

Division of Polynomials

After studying this unit, students will be able to:

- Divide a polynomial of degree up to 4 by a linear and quadratic polynomial to identify quotient and remainder.
- Recognize remainder theorem and factor theorem.
- Demonstrate and apply remainder theorem.
- Analyze and apply factor theorem to factorize a cubic polynomial.
- Apply concepts of remainder and factor theorem to real world problems.



You can use your graphing calculator to graph polynomial functions and approximate the real zeros of a functions. When using your calculator to approximate zeros, it is important to know a complete graph of the function before zooming-in on a certain point. Remember that a complete graph of the function shows all the characteristics of the graph, such as all x-and y-intercepts, relative maximum and minimum points and the behavior of the graph.



History a Mystery
Muhammad Bin Musa Al-Khawarizimi was the first Muslim mathematician who introduced Algebra and wrote a book entitled Hisab-Al-Jabr Wal Muqabala in 820 A.D. He is known as 'Father of Algebra'.

5.1 Algebraic Expressions

A statement in which variables or constants or both are connected by arithmetic operations (i.e. +, -, ×, ÷) is called an algebraic expression.

For example,

$$\frac{-5x^2 + 4}{4}, 3x + 4, x^2 - x + 9, 3(a + b) - 4, \\ 0, -5, r - \sqrt{2}t, \frac{1}{x}, \sqrt{b^2 - 4ac} \text{ etc.}$$

5.1.1 Kinds of Algebraic Expressions

Algebraic expressions are of three kinds.

1. Polynomial Expressions
2. Rational Expressions
3. Irrational Expressions

1. Polynomial Expressions (Polynomials)

Polynomials are algebraic expressions consisting of one or more terms in which exponents of the variables involved are whole numbers.

$$\text{General form of polynomial: } P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

For example,

$$0, -2, \frac{3}{4}x + \frac{3}{4}y^2z, -\sqrt{\frac{3}{9}}y^3, \sqrt{2}x^4 - \pi x^2 - \sqrt{10} \text{ etc.}$$

The expressions x^{-3} , $y^2 + \frac{1}{y^2}$, $\sqrt[3]{y^4}$, $2y^{\frac{1}{2}}$ are not polynomials because their exponents are not positive integers (whole numbers).

Types of Polynomials w.r.t. Degree

• Zero polynomial or no degree polynomial:

'0' is called a polynomial of no degree. Also, $0x^3 + 0x$ is a no degree polynomial, because coefficients are always zero in zero polynomial.

• Constant Polynomial: A polynomial having degree zero is called a constant polynomial.

e.g. $2, 2x^0, \sqrt{3}y^0, x^{2(0)} + 3x^0, \sqrt{5}$ are all constant polynomials.

• Linear Polynomial: A polynomial having degree one is called a linear polynomial.

e.g., $x, 2x - y, -7xy^0$ etc.

• Quadratic Polynomial: A polynomial having degree two is called a quadratic polynomial.

e.g., $2x^2 + 7, ax + 2xy + 3, -\frac{3}{4}xyz^0$ etc.

• Cubic Polynomial: A polynomial having degree three is called a cubic polynomial.

e.g. $9x^3 - 7x + 5, -9xzy, 3x^2y - \frac{3}{4}z$ etc.

• Bi-quadratic Polynomial: A polynomial having degree four is called Bi-quadratic Polynomial.

e.g. $x^4 - 4x - 1, 3x^2yz + 6x + 9$

Rest of the polynomials are called degree five and so on.

Enlighten Yourself



Components of an algebraic expression are:

- Numbers
- Signs of operations (+, -, ×, ÷)
- Variables ($a, b, c, \dots, x, y, z$)
- Grouping symbols —, (), { }, []



Key Facts
The highest exponent of the variable involved in a polynomial is called its degree. If more than one variables are being multiplied in terms of a polynomial, then the degree of that polynomial is the maximum sum of the exponents of the variables involved in the terms.

• Rational Expression

An algebraic expression of the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials and $Q(x) \neq 0$ (i.e. it is not a zero polynomial) is called a Rational Expression.

For example, $1\frac{1}{2}$, $\frac{3}{4x^2}$, $\frac{2x-1}{x^2+3}$, $\frac{2x+4}{x^2+5x+6}$, $\frac{x^2+x}{x^3+x}$, 5 etc.

• Irrational Expression

An algebraic expression which cannot be expressed in the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials but Q is not a zero polynomial is called an irrational expression.

For example: $\frac{x}{\sqrt{y}}$, $x^{\frac{3}{2}}y-7$, $\sqrt{x}-\frac{1}{\sqrt[3]{y}}$, $\sqrt[5]{x^2+y^2}$, $24x^{\frac{3}{2}}y^{-2}+\frac{9}{y^2}-7$ etc.

5.2 Division of Polynomial

We explain with the help of example

Divide $x^4 + 3x^2 + 1$ by $x^2 - 2x + 3$.

We use long division method.

We can write: $x^4 + 3x^2 + 1 = x^4 + 0x^3 + 3x^2 + 0x + 1$

$$\begin{array}{r}
 x^2 + 2x + 4 \\
 x^2 - 2x + 3 \overline{) x^4 + 0x^3 + 3x^2 + 0x + 1} \\
 \underline{+ x^4 - 2x^3 + 3x^2} \\
 2x^3 + 0x^2 + 0x + 1 \\
 \underline{+ 2x^3 - 4x^2 + 6x} \\
 4x^2 - 6x + 1 \\
 \underline{+ 4x^2 - 8x + 12} \\
 2x - 11
 \end{array}$$

We can say remainder is: $2x - 11$

5.3 Remainder Theorem

To understand remainder theorem, consider the following example:

Example 1: If we have two polynomials

$p(x) = x^3 - 6x^2 + 14x - 8$ and $d(x) = x - 2$, then dividing $p(x)$ by $d(x)$, we can find the quotient and remainder as follows.

$$\begin{array}{r}
 x^2 - 4x + 6 \leftarrow \text{Quotient} \\
 \text{Divisor} \rightarrow x - 2 \overline{) x^3 - 6x^2 + 14x - 8} \leftarrow \text{Dividend} \\
 \underline{+ x^3 - 2x^2} \\
 -4x^2 + 14x \\
 \underline{+ 4x^2 - 8x} \\
 6x - 8 \\
 \underline{+ 6x - 12} \\
 4 \leftarrow \text{remainder}
 \end{array}$$

So, quotient = $x^2 - 4x + 6$ and remainder = 4

Here

$$\begin{aligned}\text{quotient} \times \text{divisor} + \text{remainder} &= (x^2 - 4x + 6)(x - 2) + 4 \\ &= x(x^2 - 4x + 6) - 2(x^2 - 4x + 6) + 4 \\ &= x^3 - 4x^2 + 6x - 2x^2 + 8x - 12 + 4 \\ &= x^3 - 6x^2 + 14x - 8 \\ &= \text{dividend}\end{aligned}$$

Here, $(x - 2)$ is the divisor of $x^3 - 6x^2 + 14x - 8$.

If we put $x = 2$ in the dividend, we have,

$$\begin{aligned}p(2) &= (2)^3 - 6(2)^2 + 14(2) - 8 \\ &= 8 - 6(4) + 28 - 8 \\ &= 8 - 24 + 28 - 8 \\ &= 36 - 32 \\ &= 4 \leftarrow \text{remainder}\end{aligned}$$

Hence, $p(2)$ gives us same remainder which we have got by long division.

i.e. $p(2) = \text{remainder}$

$\therefore$ We can deduce that, if a polynomial $x^3 - 6x^2 + 14x - 8$ is divided by a polynomial $x - 2$, the remainder is 4 and the value of dividend at $x = 2$ also equals remainder.

Conclusion: If $p(x) = x^3 - 6x^2 + 14x - 8 \leftarrow \text{dividend}$

$$d(x) = x - 2 \leftarrow \text{divisor}$$

then, remainder = $p(2) = 4$

Statement: If a polynomial $p(x)$ is divided by $x - c$ (where c is a constant), then the remainder is $p(c)$.

Proof: Let $q(x)$ be the quotient and r be the remainder. When $p(x)$ is divided by $(x - c)$, then

$$p(x) = (x - c)q(x) + r \dots\dots (i)$$

Substituting $x = c$ in result (i), we have

$$\begin{aligned}p(c) &= (c - c)q(c) + r \\ &= 0 + r = r, \text{ which is the remainder.}\end{aligned}$$

Hence, $p(c)$ is the remainder when $p(x)$ is divided by $x - c$.

Example 2: Find remainder if $x^3 - 5x^2 + 7x - 6$ is divided by $x - 3$.

Solution: Let $p(x) = x^3 - 5x^2 + 7x - 6$

$$d(x) = x - 3$$

By using the Remainder Theorem,

$$p(x) = x^3 - 5x^2 + 7x - 6$$

$$\text{Remainder} = p(3)$$

$$= (3)^3 - 5(3)^2 + 7(3) - 6$$

$$= 27 - 5(9) + 21 - 6$$

$$= 27 - 45 + 21 - 6 = -3$$

We know that:

$$\text{dividend} = \text{quotient} \times \text{divisor} + \text{remainder.}$$



Key Facts
Remainder theorem provides us a helpful tool for finding remainder instead of doing long division.

Example 3: Find the value of p , when $3x^4 - 4px^2 + 5x - p$ is divided by $x + 2$ and the remainder is 4.

Solution: Let $f(x) = 3x^4 - 4px^2 + 5x - p$; $d(x) = x + 2$

By using remainder theorem,

$$\text{Remainder} = f(-2) = 3(-2)^4 - 4p(-2)^2 + 5(-2) - p$$

$$4 = 3(16) - 4p(4) - 10 - p$$

$$\begin{aligned}
 4 &= 48 - 16p - 10 - p \\
 4 &= 38 - 17p \\
 38 - 4 &= 17p \\
 34 &= 17p \\
 p &= \frac{34}{17} = 2
 \end{aligned}$$

Hence, the value of p is 2.



Pay Heed

- Degree of dividend is always greater than or equal to the degree of divisor
- Degree of remainder is always less than the degree of divisor.

5.3.1 Zeros of a Polynomial

Consider an equation, $2x + 5 = 9$

$$\begin{aligned}
 2x + 5 &= 9 \\
 2x + 5 - 5 &= 9 - 5 \\
 2x &= 4 \\
 \text{or } x &= 2
 \end{aligned}$$

Here '2' is called the root of $2x + 5 = 9$, as it satisfies the equation.

Hence, the roots of a polynomial $p(x)$ means the values of x that satisfies $p(x) = 0$. These roots are called 'zeros of the polynomial'.



Memory Plus

The zeros of $p(x) = x^2 - 9$ are the same as the solution to the equation $x^2 - 9 = 0$.

Definition: Let $P(x)$ be any polynomial and 'a' is called zero of polynomial if and only if $P(a) = 0$.

The values of x which satisfy $p(x) = 0$ are called 'Zeros of the Polynomial $p(x)$ '.

For example 5 and -5 are the zeros of the polynomial $p(x) = x^2 - 25$, because

$$\begin{aligned}
 p(5) &= (5)^2 - 25 \\
 &= 25 - 25 = 0
 \end{aligned}$$

and

$$\begin{aligned}
 p(-5) &= (-5)^2 - 25 \\
 &= 25 - 25 = 0
 \end{aligned}$$

Basically, when we are finding zeros of a polynomial, we are looking for those values of x which cause the values of polynomial equal to zero.

Example 4: Is -3 a zero of polynomial $p(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$?

Solution: '-3' will be a zero of $p(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$, If $p(-3) = 0$

$$\begin{aligned}
 p(-3) &= 2(-3)^4 + 7(-3)^3 - 4(-3)^2 - 27(-3) - 18 \\
 &= 2(81) + 7(-27) - 4(9) - 27(-3) - 18 \\
 &= 162 - 189 - 36 + 81 - 18 \\
 &= 243 - 243 = 0
 \end{aligned}$$

Hence, '-3' is the zero of $p(x)$.

Example 5: If zeros of a polynomial are 0, 6, -1, find the polynomial.

Solution: Let required polynomial be $g(x)$. Then setting $x = 0, 6, -1$, we have

$$\begin{aligned}
 g(x) &= (x - 0)(x - 6)(x + 1) \\
 &= x(x^2 - 6x + x - 6) \\
 &= x^3 - 5x^2 - 6x \text{ is the required polynomial}
 \end{aligned}$$

Example 6: If one zero of $g(x) = 2x^3 + x^2 - 2x - 1$ is $-\frac{1}{2}$, find its other zeros.

Solution: If one zero of the given polynomial is $-\frac{1}{2}$, then its other zeros can be found by

factorizing it. Setting $x = -\frac{1}{2}$, so that $2x + 1$ is the factor of $g(x)$.

First divide $g(x)$ by $2x + 1$ for getting its quadratic factor.

$$\begin{array}{r}
 x^2 - 1 \\
 2x + 1 \overline{) 2x^3 + x^2 - 2x - 1} \\
 \underline{\pm 2x^3 \pm x^2} \\
 -2x - 1 \\
 \underline{-2x - 1} \\
 + + \\
 \hline
 0
 \end{array}$$

$$\text{So, } 2x^3 + x^2 - 2x - 1 = (2x + 1)(x^2 - 1)$$

$$= (2x + 1)(x - 1)(x + 1) \quad \leftarrow \text{factorizing}$$

Hence, its other zeros are 1 and -1. $\leftarrow$ setting $x - 1 = 0$ and $x + 1 = 0$
i.e. $x = 1$ and $x = -1$

5.4 Factor Theorem

Factor Theorem is a result of Remainder Theorem and is based on the same reasoning.

Statement: A polynomial $p(x)$ has a factor $x - c$, if and only if $p(c) = 0$.

Proof: Let $q(x)$ be the quotient and r be the remainder when $p(x)$ is divided by $(x - c)$ then, $p(x) = (x - c)q(x) + r$ (i)

If $x - c$ is a factor of $p(x)$, then $r = 0$.

Now by Remainder Theorem, $r = p(c)$

$$\therefore \text{remainder} = p(c) = 0$$

Conversely, if $p(c) = 0$, that means, remainder = 0

Therefore, result (i) reduces to

$$p(x) = (x - c)q(x)$$

$\therefore (x - c)$ is a factor of $p(x)$.

Hence, $(x - c)$ will be the factor of $p(x)$ if and only if $p(c) = 0$.

Key Facts

- Every factor is a divisor but every divisor is not a factor of polynomial.
- Factor of a polynomial divides it completely.

Difference between remainder and factor theorem: The remainder theorem tells us that for any polynomial $P(x)$, if you divide it by the binomial $x - a$, the remainder is equal to the value of $P(a)$. The factor theorem tells us that if a is a zero of the polynomial $P(x)$, then $(x - a)$ is a factor of $P(x)$ and vice-versa.

Example 7: Show that $y - 1$ is a factor of $y^4 - 24y^2 - 13y + 36$.

Solution: Let $f(y) = y^4 - 24y^2 - 13y + 36$ (i)

By Factor Theorem, $y - 1$ will be a factor of $f(y)$, if $f(1) = 0$.

So, first we find $f(1)$.

$$f(1) = (1)^4 - 24(1)^2 - 13(1) + 36 \longrightarrow \text{substituting } y = 1 \text{ in (i)}$$

$$f(1) = 1 - 24 - 13 + 36$$

$$= 37 - 37$$

$$= 0$$

i.e. remainder = 0

Hence, $y - 1$ is a factor of $y^4 - 24y^2 - 13y + 36$.

Example 8: Determine the value of k for which $x + 3$ is a factor of $(x + 2)^5 + (3x + k)$.

Solution: Let $f(x) = (x + 2)^5 + (3x + k)$... (i) $d(x) = x + 3$

As $x + 3$ is a factor of $f(x)$, so by Factor Theorem,

$$f(-3) = 0, \text{ substitute } x = -3 \text{ in (i)}$$

$$\text{i.e. } (-3 + 2)^5 + [3(-3) + k] = 0$$

$$(-1)^5 + (-9 + k) = 0$$

$$-1 - 9 + k = 0$$

$$-10 + k = 0 \text{ or } \boxed{k = 10}$$

Here for $k = 10$, $(x + 3)$ is a factor of $(x + 2)^5 + (3x + k)$

Exercise 5.1

- Find the remainder of the following by using 'Remainder Theorem' when
 - $2x^3 + 3x^2 - 4x + 1$ is divided by $x + 2$.
 - $x^4 + 2x^3 - x^2 + 2x + 3$ is divided by $x - 2$.
- Show that $x - 3$ is a factor of $x^3 - 2x^2 - 5x + 6$.
- Decide whether $x - 3$ is a factor of $x^3 - 2x^2 - 5x + 1$ or not.
- If $4y^3 - 4y^2 + 10 + 2y$ is completely divisible by any of its factor such that the quotient is $4y^2 - 8y + 10$, then find other factor.
- Find the value of ' q ' if $x^3 + qx^2 - 7x + 6$ is exactly divisible by $(x + 1)$.
- Find the value of ' m ' in the polynomial $2x^3 + 3x^2 - 3x - m$ which when divided by $x - 2$ gives the remainder of 16.
- Check whether 1 and -2 are the zeros of $x^3 - 7x + 6$.
- Find zeros of the polynomial $2x^3 + 3x^2 - 11x - 6$.
- Express $f(x) = x^3 - x^2 - 14x + 8$ in the form $f(x) = (x - a)q(x) + r$, where $a = 4$.
- A rectangular room has a volume of $(x^3 + 11x^2 + 34x + 24)$ cubic feet. The height of the room is $(x + 1)$ feet. Find the area of its floor.
(Hint: Volume of room = area of floor $\times$ height.)

5.5 Factorization of a Cubic Polynomial

With the help of Factor Theorem together with some intelligent guessing, we can factorize polynomials of higher degree. Consider a cubic polynomial,

$$f(x) = x^3 - 2x^2 - 5x + 6$$

The process of factorizing above polynomial is explained as under.

Step-I: We can substitute, ± 1 , ± 2 , ± 3 and ± 4 to trace the roots by using factor theorem.

First try, $x - 1$.

Here, $x - 1$ will be the factor of $f(x)$ if $f(1) = 0 \longrightarrow$ by Factor Theorem

$$\begin{aligned} f(1) &= (1)^3 - 2(1)^2 - 5(1) + 6 \\ &= 1 - 2 - 5 + 6 \\ &= 7 - 7 = 0 \end{aligned}$$

Hence, $(x - 1)$ is a factor of $f(x)$.

Step-II: Divide $x^3 - 2x^2 - 5x + 6$ by $x - 1$ to find its other factor $q(x)$.

$$\begin{array}{r} x^2 - x - 6 \\ x-1 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{+ x^3 - x^2} \\ -x^2 - 5x + 6 \\ \underline{-x^2 + x} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

$$\text{So, } (x^3 - 2x^2 - 5x + 6) = (x - 1)(x^2 - x - 6).$$

Step-III: Factorize quadratic factor (if possible) for other linear factors.

$$\begin{aligned} &x^3 - 2x^2 - 5x + 6 \\ &= (x - 1)(x^2 - x - 6) \\ &= (x - 1)(x^2 - 3x + 2x - 6) \\ &= (x - 1)[x(x - 3) + 2(x - 3)] \quad (-1)(+2)(-3) = +6 \\ &= (x - 1)(x - 3)(x + 2) \end{aligned}$$

Hence,

$$x^3 - 2x^2 - 5x + 6 = (x - 1)(x - 3)(x + 2)$$

Example 9: If two linear factors of the polynomial $2y^3 + y^2 - 8y - 4$ are $(2y + 1)$ and $(y - 2)$, find its third factor. unknown factor

Solution: $2y^3 + y^2 - 8y - 4 = (2y + 1)(y - 2)(X)$
 $2y^3 + y^2 - 8y - 4 = (2y^2 + y - 4y - 2)(X)$
 $2y^3 + y^2 - 8y - 4 = (2y^2 - 3y - 2)(X)$
 $\frac{2y^3 + y^2 - 8y - 4}{2y^2 - 3y - 2} = (X)$

$$\begin{array}{r}
 y + 2 \\
 2y^2 - 3y - 2 \overline{) 2y^3 + y^2 - 8y - 4} \\
 \underline{+ 2y^3 - 3y^2 - 2y} \\
 4y^2 - 6y - 4 \\
 \underline{+ 4y^2 - 6y - 4} \\
 0
 \end{array}$$

Hence, missing factor is $(y + 2)$.

$$\therefore 2y^3 + y^2 - 8y - 4 = (2y + 1)(y - 2)(y + 2)$$

Example 10: Factorize $x^3 - 5x - 2$ by factor theorem.

Solution: Let $f(x) = x^3 - 5x - 2$

Step-I For $x = -2$, $f(-2) = (-2)^3 - 5(-2) - 2$
 $= -8 + 10 - 2 = 0$

Hence, $x + 2$ is one of the factors of $x^3 - 5x - 2$.

Step-II: Now, we divide $x^3 - 5x - 2$ by $(x + 2)$.

So, $x^3 - 5x - 2 = (x + 2)(x^2 - 2x - 1)$

Hence, $x^3 - 5x - 2 = (x + 2)(x^2 - 2x - 1)$

$$\begin{array}{r}
 x^2 - 2x - 1 \\
 x + 2 \overline{) x^3 - 5x - 2} \\
 \underline{+ x^3} + 2x^2 \\
 - 2x^2 - 5x - 2 \\
 \underline{- 2x^2 - 4x} \\
 - x - 2 \\
 \underline{- x - 2} \\
 0
 \end{array}$$

Key Facts



By inspecting, if $f(x)$ is of degree three, we would expect it to have three linear factors at most, so that $f(x) = (x + a)(x + b)(x + c)$, where a , b and c can be positive or negative numbers. Also, by multiplying the last term of each factor, a , b and c numerically equals the last term of the polynomial.

Exercise 5.2

Factorize the following by using factor theorem.

- $y^3 - 7y - 6$
- $2x^3 - x^2 - 2x + 1$
- $2x^3 + 5x^2 - 9x - 18$
- $3x^3 - 5x^2 - 36$
- $t^3 + t^2 + 3t - 5$
- If $(x - 2)$ is one of the factor of $2x^3 - 15x^2 + 16x + 12$, find its other factors.
- Factorize $2x^3 - 15x^2 + 27x - 10$ if $\frac{1}{2}$ is one of its zero.
- If $h(x) = 4x^3 + 4x^2 + 73x + 36$ and $h\left(\frac{-1}{2}\right) = 0$, then factorize $h(x)$.

5.6 Applications of Remainder Theorem

If you give 10 pencils to five students out of 11, each student will get 2 pencils. Only one pencil will remain with you and this leftover 1 pencil is called the remainder. 11 is the dividend, 5 is the divisor, 2 is the quotient and 1 is the remainder.

A remainder theorem formula is a powerful tool that can be used to solve a variety of mathematical problems. A remainder formula is used to differentiate the polynomials.

Suppose Nasir hits a high fast ball straight up over home plate. The function that describes the height of the ball after t seconds is

$$h(t) = -16t^2 + 80t + 5$$

The roots of the function tell us that at what times the ball is theoretically in the ground. When $t = 0$, the height of the ball is 5 feet. This is the point at which he hits the ball.

Suppose we find the height of the ball after 4 seconds.

$$h(t) = -16t^2 + 80t + 5$$

$$h(4) = -16(4)^2 + 80(4) + 5 \quad \text{replace } t \text{ with } 4.$$

$$h(4) = 69$$

After 4 seconds, the height of the ball is 69 feet.

Notice that the value of $h(4)$ is the same as the remainder when polynomial is divided by $t - 4$.

Example 11: The volume of a rectangular solid is 72 cubic units. The width is twice the height and the length is 7 units more than the height. Find the dimensions of the solid.

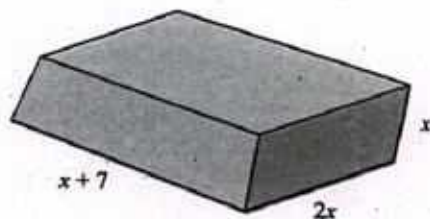
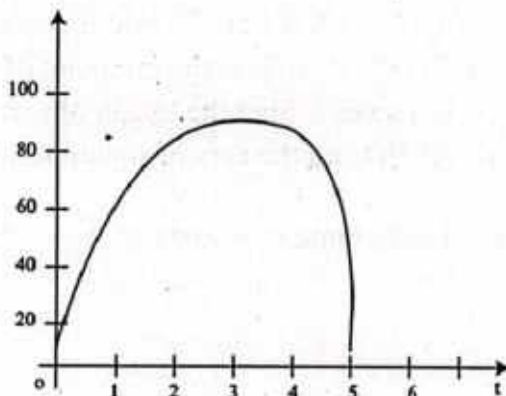
Let x be the height of the solid.

$$\text{volume} = (2x)(x+7)(x)$$

$$72 = 2x^3 + 14x^2$$

$$x^3 + 7x^2 - 36 = 0$$

Trace the possible zeros.



$$(2)^3 + 7(2)^2 - 36 = 0$$

$$0 = 0$$

So, the zero is 2.

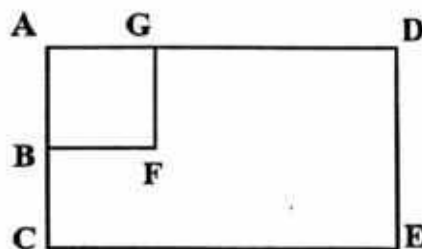
$$\text{height} = x = 2$$

$$\text{width} = 2x = 4$$

$$\text{length} = x + 7 = 9$$

Exercise 5.3

1. The volume of a wooden box is 120 cubic centimeters. The length of the box is 7 cm longer than its height, and the width is 3 cm shorter than its height. Find the dimensions of the box.
2. In the cricket match season, the number of tickets sold during the match can be modeled by $t(x) = x^3 - 12x^2 + 48x + 74$, where x is the number of games played. Find the number of tickets sold during the twelfth game of the cricket season.
3. A rectangular solid has a volume of 144 cubic units. The width is twice the height and the length is 2 units more than the width. Find the dimensions of the solid.
4. The volume of a rectangular solid is 2475 cubic units. The length of the box is three units more than twice the width of the box. The height is 2 units less than width. Find the dimensions of the box.
5. The area of rectangle ACED is represented by $6x^2 + 38x + 56$. Its width is represented by $2x + 8$. Point B is the midpoint of AC. ABFG is a square. Find the length of rectangle ACED and the area of square ABFG.
6. The volume of the box is $y^3 - 2y^2 - y + 2$. If the length of one side is $y - 2$, find the length of the other two sides.



1. Encircle the correct option in the following.

(i) Factors of $-2 - x + x^2$ are:

- (a)
- $(x-2)(x-1)$
- (b)
- $(x+1)(x+2)$
- (c)
- $(x+2)(x-1)$
- (d)
- $(x+1)(x-2)$

(ii) Divide $9y^2 + 9y - 10$ by $3y - 2$, then remainder is:

- (a) 0 (b) 1 (c) 2 (d) 3

(iii) $\frac{x^2 - x - 9}{x - 3} = x + 2 + \frac{?}{x - 3}$

- (a) -27 (b) -3 (c)
- $\frac{3}{x-3} + x + 2$
- (d) 3

(iv) If $3x^3 - 2x^2 + 5$ is divided by $x + 1$, then $x + 1$ will be its:

- (a) divisor as well as factor (b) dividend
-
- (c) quotient (d) remainder

(v) If 2 is a zero of the polynomial $x^3 + 5x^2 - 4x + k$, then the value of k will be:

- (a) -4 (b) -20 (c) 20 (d) 0

(vi) If $x - b$ is the factor of $q(x)$, then $q(b)$ is:

- (a) factor (b) divisor (c) remainder (d) dividend

(vii) If the expression $2x^3 + 3px^2 - 4x$ has a remainder of 4 when divided by $x + 2$, then $p =$

- (a) -2 (b) 1 (c) -1 (d) 0

(viii) If $f(x)$ is divided by $x - 2$, then remainder is 12. What is $f(2)$?

- (a) -12 (b)
- $f(-2)$
- (c) 12 (d) zero

2. $(64y^3 - 8) \div (4y - 2)$ 3. $(125y^3 - 8) \div (5y - 2)$

4. Is $3y - 2$ a factor of $6y^3 - y^2 - 5y + 2$?

5. If zeros of a polynomial are $4, \frac{3}{5}, -2$, find the polynomial.

6. Find the value of 'k' so that the remainder upon dividing $(x^2 + 8x + k)$ by $(x - 4)$ is zero.

7. Suppose that the quotient upon dividing one polynomial by another is

$$3x^2 - x + 32 - \frac{121}{x + 4}$$

What is the dividend?

8. If two linear factors of the polynomial $y^3 + 6y^2 - y - 30$ are $(y - 2)$ and $(y + 3)$ find its third factor.