

PERMUTATION AND COMBINATION

After studying this unit, students will be able to:

- Explain and solve problems that involve the fundamental counting principle.
- Explain and solve problems that involve permutations.
- Explain and solve problems that involve combinations.
- Apply the concept of permutation and combination to real world problems such as (cryptography, estimating the odds of winning a lottery, calculating the number of possible DNA sequences or protein structures, choosing different set of songs for certain occasions).

Fundamental Principle of Counting

The history of counting is as old as the humanity is. Counting is a basic tool. How to count correctly and quickly is very important in our daily life. For this purpose, we develop the techniques for computing number of elements in sets without listing them. To determine a general rule, we consider a coin and a dice. A coin has two outcomes that is head and tail while a die has six outcomes 1, 2, 3, 4, 5 and 6. Then the outcomes of tossing a coin and rolling a die are $(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)$.

These outcomes are 12 in number. We can also find this number 12 without listing all outcomes. We know that a coin has two outcomes while a die has six outcomes. So, the total outcomes are the product of values of two things that is $2 \times 6 = 12$.



6.1 Rule of Product

If A can happen in m ways and B can happen in n ways then the pair (A, B) can happen in $m \times n$ or mn ways. If we have three objects A, B and C which can happen in m , n and p ways respectively. Then the triplet (A, B, C) can be written in $m \times n \times p$ or mnp ways. In this unit, we will develop formulae and techniques for counting the number of objects. Then these formulae will be used to calculate the number of arrangements of the objects.

6.2 Factorial Notation

Factorial notation was introduced by French mathematician Christian Kramp in 1808. Factorial of an integer n is denoted by

$$n! = n(n-1)(n-2)\dots 3 \times 2 \times 1$$

and is defined as the product of all the positive integers from n down to 1.

Factorials for First 6 Numbers

$$0! = 1, 1! = 1$$

$$2! = 2 \times 1 = 2, 3! = 3 \times 2 \times 1 = 6$$

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

Key Facts

All the scientific calculators have a factorial key commonly locate on the top of x^{-1} have sign $x!$ is used with shift button. For example, $4! = 24$.



Example 1: Simplify the following.

$$(i) \frac{8!}{5!} \quad (ii) \frac{1}{4! \times 3!} + \frac{7}{5! \times 2!} \quad (iii) \frac{(n+1)!}{(n-2)!}$$

Solution:

$$(i) \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 8 \cdot 7 \cdot 6 = 336$$

$$(ii) \frac{1}{4! \times 3!} + \frac{7}{5! \times 2!} = \frac{5}{5 \cdot 4! \times 3!} + \frac{7 \times 3}{5! \times 3 \cdot 2!} = \frac{5}{5! \times 3!} + \frac{21}{5! \times 3!} = \frac{5+21}{120 \times 6} = \frac{26}{720} = \frac{13}{360}$$

$$(iii) \frac{(n+1)!}{(n-2)!} = \frac{(n+1)n(n-1)(n-2)!}{(n-2)!} = (n+1)n(n-1) = n^3 - n$$

Point to be Noted

$$2! \times 4! = (2 \times 1)(4 \times 3 \times 2 \times 1) = 48$$

$$(2 \times 4)! = 8! = 40320$$

In general; $(m \times n)! \neq m! \times n!$

$$(m+n)! \neq m! + n!$$

Example 2: Write the following in factorial form.

$$(i) \frac{13 \cdot 12 \cdot 11 \cdot 9}{6 \cdot 5 \cdot 4}$$

$$(ii) \frac{(n-4)(n-3)(n-1)}{n(n-1)}$$

Solution:

$$(i) \frac{13 \cdot 12 \cdot 11 \cdot 9}{6 \cdot 5 \cdot 4} = \frac{13 \cdot 12 \cdot 11 \cdot 10! \cdot 9 \cdot 8!}{6 \cdot 5 \cdot 4 \cdot 3!} \times \frac{3!}{10! \cdot 8!} = \frac{13! \cdot 9! \cdot 3!}{10! \cdot 8! \cdot 6!}$$

$$(ii) \frac{(n-4)(n-3)(n-2)}{n(n-1)} = \frac{(n-2)(n-3)(n-4)(n-5)!}{n(n-1)(n-2)!} \times \frac{(n-2)!}{(n-5)!} = \frac{(n-2)!(n-2)!}{n!(n-5)!} = \frac{[(n-2)!]^2}{n!(n-5)!}$$

Key Facts

$$(n+1)! = (n+1)n!$$

Put $n = 0$

$$(0+1)! = (0+1)0!$$

$$1! = 1 \times 0!$$

$$1 = 0!$$

$$\text{Hence } 0! = 1$$



Exercise 6.1

1. Evaluate the following:

$$(i) 10! \quad (ii) \frac{12!}{7! \cdot 3! \cdot 2!} \quad (iii) \frac{4! - 2!}{3! + 5!} \quad (iv) \frac{(n-1)!}{(n-2)!} \quad (v) \frac{8!}{(6!)^2}$$

2. Write the following in factorial form:

$$(i) 14 \cdot 13 \cdot 12 \cdot 11 \quad (ii) 1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \quad (iii) n(n^2 - 1) \quad (iv) \frac{(n-3)(n-2)(n-1)}{n(n-4)}$$

3. Prove the following:

$$(i) \frac{1}{5!} + \frac{3}{6!} + \frac{1}{7!} = \frac{4}{315} \quad (ii) \frac{(n-1)!}{(n-3)!} = n^2 - 3n + 2$$

4. Show that:

$$(i) \frac{(2n)!}{n!} = 2^n (1 \cdot 3 \cdot 5 \dots (2n-1)) \quad (ii) \frac{(2n-1)!}{(n-1)!} = 2^{n-1} (1 \cdot 3 \cdot 5 \dots (2n-1))$$

5. Find the values of n in the following.

$$(i) \frac{n}{(n-4)!} = \frac{3 \cdot 3!}{(n-3)!} \quad (ii) \frac{n!}{(n-4)!} : \frac{(n-1)!}{(n-3)!} = 36 : 2$$

6. Prove the following for $n \in \mathbb{N}$,

$$(i) (n+1)[n!n + (n-1)!(2n-1) + (n-2)!(n-1)] = (n+2)!$$

$$(ii) \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{(n+1)!}{r!(n-r+1)!}$$

$$(iii) \frac{n!}{r!} = n(n-1)(n-2) \dots (r+1)$$

$$(iv) (n-r+1) \cdot \frac{n!}{(n-r+1)!} = \frac{n!}{(n-r)!}$$

$$(v) 33! \text{ is divisible by } 2^{15}$$

$$(vi) \frac{2n!}{[(n-1)!]^2} = \frac{n(n+1)(n+2) \dots (2n-1)(2n)}{(n-1)!}$$

7. Find n , if

$$(i) \frac{n!}{(n-2)!} = 930, n \geq 2 \quad (ii) \frac{n!}{(n-5)!} = 20 \cdot \frac{n!}{(n-3)!}, n \geq 5$$

$$(iii) (n+2)! = 60 \cdot (n-1)! \quad (iv) (n+2)! = 132 \cdot n!$$

$$(v) (n+2)! = 56 \cdot n! \quad (vi) \frac{1}{9!} + \frac{1}{10!} = \frac{n}{11!}$$

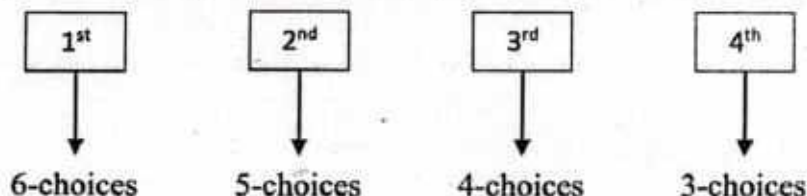
$$(vii) n! = 990 \cdot (n-3)! \quad (viii) (n+1)! = 6 \cdot (n-1)!$$

$$(ix) \frac{(n+2)!}{(2n-1)!} = \frac{(n+3)!}{(2n+1)!} \cdot \frac{72}{7} \quad (x) \frac{1}{2(n-2)!} : \frac{1}{4!(n-4)!} = 2, n \geq 4$$

6.3 Permutation

The number of ways in which r objects out of n objects $0 \leq r \leq n$ can be arranged in a definite order is called permutation.

Example 3: If a vehicle number plate consists of four digits; then the different number plates consisting of distinct digits from 1, 2, 3, 4, 5, 6 can be counted as follows.



For the first digit, we have 6 choices of digits 1, 2, 3, 4, 5, 6. For the second digit, we have 5-choices; because the selected digit cannot be selected again. Similarly, for the third digit, there are 4-choices and for the fourth digit, we are left with 3-choices.

So, the total number of plates = $6 \times 5 \times 4 \times 3 = 360$

Example 4: Suppose a student has four different subject's books and wish to arrange them on a shelf. In how many ways he can arrange the books, can be counted as follows.

For the first place he has 4 choices, for the second place he has 3 choices, for the third place he has 2 choices and for the fourth place he has 1 choice.

Hence total number of arrangements are = $4 \times 3 \times 2 \times 1 = 24 = 4!$

So, we can conclude that if we have n distinct objects then total number of arrangements are $n!$.

6.3.1. Permutation of n Distinct Objects Taken r at a Time ($0 \leq r \leq n$)

Here we are going to generalize the above discussed counting process.

Let we have n number of distinct objects and we want to arrange r of them in some order.

We denote such arrangement by ${}^n P_r$ and read it as n -permutation- r .

For the first object, we have n choices.

For the second object, we have $n - 1$ choices.

For the third object, we have $n - 2$ choices.

For the third object, we have $n - 2$ choices, and so on.

For the r^{th} object, we have $[n - (r - 1)] = (n - r + 1)$.

We write ${}^n P_r$ in factorial form as:

$$\begin{aligned}
 {}^n P_r &= n(n-1)(n-2) \dots (n-r+1) \times \frac{(n-r)(n-r-1)(n-r-2) \dots 3.2.1}{(n-r)(n-r-1)(n-r-2) \dots 3.2.1} \\
 &= \frac{n(n-1)(n-2) \dots (n-r+1)(n-r)(n-r-1)(n-r-2) \dots 3.2.1}{(n-r)(n-r-1)(n-r-2) \dots 3.2.1} = \frac{n!}{(n-r)!}
 \end{aligned}$$

$${}^n P_r = \frac{n!}{(n-r)!}$$

Deductions

(i) If $r = n$; then

$${}^n P_n = n(n-1)(n-2) \dots (n-n+1) \\ = n(n-1)(n-2) \dots 1 = n!$$

$${}^n P_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$$

(ii) Since, ${}^n P_n = n!$

$$\text{Therefore, } n! = \frac{n!}{0!} \Rightarrow 0! = \frac{n!}{n!} = 1$$

$$0! = 1$$

Example 5: In how many ways can 6 peoples out of 8 can be seated in a row?**Solution:** Total number of people = $n = 8$ People to be seated = $r = 6$

$$\begin{aligned} \text{Total number of ways} &= {}^n P_r = {}^8 P_6 = \frac{8!}{(8-6)!} = \frac{8!}{2!} \\ &= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} = 20160 \end{aligned}$$

6.3.2. Permutation of n Objects When Some of Them are Alike (not Distinct)

Let we have total ' n ' number of objects which are all not distinct. Suppose that there are n_1 objects in 1st similar objects category, n_2 objects in the 2nd similar objects category, and similarly, n_k objects in the k^{th} similar objects category.

$$\text{So, } n = n_1 + n_2 + n_3 + \dots + n_k.$$

Let X be the total number of permutations in this situation. If we consider all similar objects as distinct objects in all the categories, then number of permutations for 1st category are $n_1!$, for 2nd category $n_2!$ and for the k^{th} category $n_k!$.

$\therefore$ We have $X \cdot n_1! \cdot n_2! \dots n_k!$ number of permutations.

But total number of permutations are $n!$, therefore

$$X \cdot n_1! \cdot n_2! \dots n_k! = n!$$

$$\Rightarrow X = \frac{n!}{n_1! \cdot n_2! \dots n_k!}$$

$${}^n P_n = \frac{n!}{n_1! \cdot n_2! \dots n_k!}$$

Example 6: How many different arrangements of the letter used in the word EVENING can be made by using all the letters?**Solution:** The total number of letters in the word 'EVENING' = 7

Here, E is repeated 2 times, N is repeated 2 times

$$\text{Thus, number of permutations are} = \frac{7!}{2! \cdot 2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2! \cdot 2! \cdot 1} = 1260$$

6.3.3. Circular Permutation

Some times we have to find the number of permutations while arranging the objects about the circle.

Observe these four arrangements; $abcd, bcda, cdab, dabc$

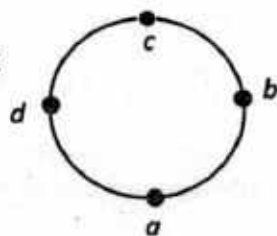
All are distinct along a line but all are same along a circle. Since

the position of each object with reference to other is same,

$$\therefore \text{Number of arrangements} = \frac{4!}{4} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4} = 6$$

So, if we have n number of objects which are all to be arranged in a circle, then

$$\text{Number of ways} = \frac{n!}{n} = \frac{n(n-1)!}{n} = (n-1)!$$



Check Point



In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

Example 7: In how many ways 8 people can sit around a circular table for dinner.

Solution:

Total number of people = $n = 8$

Number of permutations = $(8 - 1)! = 7! = 5040$ ways

Example 8: There are 5 men and 5 women in a party. Find the number of ways in which they can be seated at a round table if:

- (i) anyone can occupy any seat, (ii) men and women have alternate seats.

Solution:

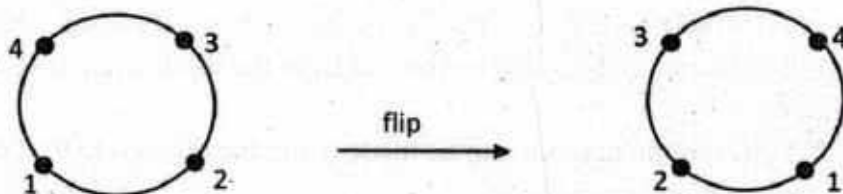
- (i) Total number of persons = $5 + 5 = 10$

Since there is no restriction, so they can sit in $(10 - 1)! = 9!$ ways.

- (ii) We can start either with men or women. If we start with men then they can be seated in $(5 - 1)!$ ways

So, the total number of ways that 5 men and 5 women be seated at round table such that they occupy alternative seats = $4! \times 5! = 24 \times 120 = 2880$

Now we consider the case in which the objects are arranged in a circular manner but we can flip or turned over them.



The arrangements which were anticlockwise are now in clockwise direction after flipping but are same. So, number of arrangements are $\frac{(4-1)!}{2} = \frac{3!}{2} = \frac{6}{2} = 3$

Use of Permutation in Cryptography

Permutations are used in the cryptography as explained in the following example.

Let we have to encrypt the word "PAK". Label P as 1, A as 2 and K as 3. Total number of encrypted words are as follow:

PAK	(1	2	3)
PKA	(1	3	2)
AKP	(2	3	1)
APK	(2	1	3)
KAP	(3	2	1)
KPA	(3	1	2)

Let we want it to be encrypted as

APK i.e.; 2 1 3

It is decrypted as 1 2 3

= PAK

Exercise 6.2

- Prove the following for $n \in \mathbb{N}$.
 - ${}^nP_r = \frac{n!}{(n-r)!}$
 - ${}^nP_n = {}^nP_{n-1}$
 - ${}^nP_r = n \cdot {}^{n-1}P_{r-1}$
 - ${}^nP_r = {}^{n-1}P_{r+r} \cdot {}^{n-1}P_{r-1}$
 - ${}^nP_n = 2 \cdot {}^nP_{n-2}$
- Find n , if:
 - ${}^nP_4 = 20 \cdot {}^nP_2$
 - ${}^{2n}P_3 = 100 \cdot {}^nP_2$
 - $16 \cdot {}^nP_3 = 13 \cdot {}^{n+1}P_3$
 - ${}^nP_5 = 20 \cdot {}^nP_3$
 - $30 \cdot {}^nP_6 = {}^{n+1}P_7$
 - ${}^nP_5 : {}^{n-1}P_4 = 6 : 1$
 - ${}^nP_4 : {}^{n-1}P_3 = 9 : 1$
 - ${}^{n-1}P_3 : {}^{n+1}P_3 = 5 : 12$
 - ${}^{2n-1}P_n : {}^{2n+1}P_{n-1} = 22 : 7$
- Find r , if:
 - ${}^6P_{r-1} = {}^5P_4$
 - ${}^{10}P_r = 2 \cdot {}^9P_r$
 - ${}^{15}P_r = 210$
 - ${}^{10}P_r = 3 \cdot {}^{10}P_{r-1}$
 - $4 \cdot {}^6P_r = {}^6P_{r+1}$
 - $2 \cdot {}^6P_{r-1} = {}^5P_r$
 - ${}^{54}P_{r+3} : {}^{56}P_{r+6} = 1 : 30800$
- How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6, if the digits are not repeated?
- How many 7-digits mobile number can be made using the digits 0 to 9, if each number starts with 5 and no digit is repeated?
- How many 4-digit numbers can be formed with the digits 1, 2, 3, 4, 5, 6 when the repetition of the digits is allowed?

7. How many numbers can be formed with the digits 1, 1, 2, 2, 3, 3, 4 so that the even digits always occupy the even places, using all the digits and no digit is repeated?
8. In how many ways can a party of 4 men and 5 women be seated at a round table so that no two women are adjacent?
9. How many different signals can be made with 2 blue, 3 yellow and 4 green flags using all at a time.
10. How many words can be formed from the letters of the word FRIDAY? How many of them will end with F?
11. How many different permutations of the word STATESMAN can be formed using all letters at a time?
12. Find the number of arrangement of letters of the word VOWEL in which vowels may occupy odd places?
13. In how many ways can letters of word MACHINE be arranged so that all the vowels are never together?
14. How many 3 letter words (with or without meaning) can be formed out of the letter of the word ENGLISH, if the repetition of the letter is not allowed.
15. Fatima wants to arrange 5 Mathematics, 3 English and 2 Urdu books on book shelf. If the books on the same subjects are together, find all possible arrangements.
16. How many odd numbers can be formed by using the digits 1, 2, 3, 4, 5, 6 when repetition of digits is not allowed.
17. How many 4-digit odd numbers can be formed using the digits 1, 2, 3, 4 and 5 if no digit is repeated.
18. How many odd numbers less than 10,000 can be formed using the digits 0, 2, 3, 5, 6 without repeating the digits.
19. The chief secretary of Sindh calls a meeting of 10 secretaries. In how many ways they be seated at a round table if three particular secretaries want to sit together?
20. Find the number of ways that 6 men and 6 women seated at a round table such that they occupy alternative seats.
21. Make all the permutations of the following words
WHY, SAD, TWO, MADE
22. Encrypt the word LAHORE by using the permutation:
(3 4 6 1 5 2)
By labelling L as 1, A as 2 and so on.
23. Decrypt the word "TNLUMA" by using the permutation:
(4 6 3 2 1 5)

6.4 Combinations

In permutation we arrange the objects in some definite order. If in the arrangements of objects their order is not important then this arrangement of objects is called **combination**.

Let, we have three objects a, b and c then $abc, acb, bac, bca, cab, cba$ all are same. In permutation we consider them all distinct, so there are 6 arrangements, but in combination all these are same (since order is not important). Hence, we consider all these arrangements same and consider them a single combination. The combination of r objects taken out of n distinct objects is denoted by ${}^nC_r$ or $\binom{n}{r}$.

6.4.1 Prove that ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

Proof: Let we have n distinct objects and we want to take r at a time where, $0 \leq r \leq n$. Let the total number of combinations be X . i.e. X we take these r objects in some order then total number of ways are $r!$. But in combinations all these $r!$ ways will be treated as same that is one way.

$$\text{i.e. } r! \text{ combination} = 1 \text{ permutations}$$

$$\Rightarrow r! X = 1 \text{ permutation}$$

Since, total number of permutations are ${}^nP_r$. So,

$$\therefore Xr! = {}^nP_r$$

$$\Rightarrow X = \frac{1}{r!} {}^nP_r = \frac{1}{r!} \cdot \frac{n!}{(n-r)!}$$

$$\Rightarrow {}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$\Rightarrow r! {}^nC_r = {}^nP_r$$



Key Facts

${}^nC_r \times r! = {}^nP_r$

Deductions

(i) $\binom{n}{n} = \binom{n}{0} = 1$

Since, $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Putting $r = n$; we have

$$\binom{n}{n} = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{1}{0!} = \frac{1}{1} = 1$$

Now taking $r = 0$; we have

$$\binom{n}{0} = \frac{n!}{0!(n-0)!} = \frac{n!}{1 \cdot n!} = \frac{1}{1} = 1$$

Thus $\binom{n}{n} = \binom{n}{0} = 1$

(ii) $\binom{n}{r} = \binom{n}{n-r}$

Since, $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

Replacing r by $n-r$, we have

$$\begin{aligned} \therefore \binom{n}{n-r} &= \frac{n!}{(n-r)![n-(n-r)]!} = \frac{n!}{(n-r)!(n-n+r)!} \\ &= \frac{n!}{(n-r)!r!} = \binom{n}{r} \end{aligned}$$

$$\therefore \binom{n}{n-r} = \binom{n}{r}$$

$$(iii) \quad \binom{n}{1} = \binom{n}{n-1} = n$$

$$\text{Since, } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Putting $r = 1$; we have

$$\binom{n}{1} = \frac{n!}{1!(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$$

Now put $r = n - 1$

$$\binom{n}{n-1} = \frac{n!}{(n-1)!(n-(n-1))!} = \frac{n(n-1)!}{(n-1)!(n-n+1)!} = \frac{n(n-1)!}{(n-1)!1!} = n$$

$$\text{Hence, } \binom{n}{1} = \binom{n}{n-1} = n$$

$$(iv) \quad \binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$$

$$\text{Since, } \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad (1)$$

Replacing r by $r - 1$

$$\binom{n}{r-1} = \frac{n!}{(r-1)!(n-(r-1))!} = \frac{n!}{(r-1)!(n-r+1)!} \quad (2)$$

Adding (1) and (2)

$$\begin{aligned} \binom{n}{r} + \binom{n}{r-1} &= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} \\ &= \frac{n!}{r(r-1)!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)(n-r)!} \\ &= \frac{n!}{(r-1)!(n-r)!} \left[\frac{1}{r} + \frac{1}{n-r+1} \right] \\ &= \frac{n!}{(r-1)!(n-r)!} \cdot \frac{n-r+1+r}{r(n-r+1)} \\ &= \frac{n!}{(r-1)!(n-r)!} \cdot \frac{n+1}{r(n-r+1)} \\ &= \frac{n!(n+1)}{(r-1)!r(n-r)!(n-r+1)} = \frac{(n+1)!}{r!(n+1-r)!} \\ &= \binom{n+1}{r} \end{aligned}$$

Example 9: 10 students applied for 6 HEC scholarships. In how many ways can these 6 be chosen?

Solution:

Total number of students = $n = 10$

Number of students to be chosen = $r = 6$

Because for this selection order of the students is not necessary.

So, this selection can be made in:

$${}^{10}C_6 = \frac{10!}{6!(10-6)!} = \frac{10!}{6!4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!4 \cdot 3 \cdot 2 \cdot 1} = 210 \text{ ways}$$

Example 10: Find the value of n if ${}^nC_2 = 10$

Solution: Given ${}^nC_2 = 10$

$$\begin{aligned} \frac{n!}{2!(n-2)!} &= 10 \\ \Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} &= 10 \times 2 \end{aligned}$$

$$\Rightarrow n(n-1) = 20$$

$$\Rightarrow n^2 - n - 20 = 0$$

$$\Rightarrow n^2 - 5n + 4n - 20 = 0$$

$$\Rightarrow n(n-5) + 4(n-5) = 0$$

$$\Rightarrow (n-5)(n+4) = 0$$

$$\Rightarrow n-5 = 0 \text{ or } n+4 = 0$$

$$\Rightarrow n = 5 \text{ or } n = -4$$

$n = -4$ is not possible as value of n is positive.

Hence, $n = 5$

Check Point



There are 8 men and 10 women and you need to form a committee of 5 men and 6 women. In how many ways can the committee be formed?

Exercise 6.3

1. Prove the following for $n \in \mathbb{N}$.

$$(i) \quad {}^nC_r = \frac{n!}{r!(n-r)!} \quad (ii) \quad n \cdot {}^{n-1}C_{r-1} = (n-r+1) {}^nC_{r-1} \quad (iii) \quad r {}^nC_r = (n-r+1) {}^nC_{r-1}$$

$$(iv) \quad {}^{n-1}C_{r-1} + {}^{n-1}C_r = {}^nC_r \quad (v) \quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \quad (vi) \quad {}^{2n}C_n = \frac{2^n \cdot [1 \cdot 3 \cdot 5 \cdots (2n-1)]}{n!}$$

$$(vii) \quad {}^nC_p = {}^nC_q \Rightarrow p = q \text{ or } p + q = n \quad (viii) \quad {}^nC_r + 2 {}^nC_{r-1} + {}^nC_{r-2} = {}^{n+2}C_r$$

$$(ix) \quad r {}^nC_r = n {}^{n-1}C_{r-1} \quad (x) \quad \text{The product of } k \text{ consecutive integers is divisible by } k!$$

2. Find n , if:

$$(i) \quad {}^nC_5 = {}^nC_8 \quad (ii) \quad {}^nC_{15} = {}^nC_7 \quad (iii) \quad {}^nC_{50} = {}^nC_1 \quad (iv) \quad {}^{2n}C_3 : {}^nC_3 = 11 : 1$$

$$(v) \quad {}^nC_6 : {}^{n-3}C_3 = 33 : 4 \quad (vi) \quad {}^{2n}C_3 : {}^nC_2 = 12 : 1$$

3. Find r , if:

$$(i) \quad {}^{15}C_{3r} = {}^{15}C_{r+3} \quad (ii) \quad {}^8C_r - {}^7C_3 = {}^7C_2 \quad (iii) \quad {}^{16}C_r = {}^{16}C_{r+4} \quad (iv) \quad {}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$$

4. Find n and r , if:

(i) ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6 : 14 : 21$

(ii) ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 3 : 4 : 5$

(iii) ${}^{n+1}C_{r+1} : {}^nC_r : {}^{n-1}C_{r-1} = 22 : 12 : 6$

(iv) ${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 1 : 2 : 3$

5. In how many ways can 11 players be chosen out of 16 if

(i) there is no restriction.

(ii) a particular player is always chosen.

6. Out of 5 men and 3 women, a committee of 3 is to be formed. In how many ways can it be formed if at least one man is selected?

7. A committee of 5 members is to be formed out of 6 men and 4 women. In how many ways can it be done if it has (i) exactly 2 women (ii) at least 2 women (iii) at most 2 women?

8. There are 10 points on a circle. Find the number of (i) lines (ii) triangles that can be drawn?

9. Find the number of diagonals in n sided polygon?

10. In how many ways a group of 10 girls can be divided into two groups of 3 and 7 girls.

11. Number of diagonals in n -sided polygon is 35. Find the number n ?

12. For the post of 6 officers, there are 100 applicants, 2 posts are reserved for serving candidates and remaining for others. There are 20 serving candidates among the applicants. In how many ways this selection can be made?

13. In an examination, a candidate has to pass in each of 6 subjects. In how many ways he cannot qualify the examination?

14. A question paper has three parts A, B and C each containing 8 questions. If a student has to choose 5 questions from A, and 3 questions each from B and C. In how many ways can he choose the questions?

Review Exercise

1. Select the correct option in the following.

(i) If ${}^3P_3 = {}^nP_4$ then value of n is:

- a. 5 b. 6 c. 7 d. 8

(ii) Number of ways of arrangement of the word "GARDEN":

- a. 480 b. 600 c. 720 d. 840

(iii) The product of r consecutive positive numbers is divisible by:

- a. $r!$ b. $(r+1)!$ c. $r! + 1$ d. $2r!$

(iv) The total number of 6-digit number in which all the odd and only odd digits appear is:

- a. $\frac{5}{2}6!$ b. $6!$ c. $\frac{1}{2}6!$ d. $\frac{3}{2}6!$

(v) Let $A = \{1, 2, 3, \dots, 20\}$. Find the number of ways that the integer chosen is a prime number is:

- a. 3 b. 5 c. 7 d. 8

(vi) From $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8\}$ if a cartesian product $A \times B$ is chosen, then the number of ways that $a + b = 9$ is:

- a. 0 b. 2 c. 3 d. 4

(vii) A student has to answer 10 out of 12 questions in an examination such that he must choose at least 4 from first five questions. The number of choices is:

- a. 30 b. 35 c. 40 d. 45

(viii) If ${}^nC_4 = {}^nC_{10}$ then value of n is:

- a. 10 b. 12 c. 13 d. 14

(ix) If ${}^{15}C_{3r} = {}^{15}C_{r+3}$ then value of r is:

- a. 1 b. 2 c. 3 d. 4

(x) The number of ways in which r letters can be posted in n letter boxes in a town is:

- a. ${}^nC_r$ b. ${}^nP_r$ c. r^n d. n^r

2. How many words can be formed by using four distinct alphabets?

3. How many 3-digit numbers are there which have 0 at unit place?

4. How many six-digit numbers can be formed using the digits 0, 2, 3, 4, 5, 7 without repeating.

5. The number of ways of arranging 7 keys in a key chain.

6. Twelve persons are seated at a round table. Find the number of ways of their arrangement if two particular persons don't want to sit together.