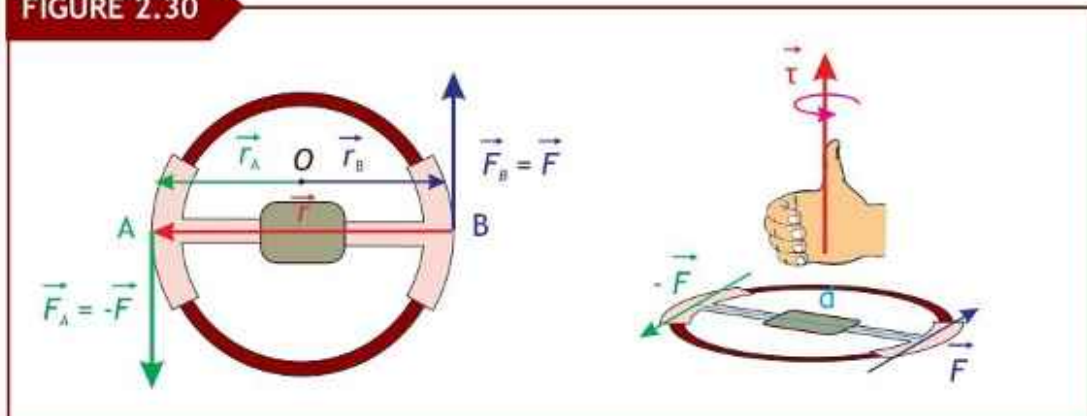
$$\tau_{\text{boom}} = \tau_{\text{counter weight}}$$



2.9.1 Moment of a Couple: Couple is defined as two parallel forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d .

Since the resultant force is zero, the only effect of a couple is to produce rotation, or if no movement is possible, there is a tendency of rotation in a specified direction. For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate as shown in Figure 2.30.

FIGURE 2.30



The moment produced by a couple is called a couple moment. We can determine its value by finding the sum of the moments of both couple forces ($\vec{F}_A = -\vec{F}$ and $\vec{F}_B = \vec{F}$) about any arbitrary point. For example, in Figure 2.30, position vectors $\vec{r}_A$ and $\vec{r}_B$ are directed from point 'O' to points 'A' and 'B' lying on the line of action of $\vec{F}_A = -\vec{F}$ and $\vec{F}_B = \vec{F}$. The couple moment determined about O is therefore

$$\vec{\tau} = \vec{r}_A \times \vec{F}_A + \vec{r}_B \times \vec{F}_B \quad \text{or} \quad \vec{\tau} = \vec{r}_A \times -\vec{F} + \vec{r}_B \times \vec{F}$$

$$\vec{\tau} = (\vec{r}_B - \vec{r}_A) \times \vec{F} \quad \text{or} \quad \vec{\tau} = \vec{r} \times \vec{F}$$

This result indicates that a couple moment is a free vector.

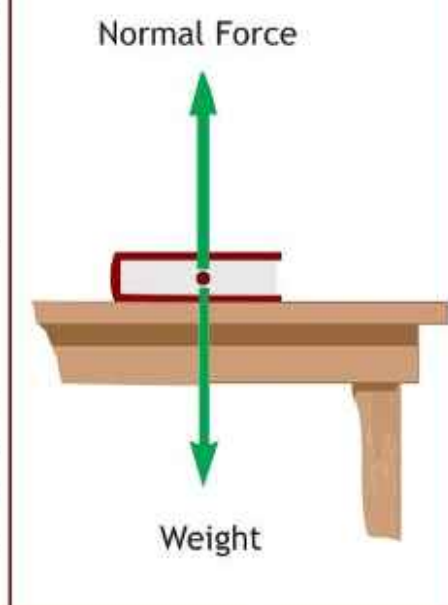
The direction and sense of the couple moment are determined by the right-hand rule. In all cases, τ will act perpendicular to the plane containing these forces.

2.10 EQUILIBRIUM

It is the state of a body in which under the action of several forces and torques acting together there is no change in translational motion as well as rotational motion is called equilibrium. An object is in equilibrium when it has zero acceleration (that is there is no change in the velocity). The study of objects in equilibrium is called **STATICS**.

2.10.1. Static equilibrium: *When a body is at rest under the action of several forces acting together the body is said to be in static equilibrium.* For example a book resting on the table is in static equilibrium, the weight mg of the book is balanced by a normal reaction force from the table surface as shown in Figure 2.31.

FIGURE 2.31

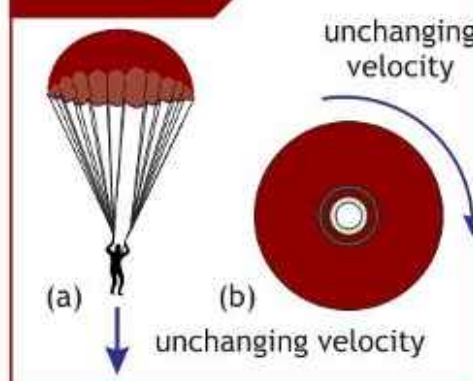


2.10.2. Dynamic equilibrium: *When a body is moving at uniform velocity under the action of several forces acting together the body is said to be in dynamic equilibrium.* It is further divided in to two types .

2.10.2.1. Dynamic Translational Equilibrium: *When a body is moving with uniform linear velocity the body is said to be in dynamic translational equilibrium.* For example a paratrooper falling down with constant velocity is in dynamic translational equilibrium as shown in Figure 2.32 (a).

2.10.2.2. Dynamic Rotational Equilibrium: *When a body is moving with uniform angular velocity the body is said to be in dynamic rotational equilibrium.* For example a compact disk (CD) rotating in CD Player with constant angular velocity is in dynamic rotational equilibrium as shown in Figure 2.32 (b).

FIGURE 2.32



2.11 CONDITIONS OF EQUILIBRIUM

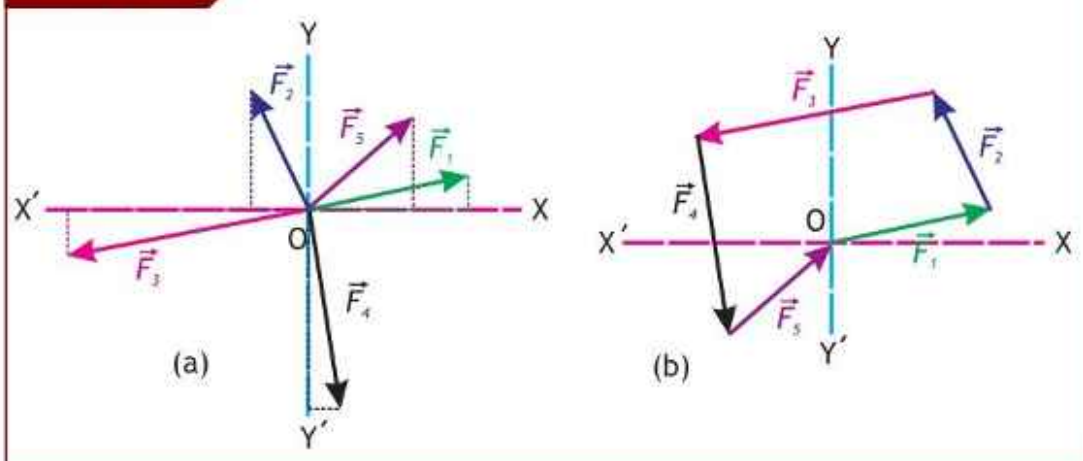
For a body to be in complete equilibrium the following two conditions must be satisfied.

2.11.1. First Condition of Equilibrium: *When the vector sum of all the forces acting on the body is ZERO then the first condition of equilibrium is satisfied.*

Mathematically if $\vec{F}_{net}$ is the sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$ then

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = 0 \quad \text{or} \quad \vec{F}_{net} = \sum_{i=1}^{i=n} \vec{F}_i = 0 \quad \text{2.15}$$

FIGURE 2.33



For example, for an object to satisfy the first condition of equilibrium, if $\vec{F}_R$ is the sum of forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ & $\vec{F}_5$ then the force polygon must close such that the resultant force F_R must be a null vector as shown in Figure 2.33 (b).

For a system of coplanar forces that lie in the x-y plane, as in Figure 2.33 (a), then each force can be resolved into its $\hat{i}$ and $\hat{j}$ components of force. For the first condition of equilibrium to be satisfied, the x and y-components of force must also be equal to zero. Hence,

$$F_R = \sum F_x \hat{i} + \sum F_y \hat{j} = 0$$

$$\sum F_x \hat{i} = 0 \quad \text{and} \quad \sum F_y \hat{j} = 0$$

2.11.2. Second Condition of equilibrium: *When the vector sum of all the Torques acting on the body is ZERO then the second condition of equilibrium is satisfied.* If $\vec{\tau}_{net}$ is the sum of torques $\vec{\tau}_1, \vec{\tau}_2, \vec{\tau}_3, \dots, \vec{\tau}_n$ then mathematically

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \dots + \vec{\tau}_n = 0 \quad \text{or} \quad \boxed{\vec{\tau}_{net} = \sum_{i=1}^n \tau_i = 0} \quad \text{2.16}$$

This idea can also be extended as

$$\sum \vec{\tau}_{Clockwise} = \uparrow \tau_{Anti-clockwise}$$

Which means that for a second condition of equilibrium to be satisfied, clockwise torques must be equal to anti-clockwise torques. Where by convention anticlockwise torques are taken as positive and clockwise torques as negative. Such that

$$\vec{\tau}_{net} = \sum \vec{\tau}_{Clockwise} + \sum \tau_{Anti-clockwise} = 0$$

2.11.3 Complete Equilibrium: When the first condition is satisfied this means that there is no net force acting on the body, so it will represent translational equilibrium only.

$$F_{net} = 0 \quad a_{net} = 0$$

therefore the first condition guarantees only translational equilibrium.

When the second condition is satisfied this means that there is no net torque acting on the body, so it will represent rotational equilibrium only.

$$\tau_{net} = 0 \quad \vec{\alpha}_{net} = 0$$

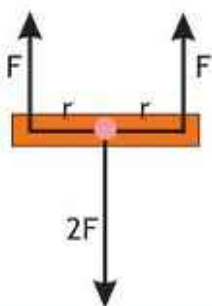
therefore the second condition guarantees only rotational equilibrium.

For complete equilibrium both the first and second conditions of equilibrium must be satisfied. For example in case of couple we have seen that the first condition is satisfied, but still the object has tendency to rotate, therefore the object is not in equilibrium with respect to rotation, hence we cannot say that object is in complete equilibrium.

The Figure 2.34 shows an object is acted upon by forces, and conditions of equilibrium are discussed.

FIGURE 2.34

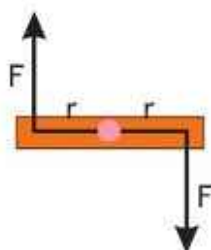
In these diagrams the axis of rotation is taken perpendicular to figure and is represented by a red dot.



(a) This body is in static equilibrium.

First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

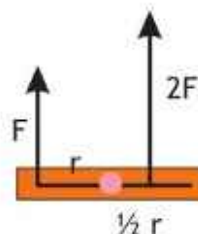
Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.



(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.

First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.



(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.

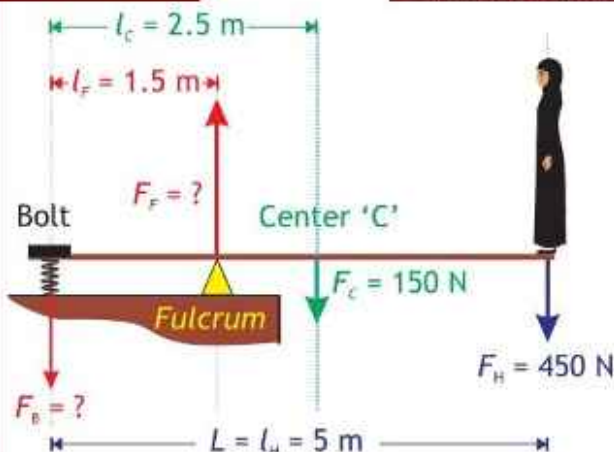
First condition NOT satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied: Net torque about the axis = 0, so body at rest has no tendency to start rotating.

To be in static equilibrium, a body at rest must satisfy both conditions for equilibrium: It can have no tendency to accelerate as a whole or to start rotating.

Example 2.5

Hina weighing 450 N is standing at the edge of the uniform diving board 5 m in length. Weight of the board is 150 N, and is bolted down at the left end, while being supported 1.50 m away by a fulcrum, as in Figure 2.35. Find the forces that the bolt and the fulcrum, exert on the board.

FIGURE 2.35**DIVING BOARD****GIVEN**

Weight of Hina $F_H = 450 \text{ N}$
 Weight of Board $F_C = 150 \text{ N}$
 Center of gravity of board $l_C = 2.5 \text{ m}$
 Distance of fulcrum $l_F = 2.5 \text{ m}$
 Length of board $L = l_H = 5 \text{ m}$

REQUIRED

Force $F_B = ?$ and Force $F_F = ?$

SOLUTION

For the axis of rotation at point A, let the torque produced by support fulcrum is τ_F , the torque produced by weight of board is τ_C and the torque produced by weight of girl Hina is τ_H .

By second condition of equilibrium $\sum \tau = 0$

$$\text{therefore } +\tau_F - \tau_C - \tau_H = 0$$

The sign convention is adopted, Hence $(F_F)(l_F) - (F_C)(l_C) - (F_H)(l_H) = 0$

$$\text{or } (F_F)(l_F) = (F_C)(l_C) + (F_H)(l_H) \quad \text{or} \quad F_F = \frac{(F_C)(l_C) + (F_H)(l_H)}{(l_F)}$$

$$\text{putting values } F_F = \frac{(150 \text{ N})(2.5 \text{ m}) + (450 \text{ N})(5 \text{ m})}{1.5 \text{ m}}$$

$$\text{or } F_F = \frac{375 \text{ Nm} + 2250 \text{ Nm}}{1.5 \text{ m}} \quad \text{or} \quad F_F = \frac{2625 \text{ Nm}}{1.5 \text{ m}}$$

hence

$$F_F = 1750 \text{ N}$$

Answer

Now the force due to bolt ' F_b ' can be easily found out by solving first condition of equilibrium along y-axis. Such that

$$\sum F_y = 0 \quad \text{or} \quad -F_B + F_F - F_C - F_H = 0$$

$$\text{or} \quad F_B = F_F - F_C - F_H$$

$$\text{putting values} \quad F_B = 1750 \text{ N} + 150 \text{ N} + 450 \text{ N}$$

hence

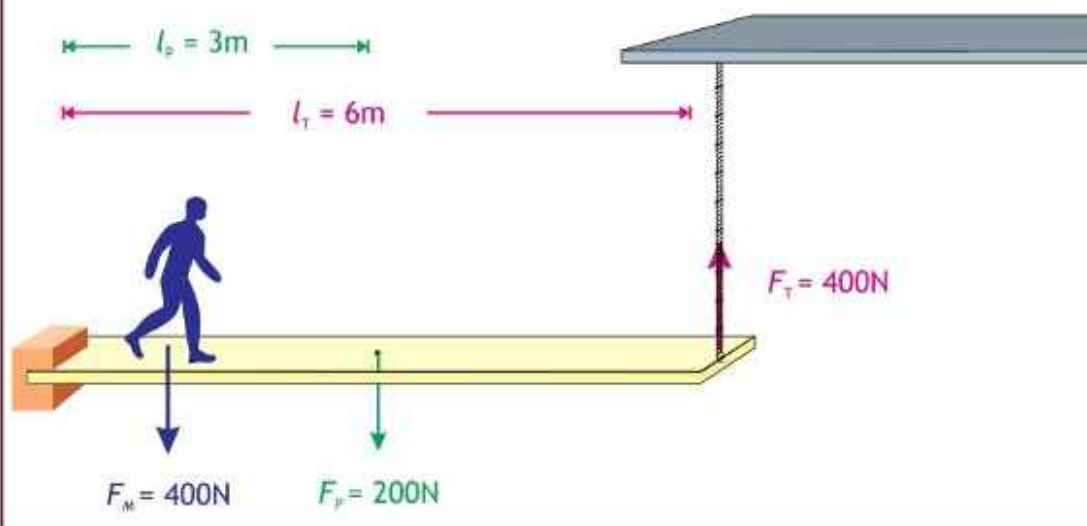
$$F_B = 1150 \text{ N}$$

Answer**EXTENSION EXERCISE**

Solve example 2.6 by taking fulcrum as the axis of rotation.

Assignment 2.5**BREAKING TENSION**

A uniform plank of weight 200 N and length 6 m is supported by a rope as shown in the figure. If the breaking tension in the rope is 400 N. How far can a boy of weight 400 N walk towards the support? (4.5 m)



Scalar: a quantity with magnitude but no direction

Vector: a quantity that has both magnitude and direction and obey vectors addition rules.

Magnitude (of a vector): the length or size of a vector; magnitude is a scalar quantity.

Direction (of a vector): the orientation of a vector in space.

Free-body diagram (FBD): includes vector arrows representing every force acting on the chosen object due to some other object, but no forces acting on other objects.

Vector addition: the rules that apply to adding vectors together.

Resultant vector: the combined effect of two or more vectors.

Component (of a vector): a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components.

Vector addition (by rectangular components): To add vectors algebraically, add their components to find the components of the sum

for example if $\vec{A} + \vec{B} = \vec{C}$ then $A_x + B_x = C_x$ and $A_y + B_y = C_y$

Product of vectors: When two vectors are multiplied (1) result obtained can be a scalar quantity we call it scalar product, and (2) result obtained can also be a vector quantity we call it vector product.

Equilibrium: the state of a body in which under the action of several forces and torques acting together there is no change in translational motion as well as rotational motion

Conditions of equilibrium: the two necessary conditions for an object to be in equilibrium are (1) the vector sum of all the forces on it must be zero, and (2) the sum of all the torques (calculated about any arbitrary axis) must also be zero. We can write

$$\vec{F}_{net} = \sum_{i=1}^{i=n} \vec{F}_i = 0 \quad \text{and} \quad \vec{\tau}_{net} = \sum_{i=1}^{i=n} \vec{\tau}_i = 0$$

EXERCISE

MULTIPLE CHOICE QUESTIONS

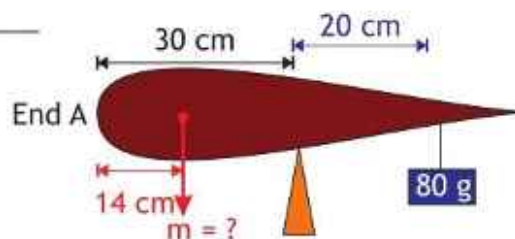
Choose the best possible answer

- 1 Two vectors lie with their tails at the same point. When the angle between them is increased by 20° their scalar product has the same magnitude but changes from positive to negative. The original angle between them was:
A. 0° B. 60° C. 70° D. 80°
- 2 The minimum number of vectors of unequal magnitude required to produce a zero resultant is
A. 2 B. 3 C. 4 D. 5
- 3 If the resultant of two vectors, each of magnitude A is also a magnitude of A, the angle between the two vectors will be:
A. 30° B. 45° C. 60° D. 120°
- 4 The magnitude of vector $A = 2\hat{i} + \hat{j} + 2\hat{k}$ is
A. 9 B. 5 C. 3 D. 1
- 5 When $F_x = 3\text{ N}$ and $F = 5\text{ N}$ then $F_y =$
A. 6 N B. 4 N C. 2 N D. 0 N
- 6 A meter stick is supported by a knife-edge at the 50-cm mark. Arif hangs masses of 0.40 kg and 0.60 kg from the 20-cm and 80-cm marks, respectively. Where should Arif hang a third mass of 0.30 kg to keep the stick balanced?
A. 20 cm B. 70 cm C. 30 cm D. 25 cm
- 7 If $A_x = 1.5\text{ cm}$, $A_y = -1.0\text{ cm}$, into which quadrant do the vector A point?
A. I B. II C. III D. IV
- 8 $\vec{A} \cdot (\vec{A} \times \vec{B}) = ?$
A. 0 B. 1 C. AB D. A^2B
- 9 Two forces of magnitude 20 N and 50 N act simultaneously on a body. Which one of the following forces cannot be a resultant of the two forces?
A. 20 N B. 30 N C. 40 N D. 70 N
- 10 If the dot product of two nonzero vectors A and B is zero then the magnitude of their cross product is ____
A. 0 B. 1 C. AB D. -AB

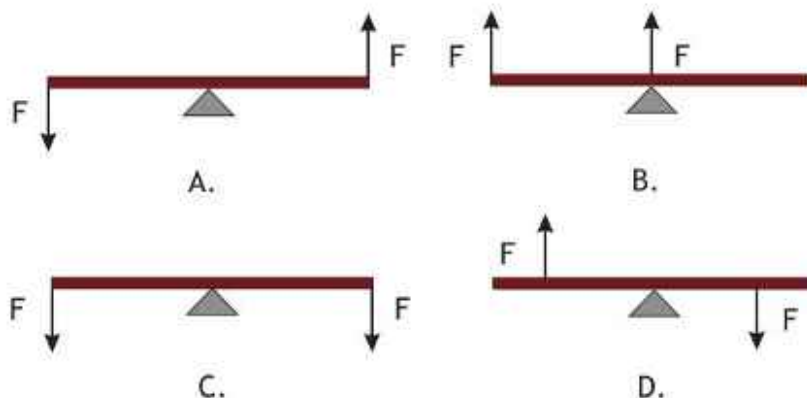
- 11 The sum of magnitudes of two forces is 16N. If the resultant force is 8N and its direction is perpendicular to minimum force then the forces are
 A. 6N and 10N B. 8N and 8N C. 4N and 12N D. 2N and 14N

- 12 Find the mass of the uneven rod shown in the figure. If its center of gravity is 14 cm from end A is ____

- A. 100 g
 B. 150 g
 C. 80 g
 D. 5 g

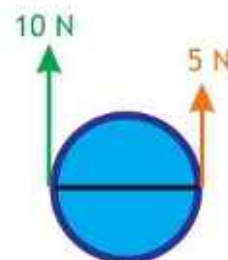


- 13 The following diagrams show a uniform rod with its midpoint on the pivot. Two equal forces F are applied on the rod, as shown in the Figure. Which diagram shows the rod in equilibrium?



- 14 For which angle the equation $|\vec{A} \cdot \vec{B}| = |\vec{A}| |\vec{B}|$ is correct.
 A. 30° B. 45° C. 60° D. 90°

- 15 What is the net torque on wheel radius 2 m as shown?
 A. 10 N anticlockwise
 B. 10 Nm anticlockwise
 C. 10 Nm clockwise
 D. 5 Nm clockwise



CONCEPTUAL QUESTIONS

Give a short response to the following questions

- 1 Is it possible to add three vectors of equal magnitude but different directions to get a null vector? Illustrate with a diagram.
- 2 The magnitudes of three vectors are 2 m, 3 m, and 5 m, respectively. The directions are at your disposal. Can these three vectors be added to yield zero? Illustrate with a diagram.
- 3 What units are associated with the unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$?
- 4 Can a scalar product of two vectors be negative? Provide a proof and give an example.
- 5 A and B are two nonzero vectors. How can their scalar product be zero? And how can their vector product be zero?
- 6 Suppose you are given a known nonzero vector A . The scalar product of A with an unknown vector B is zero. Likewise, the vector product of A with B is zero. What can you conclude about B ?
- 7 Why a particle experiencing only one force cannot be in equilibrium?
- 8 To open a door that has the handle on the right and the hinges on the left a torque must be applied. Is the torque clockwise or counterclockwise when viewed from above? Does your answer depend on whether the door opens toward or away from you?
- 9 Explain the warning 'Never use a large wrench to tighten a small bolt'.
- 10 A central force is one that is always directed toward the same point. Can a central force give rise to a torque about that point?

COMPREHENSIVE QUESTIONS

Give extended response to the following questions

- 1 How are vectors added and subtracted geometrically?
- 2 If a vector is multiplied by a positive scalar, how is the result related to the original vector? What if the scalar is zero? Negative?
- 3 What are rectangular components of a vector? How rectangular components are used to represent a vector?

- 4 Explain addition of vectors by rectangular components.
- 5 Define the dot product (scalar product) of two vectors. What geometric interpretation does the dot product have? Give examples.
- 6 Define the cross product (vector product) of two vectors. What geometric interpretation does the cross product have? Give examples.
- 7 Define torque. Show that torque is the vector product of force and position vector.
- 8 What is mechanical equilibrium? Explain different types of equilibrium.
- 9 What type of equilibrium is guaranteed by each condition of equilibrium?

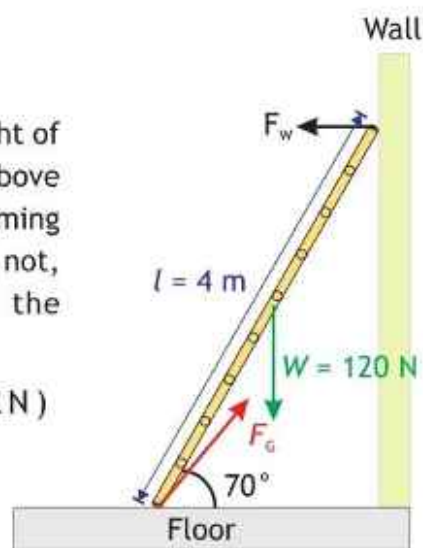
NUMERICAL QUESTIONS

- 1 A person throws a ball straight up with a speed of 12 m/s. If the bus is moving at 25 m/s, what is the velocity of the ball to an observer on ground? ($v_r = 28 \text{ m/s}$, $\theta_r = 26^\circ$)
- 2 A football leaves the foot of a punter at an angle of 54° positive x-direction) at a speed of 21 m/s. Determine the horizontal and vertical components of the velocity. (12m/s and 17m/s)
- 3 A 1.84-kg school bag hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50^\circ$. Find the tension T in the clothesline. (148 N)
- 4 Find the magnitude and direction of vector represented by the following pair of components
(a) $A_x = -2.3 \text{ cm}$, $A_y = +4.1 \text{ cm}$ (b) $A_x = +3.9 \text{ m}$, $A_y = -1.8 \text{ m}$
((a) $A=4.7$ and $\theta_A = 119.3^\circ$ (b) $A=4.3$ and $\theta_A = 335.2^\circ$)
- 5 Vector F having magnitude 5.5 N makes 10° with x-axis and vector r with magnitude 4.3 m makes 80° with x-axis. What is the magnitude of their dot and cross products? (8.1 Nm and 22.2 Nm)
- 6 The magnitude of dot and cross product of two vectors $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors. (30°)

- 7 A uniform rod 1 m long with weight 6 N can be supported in a horizontal position on a sharp edge with weights of 10 N and 15 N suspended from its ends. What is the position of point of balance? (0.41 m)

- 8 A 4.0-m-long uniform ladder with weight of 120 N leans against a wall making 70° above a cement floor as shown in Figure. Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

(122 N)



- 9 The 450-kg uniform I-beam supports the load of 220 kg as shown. Determine the reactions at the supports. (2850 N)

