

Exercise 7.1

1- Write the first three terms of the following:

$$(i) a_n = n + 3 \quad (ii) a_n = (-1)^n n^3 \quad (iii) a_n = 3n + 5$$

$$(iv) a_n = \frac{n+1}{2n+5} \quad (v) a_n = \frac{1}{(2n-1)^2} \quad (vi) a_n = n + 3 = 2$$

$$(vii) a_n = \frac{1}{3^n} \quad (viii) a_n = 3n - 5 \quad (ix) a_n = (n+1)a_{n-1}, a_1 = 1$$

Solution:(i)

$$\begin{aligned} a_n &= n + 3 \\ \text{Putting values } n &= 1, 2, 3 \\ a_1 &= 1 + 3 = 4 \\ a_2 &= 2 + 3 = 5 \\ a_3 &= 3 + 3 = 6 \end{aligned}$$

First three terms 4, 5, 6

Solution:(ii)

$$\begin{aligned} a_n &= (-1)^n n^3 \\ \text{Putting values } n &= 1, 2, 3 \\ a_1 &= (-1)^1 (1)^3 \\ &= (-1)(1) = -1 \\ a_2 &= (-1)^2 (2)^3 \\ &= (+1)(8) = 8 \\ &= (+1)(8) = 8 \\ a_3 &= (-1)^3 (3)^3 \\ &= (-1)(27) = -27 \end{aligned}$$

First three terms -1, 8, -27

Solution:(iii)

Putting $n = 1, 2, 3$

$$\begin{aligned} a_1 &= 3(1) + 5 \\ &= 3 + 5 = 8 \end{aligned}$$

$$\begin{aligned} a_2 &= 3(2) + 5 \\ &= 6 + 5 = 11 \end{aligned}$$

$$\begin{aligned} a_3 &= 3(3) + 5 \\ &= 9 + 5 = 14 \end{aligned}$$

First three terms are 8, 11, 14.

Solution:(iv)

$$a_n = \frac{n+1}{2n+5}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1+1}{2(1)+5} = \frac{2}{2+5} = \frac{2}{7}$$

$$a_2 = \frac{2+1}{2(2)+5} = \frac{3}{4+5} = \frac{3}{9}$$

$$a_3 = \frac{3+1}{2(3)+5} = \frac{4}{6+5} = \frac{4}{11}$$

First three terms are $\frac{2}{7}, \frac{3}{9}, \frac{4}{11}$

Solution:(v)

$$a_n = \frac{1}{(2n-1)^2}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1}{\{2(1)-1\}^2} = \frac{1}{(2-1)^2} = \frac{1}{(1)^2} = 1 \quad (i)$$

$$a_2 = \frac{1}{\{2(2) - 1\}^2} = \frac{1}{(4-1)^2} = \frac{1}{(3)^2} = \frac{1}{9} \quad (\text{ii})$$

$$a_3 = \frac{1}{\{2(3) - 1\}^2} = \frac{1}{(6-1)^2} = \frac{1}{(5)^2} = \frac{1}{25} \quad (\text{iii})$$

First three terms are $1, \frac{1}{9}, \frac{1}{25}$

Solution:(vi)

$$a_n = n + 3$$

Putting $n = 1, 2, 3$

$$a_1 = 1 + 3 = 4$$

$$a_2 = 2 + 3 = 5$$

$$a_3 = 3 + 3 = 6$$

First three terms are 4, 5, 6

Solution:(vii)

$$a_n = \frac{1}{3^n}$$

Putting $n = 1, 2, 3$

$$a_1 = \frac{1}{3^1} = \frac{1}{3}$$

$$a_2 = \frac{1}{3^2} = \frac{1}{9}$$

$$a_3 = \frac{1}{3^3} = \frac{1}{27}$$

First three terms are $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

Solution:(viii)

$$a_n = 3n - 5$$

Putting $n = 1, 2, 3$

$$a_1 = 3(1) - 5 = 3 - 5 = -2$$

$$a_2 = 3(2) - 5 = 6 - 5 = 1$$

$$a_3 = 3(3) - 5 = 9 - 5 = 4$$

First three terms are $-2, 1, 4$

Solution: (ix)

$$\text{Putting } a_n = (n + 1) \\ n = 2, 3$$

$$a_1 = 1$$

$$a_2 = 2 + 1 = 3$$

$$a_3 = 3 + 1 = 4$$

First three terms are $1, 3, 4$

2- Find the terms indicated in the following sequences.

$$(i) 2, 6, 11, 17, \dots, a_8 \quad (ii) 1, 3, 12, 60, \dots, a_7 \quad (iii) 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots, a_6$$

$$(iv) -1, 1, 3, 5, \dots, a_9 \quad (v) \frac{1}{3}, \frac{2}{5}, \dots, a_5 \quad (vi) 1, -3, 5, -7, \dots, a_9$$

$$2(i) 2, 6, 11, 17, \dots, a_8$$

Solution:

$$a_1 = 2$$

$$a_2 = 6$$

$$a_3 = 11$$

$$a_4 = 17$$

$$a_2 - a_1 = 6 - 2 = 4$$

$$a_3 - a_2 = 11 - 6 = 5$$

$$a_4 - a_3 = 17 - 11 = 6$$

$$a_5 - a_4 = 24 - 17 = 7$$

$$a_6 - a_5 = 32 - 24 = 8$$

$$a_7 - a_6 = 41 - 32 = 9$$

$$a_8 - a_7 = 51 - 41 = 10$$

$$a_8 = 51$$

2(ii) 1, 3, 12, 60, ... a_7

Solution:

$$\begin{aligned} a_1 &= 1 \\ \therefore a_2 &= 3 = 1 \times 3 \\ a_3 &= 12 = 3 \times 4 \\ a_4 &= 60 = 12 \times 5 \\ a_5 &= 360 = 60 \times 6 \\ a_6 &= 2520 = 360 \times 7 \\ a_7 &= 2520 \times 8 = 20160 \end{aligned}$$

2(iii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots, a_6$

Solution:-

$$\begin{aligned} a_1 &= 1 \\ a_2 &= \frac{1}{3} = 1 \times \frac{1}{3} \\ a_3 &= \frac{1}{9} = \frac{1}{3} \times \frac{1}{3} \\ a_4 &= \frac{1}{27} = \frac{1}{9} \times \frac{1}{3} \\ a_5 &= \frac{1}{81} = \frac{1}{27} \times \frac{1}{3} \\ a_6 &= \frac{1}{243} = \frac{1}{81} \times \frac{1}{3} = \frac{1}{243} \end{aligned}$$

2(iv) -1, 1, 3, 5, ... a_9

Solution:

$$a_1 = -1$$

$$a_2 = 1$$

$$a_3 = 3$$

$$a_4 = 5$$

$$a_2 - a_1 = 1 - (-1)$$

$$= 1 + 1 = 2$$

$$a_3 - a_2 = 3 - 1 = 2$$

$$a_4 - a_3 = 5 - 3 = 2$$

$$a_5 = 5 + 2 = 7$$

$$a_6 = 7 + 2 = 9$$

$$a_7 = 9 + 2 = 11$$

$$a_8 = 11 + 2 = 13$$

$$a_9 = 13 + 2 = 15$$

$$2(v) \quad \frac{1}{3}, \frac{2}{5}, \dots, a_5$$

Solution:

$$\frac{1}{3}, \frac{2}{5}, \dots, a_5$$

$$a_1 = \frac{1}{3}$$

$$a_2 = \frac{2}{5}$$

$$a_3 = \frac{3}{7}$$

$$a_4 = \frac{4}{9}$$

$$a_5 = \frac{5}{11}$$

$$2(vi) \quad 1, -3, 5, -7, \dots, a_9$$

Solution:

$$a_1 = 1$$

$$a_2 = -3$$

$$a_3 = 5$$

$$a_4 = -7$$

$$a_5 = 9$$

$$a_6 = -11$$

$$a_7 = 13$$

$$a_8 = -15$$

$$a_9 = 17$$

3- Find the next four terms of the following sequences.

(i) 12, 16, 21, 27, ... (ii) 1, 3, 7, 15, 31, ... (iii) -1, 2, 12, 40, ...

(iv) 9, 11, 14, 17, 19, 22, ... (v) 4, 8, 12, 16, ... (vi) -2, 0, 2, 4, 6, 8, 10, ...

3(i) 12, 16, 21, 27, ...

Solution:

$$16 - 12 = 4$$

$$21 - 16 = 5$$

$$27 - 21 = 6$$

$$34 - 27 = 7$$

$$42 - 34 = 8$$

$$51 - 42 = 9$$

$$61 - 51 = 10$$

$$27 + 7 = 34$$

$$34 + 8 = 42$$

$$42 + 9 = 51$$

$$51 + 10 = 61$$

$$34, 42, 51, 61$$

3(ii) 1, 3, 7, 15, 31, ...

Solution:

$$3 - 1 = 2$$

$$7 - 3 = 4$$

$$15 - 7 = 8$$

$$31 - 15 = 16$$

$$63 - 31 = 32$$

$$127 - 63 = 64$$

$$31 + 32 = 63$$

$$63 + 64 = 127$$

$$127 + 128 = 255$$

$$225 - 127 = 128$$

$$511 - 255 = 256$$

$$255 + 256 = 511$$

$$1, (1+2), (3+2^2) + (7+2^3) + (15+2^4) \\ + (31+2^5) + (63+2^6) + (127+2^7) \dots$$

3(iii) -2, 2, 12, 40, ...

Solution:

$$2 - (-2) = 2 + 2 = 4 = 3^1 + 1$$

$$12 - 2 = 10 = 3^2 + 1$$

$$40 - 12 = 28 = 3^3 + 1$$

$$112 - 40 = 72 = 3^4 + 1$$

$$356 - 112 = 244 = 3^5 + 1$$

$$1086 - 356 = 730 = 3^6 + 1$$

$$3274 - 1086 = 2188 = 3^7 + 1$$

$$112, 356, 1086, 3274$$

$$3(\text{iv}) \quad 9, \quad \frac{3}{9} \quad \frac{3}{11} \quad \frac{3}{14} \quad \frac{3}{17} \quad \frac{3}{19} \quad \frac{3}{22} \quad \frac{3}{25} \quad \frac{3}{27} \quad \frac{3}{30} \quad \frac{3}{33}$$

3(v) 4, 8, 12, 16, ...

Solution:

$$8 - 4 = 4$$

$$12 - 8 = 4$$

$$16 - 12 = 4$$

$$20 - 16 = 4$$

$$24 - 20 = 4$$

$$28 - 24 = 4$$

$$32 - 28 = 4$$

3(vi) -2, 0, 2, 4, 6, 8, 10

Solution:

$$0 - (-2) = 0 + 2 = 2$$

$$2 - 0 = 2$$

$$4 - 2 = 2$$

$$6 - 4 = 2$$

$$8 - 6 = 2$$

$$10 - 8 = 2$$

$$12 - 10 = 2$$

$$14 - 12 = 2$$

$$16 - 14 = 2$$

$$18 - 16 = 2$$

$$-2, 0, 2, 4, 6, 8, 10, \dots$$

Exercise 7.2

1- Find the specified term of the following A.P.

(i) 3, 7, 11, ... 61st term. (ii) -4, -7, -10, ... a_{19}

(iii) 6, 4, 2, ... 45th term. (iv) 9, 14, 19, ... a_{14}

(v) 11, 6, 1, ... a_{18}

Solution:-

61st 3, 7, 11, ...

Here

$$a_1 = 3$$

$$d = 7 - 3 = 4$$

$$a_{61} = ?$$

$$n = 61$$

We know that $a_n = a + (n-1)d$

Putting values a , d and n in the formula

$$\begin{aligned} a_{61} &= 3 + (61-1)(4) \\ &= 3 + 60 \times 4 \\ &= 3 + 240 \\ &= 243 \end{aligned}$$

1(ii) -4, -7, -10, ... a_{19}

Solution:

Here $a = a_1 = -4$

$$d = (-7) - (-4)$$

$$= -7 + 4 = -3$$

$$a_{19} = ?$$

$$n = 19$$

We know that $a_n = a + (n-1)d$

Putting values of a , d and n in the formula

$$\begin{aligned} a_{19} &= -4 + (19-1)(-3) \\ &= -4 + 18(-3) \\ &= -4 - 54 \\ &= -58 \\ a_{19} &= -58 \end{aligned}$$

Solution:

Here $a = a_1 = 4$
 $d = 4 - 6 = -2$
 $a_{45} = ?$
 $n = 45$

We know that $a_n = a + (n-1)d$

Putting values of a , d and n in the formula

$$\begin{aligned} a_{45} &= 4 + (45 - 1)(-2) \\ &= 4 + (44)(-2) \\ &= 4 - 88 \\ &= -84 \end{aligned}$$

Hence 45th term is -84

1(iv) 9, 14, 19, ... a_{14}

Solution:

Here $a = a_1 = 9$
 $d = 14 - 9 = 5$
 $a_{14} = ?$
 $n = 14$

We know that $a_n = a + (n-1)d$

Putting values of a , d and n in the formula

$$\begin{aligned} a_{14} &= 9 + (14 - 1)(5) \\ &= 9 + (13)(5) \\ &= 9 + 65 \\ &= 74 \end{aligned}$$

1(v) 11, 6, 1, ... a_{18}

Solution:

$$\begin{aligned} a &= a_1 = 11 \\ d &= 6 - 11 = -5 \\ a_{18} &= ? \\ n &= 18 \end{aligned}$$

We know that $a_n = a + (n-1)d$

Putting values in a , d and n in the formula

$$\begin{aligned} a_{18} &= 11 + (18 - 1)(-5) \\ &= 11 + (17)(-5) \end{aligned}$$

$$= 11 - 85$$

$$= -74$$

2- Find the missing element using the formula of A.P

$$a_n = a + (n - 1) d$$

$$(i) \quad a = 2, a_n = 402, n = 26$$

$$(ii) \quad a_n = 81, d = -3, n = 18$$

$$(iii) \quad a = 5, a_n = 61, n = 15$$

$$(iv) \quad a = 16, a_n = 0, d = -\frac{1}{4}$$

$$(v) \quad a = 10, a_n = 400, d = 5$$

$$(vi) \quad a_n = 261, d = 4, n = 18$$

Solution: (i) $a_n = a + (n - 1) d$

$$a = 2$$

$$a_n = 402$$

$$n = 26$$

$$d = ?$$

Here we find d ?

$$a_n = a + (n - 1) d$$

Putting values of a, a_n, n

$$402 = 2 + (26 - 1) d$$

$$402 = 2 + 25 d$$

$$2 + 25 d = 402 \quad \text{OR}$$

$$25 d = 402 - 2$$

$$25 d = 400$$

$$\text{Hence} \quad d = \frac{400}{25} = 16$$

Solution: (ii) $a_n = a + (n - 1) d$

$$a_n = 81$$

$$d = -3$$

$$n = 18$$

Here a is unknown

$$a_n = a + (n - 1) d$$

Putting values of a, a_n, n

$$81 = a + (18 - 1)(-3)$$

$$81 = a + (17)(-3)$$

$$81 = a - 51$$

$$a - 51 = 81 \quad \text{OR}$$

$$\begin{aligned} \text{Therefore } a &= 81 + 51 \\ &= 132 \end{aligned}$$

Solution: (iii) $a_n = a + (n - 1)d$ formula

$$a_n = 5 \quad \text{and}$$

$$a_n = 61$$

$$n = 15$$

Here d is unknown

$$a_n = a + (n - 1)d$$

Putting values of a , a_n , and n .

$$61 = 5 + (15 - 1)d$$

$$61 = 5 + 14d$$

$$5 + 14d = 61 \quad \text{OR}$$

$$14d = 61 - 5$$

$$14d = 56 \quad \text{Therefore}$$

$$d = \frac{56}{14}$$

$$d = 4$$

Solution: (iv) $a_n = a + (n - 1)d$ formula

$$a = 16 \quad \text{and}$$

$$a_n = 0$$

$$d = -\frac{1}{4}$$

Here n is unknown.

$$a_n = a + (n - 1)d$$

Putting values of a , a_n and d .

$$0 = 16 + (n - 1)\left(-\frac{1}{4}\right)$$

Multiplying by 4.

$$0 = 64 + (n - 1)(-1)$$

$$0 = 64 - n + 1$$

$$n = 64 + 1 \quad \text{OR}$$

$$n = 65 \quad \text{OR}$$

Solution: (v) $a_n = a + (n - 1) d$ formula

$$a = 10 \quad \text{and}$$

$$a_n = 400$$

$$d = 5$$

Here n is unknown

$$a_n = a + (n - 1) d$$

Putting values of a , a_n and d .

$$400 = 10 + (n - 1)(5)$$

$$400 - 10 = 5n - 5$$

$$5n - 5 = 390 \quad \text{OR}$$

$$5n = 390 + 5$$

$$5n = 395$$

$$n = \frac{395}{5}$$

$$= 79$$

Solution: (vi) $a_n = a + (n - 1) d$ formula

$$a_n = 261 \quad \text{and}$$

$$d = 4$$

$$n = 18$$

Here a is unknown

$$a_n = a + (n - 1) d$$

Putting values of a_n , d and n .

$$261 = a + (18 - 1)(4)$$

$$= a + 17 \times 4$$

$$261 = a + 68$$

$$261 - 68 = a$$

$$193 = a \quad \text{OR}$$

$$a = 193$$

- 3- Find the 15th terms of an A.P where the 3rd term is 8
and the common difference is $\frac{1}{3}$.

Solution:

$$a_3 = 8$$

$$d = \frac{1}{3}$$

$$a_{15} = ?$$

First we find a

$$a_n = a + (n - 1) d$$

$$a_3 = a + (3 - 1) d$$

$$8 = a + (2) \left(\frac{1}{3} \right)$$

$$8 = a + \left(\frac{2}{3} \right)$$

$$a = 8 - \frac{2}{3} \quad \text{OR}$$

$$a = 8 - \frac{2}{3}$$

$$a = \frac{24 - 2}{3} = \frac{22}{3} \quad (A)$$

$$a_n = a + (n - 1) d \quad \text{formula}$$

$$a_{15} = \frac{22}{3} + (15 - 1) \left(\frac{1}{3} \right) \quad \text{from A}$$

$$= \frac{22}{3} + (14) \frac{1}{3}$$

$$\begin{aligned}
 &= \frac{22}{3} + \frac{14}{3} \\
 &= \frac{22+14}{3} \\
 &= \frac{36}{3} = 12
 \end{aligned}$$

4- Which term of an A.P 6, 2, -2..... is -146?

Solution:

Here term 6, 2, -2, ...

$$a = 6$$

$$d = 2 - 6 = -4$$

$$a_n = -146$$

$$n = ?$$

$$a_n = a + (n - 1) d \quad \text{formula}$$

Putting values a , a_n and d in formula.

$$-146 = 6 + (n - 1)(-4) \quad \text{formula}$$

$$-146 = 6 - 4n + 4$$

$$4n = 6 + 4 + 146 \quad \text{OR}$$

$$4n = 156$$

$$n = \frac{156}{4}$$

$$n = 39$$

5- Which term of an A.P 5, 2, -1 is -134?

Solution:

$$a = 5$$

$$d = 2 - 5 = -3$$

$$n = -118$$

$$n = ?$$

$$a_n = a + (n - 1) d \quad \text{formula}$$

Putting values of a , d and n .

$$-118 = 5 + (n - 1)(-3)$$

$$-118 = 5 + 3n + 3$$

$$3n = 5 + 3 + 118 \quad \text{Or}$$

$$3n = 126$$

$$n = \frac{126}{3}$$

$$n = 42$$

- 6- How many terms are there in an A.P, in which $a_1 = a = 11$, $a_n = 68$, $d = 3$.

Solution:- Here term.....

$$d = 3$$

$$a_n = 68$$

$$a = 11$$

$$n = ?$$

$$a_n = a + (n - 1) d \quad \text{formula}$$

Putting values of a , a_n and d in the formula.

$$68 = 11 + (n - 1)(3)$$

$$68 = 11 + 3n - 3$$

$$68 = 8 + 3n$$

$$8 + 3n = 68$$

$$3n = 68 - 8 \quad \text{Or}$$

$$3n = 60$$

$$n = \frac{60}{3}$$

$$n = 20$$

- 7- Find the 11th term of a_n A.P $2 - x$, $3 - 2x$, $4 - 3x$

Solution:- In term of a_n A.P $2 - x$, $3 - 2x$, $4 - 3x$

$$a = 2 - x$$

$$d = (3 - 2x) - (2 - x)$$

$$d = 3 - 2x - 2 + x = 1 - x$$

$$a_{11} = ?$$

$$\therefore n = 11$$

$$a_n = a + (n - 1) d \quad \text{formula}$$

$$a_{11} = (2 - x) + (11 - 1)(1 - x)$$

$$= 2 - x + 10(1 - x)$$

$$= 2 - x + 10 - 10x$$

Hence $a_{11} = 12 - 11x$

8- Find the n^{th} term of an A.P, where $a_{n-5} = 3n + 9$

$$a_n = ?$$

Solution:-

$$a_{n-5} = 3n + 9 \quad \text{We put } n + 5 \text{ in place of } n.$$

$$a_{n+5-5} = 3(n+5) + 9$$

$$a_n = 3n + 15 + 9$$

$$a_n = 3n + 24$$

9- Find the n^{th} term of an A.P: $\left(\frac{3}{4}\right)^2, \left(\frac{3}{7}\right)^2, \left(\frac{3}{10}\right)^2, \dots$

Solution:

$$a = 4$$

$$d = 7 - 4 = 3$$

$$a_n = ?$$

$$a_n = a + (n-1)d \quad \text{formula}$$

$$= 4 + (n-1)(3)$$

$$= a + 3n - 3$$

$$= 4 + 3n - 3$$

$$= 3n + 1$$

The n^{th} term of the sequence $\left(\frac{3}{3n+1}\right)^2$

10- If the n^{th} term of an A.P is $3n - 5$. Find the A.P

Solution:-

$$a_n = 3n - 5$$

Putting $n = 1, 2, 3, 4, \dots$

$$a_1 = 3(1) - 5 = 3 - 5 = -2$$

$$a_2 = 3(2) - 5 = 6 - 5 = 1$$

$$a_3 = 3(3) - 5 = 9 - 5 = 4$$

$$a_4 = 3(4) - 5 = 12 - 5 = 7$$

$-2, 1, 4, 7, \dots$ is required

Exercise 7.3

1- Find A.M between:

(i) $-3, 7$

(ii) $x-1, x+7$

(iii) $\sqrt{7}, 3\sqrt{7}$

(iv) x^2+x+1, x^2-x+1

Solution:-(i) $-3, 7$

Here $a = -3$

$b = 7$

$$(A.M) \quad A = \frac{-3+7}{2}$$

$$\text{Here} \quad A = \frac{4}{2} = 2$$

Solution:-(ii) $x-1, x+7$

Here $a = x-1$

$b = x+7$

$$\begin{aligned} (A.M) \quad A &= \frac{a+b}{2} \\ &= \frac{(x-1)+(x+7)}{2} \\ &= \frac{x-1+x+7}{2} \\ &= \frac{2x+6}{2} \\ &= \frac{2(x+3)}{2} \end{aligned}$$

$$A = x+3$$

Solution:-(iii) $\sqrt{7}, 3\sqrt{7}$

Here $a = \sqrt{7}$

$b = 3\sqrt{7}$

$$\begin{aligned} \text{(A.M)} \quad A &= \frac{a+b}{2} \\ &= \frac{\sqrt{7} + 3\sqrt{7}}{2} \\ &= \frac{\sqrt{7}(1+3)}{2} \\ &= \frac{\sqrt{7} \cdot 4}{2} \end{aligned}$$

Hence $A = 2\sqrt{7}$

Solution:-(iv) $x^2 + x + 1, x^2 - x + 1$

Here $a = x^2 + x + 1$

$b = x^2 - x + 1$

$$\begin{aligned} \text{(A.M)} \quad A &= \frac{a+b}{2} \\ &= \frac{(x^2 + x + 1) + (x^2 - x + 1)}{2} \\ &= \frac{x^2 + x + 1 + x^2 - x + 1}{2} \\ &= \frac{2x^2 + 2}{2} \\ &= \frac{2(x^2 + 1)}{2} \end{aligned}$$

Hence $A = x^2 + 1$

2- If 3 and 6 are two A.Ms between a and b , find a and b .

Solution:-

Here A. Ms between $a, 3, 6, b$

$$d = 6 - 3 = 3$$

$$\begin{aligned} b &= a_4 = a_3 + d \\ &= 6 + 3 \end{aligned}$$

$$b = 9$$

$$\text{and } \begin{aligned} a &= a_2 - d \\ &= 3 - 3 \end{aligned}$$

$$a = 0$$

$$a = 0, b = 9$$

We can also find a and b using this method.

$$a + d = 3$$

$$a + 3 = 3$$

$$a = 3 - 3$$

$$a = 0$$

$$6 + d = b$$

$$\text{and } 6 + 3 = b$$

$$9 = b$$

$$b = 9$$

3- Find three A.Ms between 11 and 19.

Solution:-

$$\text{Here } a_1 = 11$$

$$a_5 = 19$$

$$n = 5$$

$$\text{Now } a_n = a + (n - 1)d$$

$$a_5 = a + (5 - 1)d$$

$$\text{Now } a_n = 11 + 4d$$

$$19 = 11 + 4d$$

$$19 - 11 = 4d$$

$$8 = 4d$$

$$d = 2$$

$$A_1 = a + d$$

$$= 11 + 2 = 13$$

$$A_2 = A_1 + d$$

$$= 13 + 2 = 15$$

$$A_3 = A_2 + d$$

$$= 15 + 2 = 17$$

required three A. Ms are 13, 15, 17.

4 Find three A.Ms between $2\sqrt{3}$ and $6\sqrt{3}$

Solution:-

Let A_1, A_2, A_3 be three A.Ms between $2\sqrt{3}$ and $6\sqrt{3}$. Such that $2\sqrt{3}, A_1, A_2, A_3, 6\sqrt{3}$ is an A.P.

$$a_1 = 2\sqrt{3}$$

$$a_5 = 6\sqrt{3}$$

$$n = 5$$

Now $a_n = a + (n - 1)d$

$$6\sqrt{3} - 2\sqrt{3} = 4d$$

Or $4d = 6\sqrt{3} - 2\sqrt{3}$

$$= \sqrt{3}(6 - 2)$$

$$4d = \sqrt{3}(4)$$

$$d = \sqrt{3}$$

$$A_1 = a + d$$

Now $= 2\sqrt{3} + \sqrt{3}$

$$\sqrt{3} + (2 + 1)$$

$$A_1 = 3\sqrt{3}$$

and $A_2 = a + 2d$

$$= 2\sqrt{3} + 2\sqrt{3}$$

$$= \sqrt{3}(2 + 2) = 4\sqrt{3} A_2$$

$$A_3 = a + 3d$$

$$= 2\sqrt{3} + 3\sqrt{3}$$

$$= \sqrt{3}(2 + 3)$$

$$A_3 = 5\sqrt{3}$$

$$3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$$

Thus $3\sqrt{3}, 4\sqrt{3}, 5\sqrt{3}$ are the required three A.Ms between $6\sqrt{3}$ and $2\sqrt{3}$.

5- Find six A.Ms between 5 and 8.

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six A.Ms between 5 and 8. Such that 5, $A_1, A_2, A_3, A_4, A_5, A_6$, 8 is an A.P.

Here $a_1 = 5$

$$a_8 = 8$$

$$n = 8$$

$$a_n = a + (n - 1)d$$

$$8 = 5 + (8 - 1)d$$

$$8 - 5 = 7d$$

$$3 = 7d$$

$$7d = 3 \quad \text{Or}$$

$$d = \frac{3}{7} \quad \text{Or}$$

Now $A_1 = a + d$

$$= 5 + \frac{3}{7} = \frac{38}{7}$$

$$A_2 = a + 2d$$

$$= 5 + 2 \times \frac{3}{7}$$

$$= 5 + \frac{6}{7} = \frac{41}{7}$$

$$A_3 = a + 3d$$

$$= 5 + 3 \times \frac{3}{7}$$

$$= 5 + \frac{9}{7} = \frac{44}{7}$$

$$A_4 = a + 4d$$

$$= 5 + 4 \times \frac{3}{7}$$

$$= 5 + \frac{12}{7} = \frac{47}{7}$$

$$A_5 = a + 5d$$

$$= 5 + 5 \left(\frac{3}{7} \right)$$

$$= 5 + \frac{15}{7} = \frac{50}{7}$$

$$A_6 = a + 6d$$

$$= 5 + 6 \left(\frac{3}{7} \right)$$

$$= 5 + \frac{18}{7}$$

$$= \frac{35 + 18}{7} = \frac{53}{7}$$

Thus $\frac{38}{7}, \frac{41}{7}, \frac{44}{7}, \frac{47}{7}, \frac{50}{7}, \frac{53}{7}$ are six required A.Ms.

between 5 and 8.

6 Find seven A.Ms between 8 and 12.

Solution:-

Let $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ be seven required A.Ms, between 8 and 12.

Such that $8, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 12$ is an A. P.

$$a_1 = 8$$

$$a_9 = 12$$

$$n = 9$$

$$a_n = a + (n - 1)d$$

$$12 = 8 + (a - 1)d$$

$$12 - 8 = 8d$$

$$4 = 8d$$

$$8d = 4 \quad \text{Or}$$

$$d = \frac{4}{8}$$

$$d = \frac{1}{2} \quad \text{Or}$$

$$A_1 = a + d$$

$$= 8 + \frac{1}{2}$$

$$= \frac{16 + 1}{2} = \frac{17}{2}$$

$$A_2 = 8 + 2d \quad \text{Now}$$

$$= 8 + 2\left(\frac{1}{2}\right)$$

$$= 8 + \frac{2}{2}$$

$$= \frac{16 + 2}{2} = \frac{18}{2}$$

$$A_3 = a + 3d$$

$$= 8 + 3\left(\frac{1}{2}\right)$$

$$= 8 + \frac{3}{2}$$

$$= \frac{16+3}{2} = \frac{19}{2}$$

$$A_4 = a + 4d$$

$$= 8 + 4\left(\frac{1}{2}\right) = \frac{8}{1} + \frac{4}{2}$$

$$= \frac{16+4}{2} = \frac{20}{2}$$

$$A_5 = a + 5d$$

$$= 8 + 5\left(\frac{1}{2}\right)$$

$$= \frac{16+5}{2} = \frac{21}{2}$$

$$A_6 = a + 6d$$

$$= 8 + 6\left(\frac{1}{2}\right)$$

$$= \frac{16+6}{2} = \frac{22}{2}$$

$$A_7 = a + 7d$$

$$= 8 + 7\left(\frac{1}{2}\right)$$

$$= \frac{16+7}{2} = \frac{23}{2}$$

Thus $\frac{17}{2}, \frac{18}{2}, \frac{19}{2}, \frac{20}{2}, \frac{21}{2}, \frac{22}{2}, \frac{23}{2}$, are the required

seven A.Ms between 8 and 12.

- 7- If the A.M between 5 and b is 10, then find the value of b .

Solution:-

a, A, b is an A. P.

$$A = \frac{a+b}{2}$$

Here $a = 5$

$$b = b$$

$$A = 10$$

$$A = \frac{a+b}{2}$$

$$10 = \frac{5+b}{2}$$

$$\text{Or } 20 = 5 + b$$

$$\text{Or } 5 + b = 20$$

$$b = 20 - 5$$

$$b = 15$$

- 8- If the A.M between a and 10 is 40, then find the value of " a ".

Solution:-

a, A, b is an A. P.

$$A = \frac{a+b}{2}$$

Here $a = a$

$$b = 10$$

$$A = 40$$

$$A = \frac{a+b}{2} \quad \text{and}$$

$$40 = \frac{a + 10}{2}$$

$$40 \times 2 = a + 10$$

$$80 = a + 10$$

$$a + 10 = 80$$

$$a = 80 - 10 \quad \text{Or}$$

$$a = 70$$

9. If the three A.Ms between a and b are 5, 9 and 13, find a and b .

Solution:-

5, 9 and 13 is an A.P between a and b .

$a, 5, 9, 13, b$ is A. Ms

$$d = 9 - 5 = 4$$

$$= 13 - 9 = 4$$

$$a + d = 5$$

$$a + 4 = 5$$

$$a = 5 - 4$$

$$a = 1 \quad (i)$$

$$13 + d = b \quad \text{and}$$

$$13 + 4 = b$$

$$17 = b$$

$$b = 17 \quad (ii)$$

$$a = 1, b = 17$$

Exercise 7.4

1- Find the 7th term of a G.P 2, 8, 32, ...

Solution: G. P sequence is 2, 8, 32, ...

Given $a = 2$

$$r = \frac{8}{2} = 4$$

$$n = 7$$

$$a_n = ar^{n-1} \quad \text{formula}$$

$$a_7 = ?$$

$$\begin{aligned} a_7 &= 2(4)^{7-1} \\ &= 2(4)^6 \\ &= 2 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \\ &= 2 \times 4096 \end{aligned}$$

$$a_7 = 8192$$

2- Find the 11th term of a G.P 2, 6, 18, ...

Solution:- G. P sequence is 2, 6, 18, ...

Given $a = 2$

$$r = \frac{6}{2} = 3$$

$$n = 11$$

$$a_{11} = ?$$

$$a_n = ar^{n-1} \quad \text{formula}$$

$$a_{11} = 2 \times 3^{11-1}$$

$$= 2 \times 3^{10}$$

$$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= 2 \times 9 \times 9 \times 9 \times 9 \times 9$$

$$= 2 \times 81 \times 81 \times 9$$

$$a_{11} = 118098$$

- 3- Find the 6th term of a G.P $-\frac{3}{2}, 3, -6, \dots$

Solution:-

In term of a G. P $-\frac{3}{2}, 3, -6, \dots$

$$a = -\frac{3}{2}$$

$$r = -\frac{6}{3} = -2$$

$$n = 6$$

$$a_6 = ?$$

$$a_n = ar^{n-1} \quad \text{formula}$$

$$a_6 = \left(-\frac{3}{2}\right)(-2)^{6-1} \quad \text{Now}$$

$$= \left(-\frac{3}{2}\right)(-2)(-2)(-2)(-2)(-2)$$

$$= \left[\left(-\frac{3}{2}\right)(-2)\right][(-2)(-2)][(-2)(-2)]$$

$$= 3 \times 4 \times 4$$

$$= 48$$

- 4- Find the 5th term of a G.P 4, -12, 36, ...

Solution:-

In term of a G. P 4, -12, 36, ...

$$a = 4$$

$$d = \frac{-12}{4} = -3$$

$$n = 5$$

$$a_5 = ?$$

$$a_n = ar^{n-1} \quad \text{formula}$$

$$a_5 = 4(-3)^{5-1}$$

$$= 4(-3)^4$$

$$= 4(-3)(-3)(-3)(-3)$$

$$= 4 \times 9 \times 9$$

$$a_5 = 324$$

5- Find the missing elements of the G.P

(i) $r = 10, a_n = 100, a = 1$

(ii) $a_n = 400, r = 2, a = 25$

(iii) $a = 128, r = \frac{1}{2}, a_n = \frac{1}{4}$

Solution: 5(i)

$$r = 10$$

$$a_n = 100$$

$$a = 1$$

$$n = ?$$

$$a_n = ar^{n-1} \quad \text{formula}$$

Putting values of a, a_n, r

$$100 = (1)(10)^{n-1}$$

$$(10)^2 = (10)^{n-1}$$

$$(10)^2 = (10)^{n-1}$$

Therefore $n-1 = 2$

$$n = 2 + 1 = 3$$

Solution: 5(ii)

$$a_n = 400$$

$$r = 2$$

$$a = 25$$

$$n = ?$$

$$a_n = ar^{n-1} \quad \text{formula}$$

Putting values a, r, a_n

$$400 = 25(2)^{n-1}$$

$$\frac{400}{25} = 2^{n-1}$$

$$16 = 2^{n-1}$$

$$2^4 = 2^{n-1}$$

Therefore $n-1 = 4$

$$n = 4 + 1 = 5$$

Solution: 5(iii)

$$a = 128$$

$$r = \frac{1}{2}$$

$$a_n = \frac{1}{4}$$

$$n = ?$$

$$a^n = ar^{n-1} \quad \text{Formula}$$

Putting values a, r, a_n

$$\frac{1}{4} = 128 \left(\frac{1}{2} \right)^{n-1}$$

$$\frac{1}{4 \times 128} = \left(\frac{1}{2} \right)^{n-1}$$

$$\frac{1}{32 \times 4 \times 4} = \left(\frac{1}{2} \right)^{n-1}$$

$$\frac{1}{2^2 \times 2^2 \times 2^2 \times 2^3} = \left(\frac{1}{2} \right)^{n-1}$$

$$\frac{1}{2^9} = \left(\frac{1}{2} \right)^{n-1}$$

$$\left(\frac{1}{2} \right)^9 = \left(\frac{1}{2} \right)^{n-1}$$

$$n - 1 = 9$$

$$n = 9 + 1$$

Hence $n = 10$

- 6- Find the 11th term of a G.P whose 5th term is 9 and common ratio is 2.

Solution:

$$r = 2$$

$$a_5 = 9$$

$$a_{11} = ?$$

$$a_n = ar^{n-1} \quad \text{formula}$$

$$a_5 = ar^{5-1} = ar^4 \quad \text{Putting values of } a_5 \text{ and } r.$$

$$a_5 r$$

$$9 = a(2)^4$$

$$9 = a \times 16$$

$$\frac{9}{16} = a$$

$$a = \frac{9}{16}$$

Now we want to find $a_{11} = ar^{11-1}$
putting values of a and r .

$$a_{11} = \frac{9}{16} \times 2^{10}$$

$$a_{11} = \frac{9}{16} \times 1024$$

$$a_{11} = 9 \times 64$$

$$= 576$$

- 7- Find the 13th term of a G.P whose 7th term is 25 and common ratio is 3.

Solution:

$$a_7 = 25$$

$$r = 3$$

$$a_{13} = ?$$

$$a_n = ar^{n-1} \quad \text{formula}$$

$$a_7 = ar^{7-1} = ar^6 \quad \text{putting values of } a_7 \text{ and } r.$$

$$25 = a(3)^6$$

$$25 = a \times 729$$

$$\frac{25}{729} = a$$

$$a = \frac{25}{729}$$

$$a_{13} = ar^{13-1} \quad \text{Now}$$

Putting values of a , a_{13}

$$= \frac{25}{729} (3)^{13-1}$$

$$= \frac{25}{729} \times 3^{12}$$

$$= \frac{25}{3^6} \times 3^6 \times 3^6$$

$$= 25 \times 3^6$$

$$= 25 \times 729 = 18225$$

- 8- If a, b, c, d are in G.P, show that, $a-b, b-c, c-d$, are in G.P

Solution:-

a, b, c, d is G.P therefore

$$(A) \quad \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$$

$$\frac{c}{b} = \frac{d}{c} \quad \text{and}$$

$$\frac{b}{a} = \frac{d}{c}$$

If $a-b, b-c, c-d$ is G.P.

Then
$$\frac{b-c}{a-b} = \frac{c-d}{b-c}$$

$$(b-c)(b-c) = (a-b)(c-d)$$

$$b^2 - bc - bc + c^2 = ac - ad - bc + bd$$

$$(i) \quad b^2 = ca \quad \left(\frac{b}{a} = \frac{c}{b} \right)$$

$$\left(\frac{c}{b} = \frac{d}{c} \right) \quad c^2 = bd \quad (ii)$$

$$\left(\frac{b}{a} = \frac{d}{c} \right) \quad bc = ad \quad (iii)$$

$$b^2 - bc - bc + c^2$$

$$= ca - bc - ad + bd \quad \text{From (i), (ii), (iii)}$$

$$a - b, b - c, c - d \quad \text{is G.P}$$

Second Law:

a, b, c, d is G.P

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} \quad \text{Therefore}$$

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} \quad (A) \quad \text{Or}$$

Now if $a-b, b-c, c-d$ is G.P

$$\text{Then } \frac{b-c}{a-b} = \frac{c-d}{b-c} \quad \text{prove}$$

$$\text{Now } \frac{a}{b} = \frac{b}{c} \quad \text{from A}$$

$$\frac{a}{b} - 1 = \frac{b}{c} - 1$$

$$\frac{a-b}{b} = \frac{b-c}{c}$$

$$\frac{a-b}{b-c} = \frac{b}{c}$$

$$\frac{b-c}{a-b} = \frac{c}{b} \quad (i)$$

$$\frac{b}{c} = \frac{c}{d}$$

$$\frac{b}{c} - 1 = \frac{c}{d} - 1$$

$$\frac{b-c}{c} = \frac{c-d}{d}$$

$$\frac{b-c}{c-d} = \frac{c}{d}$$

$$\frac{c-d}{b-c} = \frac{d}{c}$$

$$\frac{c-d}{b-c} = \frac{c}{b} \quad (ii) \left(\frac{c}{b} = \frac{d}{c} \right) \quad (A)$$

$$\frac{b-c}{a-b} = \frac{c-d}{b-c} \quad (i), (ii)$$

9- Find the n^{th} term of a G.P, if $\frac{a_5}{a_3} = \frac{4}{9}$ and a

Solution:

$$\frac{a_5}{a_3} = \frac{4}{9}$$

$$\frac{ar^{5-1}}{ar^{3-1}} = \frac{4}{9}$$

$$\frac{r^4}{r^2} = \frac{4}{9}$$

$$r^2 = \left(\frac{2}{3}\right)^2$$

$$r = \frac{2}{3} \quad (A)$$

$$a_2 = ar^{2-1} \quad (i)$$

$$a_2 = \frac{4}{9} \quad (ii)$$

$$ar = \frac{4}{9} \quad \text{from (i) and (ii)}$$

$$a \times \frac{2}{3} = \frac{4}{9} \quad \text{from (A)}$$

$$a = \frac{4}{9} \times \frac{3}{2}$$

$$a = \frac{2}{3}$$

Now $a_n = ar^{n-1}$

$$= \frac{2}{3} \left(\frac{2}{3} \right)^{n-1}$$

$$a_n = \left(\frac{2}{3} \right)^{n-1+1} = \left(\frac{2}{3} \right)^n$$

10- Find three consecutive numbers in G.P, whose sum is 26 and their product is 216.

Solution:-

Three consecutive numbers according to the given condition a, ar, ar^2 .

$$a + ar + ar^2 = 26$$

$$(a)(ar)(ar^2) = 216 \quad \text{and}$$

$$a^3 r^3 = 216$$

$$(ar)^3 = (6)^3$$

$$ar = 6 \quad (A)$$

$$a = \frac{6}{r} \quad \text{Putting (i)}$$

$$a + ar + ar^2 = 26$$

$$a(1 + r + r^2) = 26$$

$$a = \frac{6}{r} \quad (i)$$

$$(1 + r + r^2) = 26 \frac{6}{r}$$

$$6(1 + r + r^2) = 26r$$

$$6 + 6r + 6r^2 - 26r = 0$$

$$6r^2 - 20r + 6 = 0 \quad \text{Dividing by 2}$$

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - r - 9r + 3 = 0$$

$$r(3r - 1) - 3(3r - 1) = 0$$

$$(3r - 1)(r - 1) = 0$$

$$3r - 1 = 0$$

$$3r = 1$$

$$r = \frac{1}{3} \quad (\text{ii})$$

and

$$r - 3 = 0$$

$$r = 3 \quad \text{then (B)}$$

$$ar = 6 \quad \text{Now (A)}$$

$$a(3) = 6 \quad \text{Now (B)}$$

$$a = \frac{6}{3}$$

$$a = 2 \quad (\text{C})$$

Take 3 or $a = 2, r = \frac{1}{3}$

$$a = 2$$

$$ar = 2 \times \frac{1}{3} = \frac{2}{3}$$

$$ar^2 = 2 \times \left(\frac{1}{3}\right)^2 = 2 \times \frac{1}{9} \\ = \frac{2}{9}$$

Three consecutive numbers are $2, \frac{2}{3}, \frac{2}{9}$

$$\text{If } r = 3$$

$$\text{Then } a = 2$$

$$ar = 2 \times 3 = 6$$

$$ar^2 = 2 \times 3^2 = 2 \times 9 = 18$$

Consecutive number 2, 6, 18

11- Find the 30th term of a G.P $x, 1, \frac{1}{x}, \dots$

Solution:

G. P is $x, 1, \frac{1}{x}, \dots$

$$a = x \quad (i)$$

$$r = \frac{1}{x} \quad (ii)$$

$$a_n = ar^{n-1}$$

$$a_{30} = ar^{30-1}$$

from (i) and (ii)

$$= x \times \left(\frac{1}{x} \right)^{29}$$

$$= x \times x^{-29}$$

$$= x^{1-29}$$

$$= x^{-28}$$

$$= x^{\frac{1}{28}}$$

12- Find the p^{th} term of a G.P $x, x^3, x^5, \dots$

Solution:-

G.P is $x, x^3, x^5, \dots$

Therefore $a = x$

$$r = \frac{x^3}{x}$$

$$x^2 =$$

$$a_n = ar^{n-1} \quad \text{formula}$$

$$a_p = (x)(x^2)^{p-1}$$

$$= (x)x^{2(p-1)}$$

$$= x \cdot x^{2p-2}$$

$$= x^{2p-2+1}$$

$$a_p = x^{2p-1}$$

Exercise 7.5

G.M between a, b is G . $G = \pm\sqrt{ab}$

1- Find G.M between: (i) 9 and 5 (ii) 4 and 9
(iii) -2 and -8.

(i) 9 and 5

(ii) 4 and 9

(iii) -2 and -8

Solution:(i)

Given 9 and 5

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{9 \times 5} \\ &= \pm 3\sqrt{5} \end{aligned}$$

Solution:- (ii)

4 and 9

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{(4)(9)} \\ &= \pm\sqrt{36} \\ &= \pm 6 \end{aligned}$$

Solution:- (iii)

-2 and -8

$$\begin{aligned} G &= \pm\sqrt{ab} \\ &= \pm\sqrt{(-2)(-8)} \\ &= \pm\sqrt{16} \\ &= \pm 4 \end{aligned}$$

2- Insert two G.Ms between: (i) 1 and 8 (ii) 3 and 81

(i) 1 and 8

(ii) 3 and 81

Solution:-

Let G_1 and G_2 be the two G.Ms between 1 and 8 such that 1, G_1 , G_2 , 8 is a G.P.

Here $a = 1$

$$n = 4$$

$$a_4 = 8$$

Since $a_n = ar^{n-1}$

$$a_4 = ar^{4-1}$$

$$8 = ar^3$$

$$8 = 1 \times r^3$$

$$2^3 = r^3$$

$$r = 2$$

Thus $G_1 = ar$

$$= 1 \times 2 = 2$$

$$G_2 = ar^2$$

$$= 1 \times 2^2$$

$$= 1 \times 4 = 4$$

Solution:- (ii) 3 and 81

Let G_1 and G_2 be the two GMs between 3 and 81, such that 3, G_1 , G_2 , 81 is a G.P.

$$n = 4$$

Here $a = 3$

$$n = 4$$

$$a_4 = 81$$

Since $a_n = ar^{n-1}$

$$a_4 = ar^{4-1}$$

$$81 = 3r^3$$

$$r^3 = \frac{81}{3} = 27$$

$$r^3 = 3^3$$

$$r = 3$$

Thus $G_1 = ar$

$$= 3 \times 3$$

$$= 9$$

$$\begin{aligned}
 G_2 &= ar^2 \\
 &= 3(3)^2 \\
 &= 3 \times 9 = 27
 \end{aligned}$$

3- **Insert three G.Ms between: (i) 1 and 16 (ii) 2 and 32**

(i) **1 and 16**

(ii) **2 and 32**

Solution:-

Let G_1, G_2 and G_3 be the three G.Ms between 1 and 16, that are required. G.Ms 1, G_1, G_2, G_3 16 is a G. P.

Here $a = 1$

$$n = 5$$

$$a_5 = 16$$

Since $a_n = ar^{n-1}$

$$a_5 = (1)r^{5-1}$$

$$16 = r^4$$

$$r^3 = r^4$$

Thus $2^4 = r^4$

$$r = 2$$

$$G_1 = ar$$

$$= 1 \times 2 = 2$$

$$G_2 = ar^2 = 1 \times 2^2 = 4$$

$$G_3 = ar^3 = 1 \times 2^3 = 8$$

2, 4, 8 are required three G. Ms

Solution:- 2 and 32

Let G_1, G_2, G_3 be the three G.Ms between 2 and 32 that are required. G.Ms 2, G_1, G_2 and G_3 32 is a G. P.

Here $a = 2$

$$n = 5$$

$$a_5 = 32$$

Since $a_n = ar^{n-1}$

$$a_5 = ar^{5-1}$$

$$32 = 2r^4$$

$$r^4 = 16$$

Thus $r^4 = 2^4$

$$r = 2$$

$$\begin{aligned}\text{Now } G_1 &= ar \\ &= 2 \times 2 = 4\end{aligned}$$

$$\begin{aligned}G_2 &= ar^2 \\ &= 2 \times 2^2 = 8\end{aligned}$$

$$\begin{aligned}G_3 &= ar^3 \\ &= 2 \times 2^3 = 16\end{aligned}$$

4, 8 and 16 are required three G.Ms.

4- *Insert four real geometric means between 3 and 96.*

Solution:- 3 and 96

Let G_1, G_2, G_3 and G_4 be the four G.Ms between 3 and 96 that are required. G. Ms 3, G_1, G_2, G_3 and $G_4, 96$ is a G. P.

$$\text{Here } a = 3$$

$$n = 6$$

$$a_6 = 96$$

$$\text{Since } a_n = ar^{n-1}$$

$$a_6 = ar^{6-1}$$

$$96 = 3r^5$$

$$\frac{96}{3} = r^5$$

$$r^5 = \frac{96}{3} = 32$$

$$r^5 = 2^5$$

$$r = 2$$

$$\begin{aligned}G_1 &= ar \\ &= 3 \times 2 = 6\end{aligned}$$

$$\begin{aligned}G_2 &= 3r^2 \\ &= 3(2)^2 = 12\end{aligned}$$

$$\begin{aligned}G_3 &= 3r^3 \\ &= 3(2)^3 = 24\end{aligned}$$

$$\begin{aligned}G_4 &= 3(r)^4 \\ &= 3(2)^4 = 48\end{aligned}$$

6, 12, 24, 48 are four required G.Ms.

- 5- The A.M between two numbers is 5 and their positive G.M is 4. Find the numbers.

Solution:-

Let numbers are a and b .

$$(i) \text{ A is A.M } A = \frac{a+b}{2}$$

$$(ii) \text{ G is G.M } G = \sqrt{ab}$$

According to condition $A = 5, G = 4$

$$5 = \frac{a+b}{2} \quad \text{from (i) and (ii)}$$

$$4 = \sqrt{ab}$$

$$16 = ab \quad \text{from (iii)}$$

$$10 = a + b \quad \text{from (iv)}$$

$$10 - b = a \quad \text{Or}$$

$$a = 10 - b \quad \text{Or}$$

Putting values of a in (iii)

$$16 = (10 - b)(b)$$

$$16 = 10b - b^2$$

$$b^2 - 10b + 16 = 0 \quad \text{Or}$$

$$b^2 - 2b - 8b + 16 = 0$$

$$b(b-2) - 8(b-2) = 0$$

$$(b-2)(b-8) = 0$$

$$b - 2 = 0$$

$$b = 2$$

$$b - 8 = 0$$

$$b = 8$$

$$a + b = 10$$

$$a + 2 = 10$$

$$a = 10 - 2 = 8$$

$$a + 8 = 10$$

$$a = 10 - 8 = 2$$

Now from (iv)

$$b = 8$$

Hence Numbers are 2,8 or 8,2

- 6- The positive G.M between two numbers is 6 and the A.M between them is 10. Find the numbers.

Solution:-

$$A \text{ is A.M} = \frac{a+b}{2} \quad (i)$$

$$G \text{ is G.M} = \sqrt{ab} \quad (ii)$$

From (i) and (ii)

$$10 = \frac{a+b}{2} \quad \text{Or}$$

$$20 = a+b \quad (iii)$$

$$6 = \sqrt{ab} \quad \text{Or}$$

$$36 = ab \quad (iv)$$

$$a = 20 - b \quad \text{from (iii)}$$

Putting values of a in (iv)

$$36 = (20 - b)(b)$$

$$36 = 20b - b^2$$

$$b^2 - 20b + 36 = 0 \quad \text{Or}$$

$$b^2 - 2b - 18b + 36 = 0$$

$$b(b-2) - 18(b-2) = 0$$

$$(b-2)(b-18) = 0$$

$$\text{If } b-2 = 0$$

$$b-18 = 0$$

$$b-2 = 0$$

$$b = 2$$

Putting $b = 2$ in (iv)

$$36 = 2a$$

$$a = 18$$

$$b-18 = 0 \quad \text{and}$$

$$b = 18$$

Putting $b = 18$ in (iv)

$$36 = 18a$$

$$a = 2$$

Thus numbers are 2, 18 or 18, 2

- 7- Show that the A.M between the two numbers 4 and 8 is greater than their geometric mean.

Solution:-

Here $a = 4$

$$b = 8$$

$$(A. M) = \frac{4 + 8}{2} = 6$$

$$(G. M) = \sqrt{ab}$$

$$G = \sqrt{4 \times 8}$$

$$= \sqrt{32}$$

Now $\sqrt{32}$ is less than 6.

Proved $6 > \sqrt{32}$

Therefore $A > G$

- 8- Insert four geometric means between 160 and 5.

Solution:-

numbers are 5 and 160.

Let G_1, G_2, G_3 and G_4 the four G.Ms between 160 and 5.

Here $n = 6$

$$a = 5$$

$$a_6 = 160$$

Since $a_n = ar^{n-1}$

$$a_6 = 5r^{6-1}$$

$$160 = 5r^5$$

$$r^5 = \frac{160}{5}$$

$$r^5 = 32 = 2^5$$

$$r = 2$$

$$G_1 = ar$$

$$\begin{aligned}
 &= 5 \times 2 = 10 \\
 G_2 &= ar^2 \\
 &= 5 \times 2^2 = 20 \\
 G_3 &= ar^3 \\
 &= 5 \times 2^3 = 40 \\
 G_4 &= 5 \times 2^4 = 80
 \end{aligned}$$

Hence required G.Ms are 10, 20, 40, 80.

9- *Insert three geometric means between 486 and 6.*

Solution:- Let G_1, G_2 and G_3 the three G. Ms between 186 and 6.

Here $n = 5$

$$a = 486$$

$$a_5 = 6$$

Since $a_n = ar^{n-1}$

$$a_5 = ar^{5-1}$$

$$6 = 486r^4$$

$$\frac{6}{486} = r^4$$

$$\frac{1}{81} = r^4$$

$$r^4 = \frac{1}{81} = \left(\frac{1}{3}\right)^4$$

$$r = \frac{1}{3}$$

$$G_1 = ar$$

$$= 486 \times \frac{1}{3} = 162$$

$$G_2 = 486 \times \left(\frac{1}{3}\right)^2$$

$$= 486 \times \frac{1}{9} = 54$$

$$G_3 = 486 \times \left(\frac{1}{3}\right)^3$$

$$= 486 \times \frac{1}{27} = 18$$

Hence required G.Ms are 162, 54, 18.

10- Insert four geometric means between $\frac{1}{8}$ and 128.

Solution:- $\frac{1}{8}, 128$

Let G_1, G_2, G_3, G_4 be the four G.Ms between $\frac{1}{8}$ and 128.

Here $n = 6$

$$a = \frac{1}{8}$$

$$a_6 = 128$$

$$a_n = ar^{n-1}$$

$$a_6 = ar^{6-1}$$

$$128 = \frac{1}{8} r^5$$

$$128 \times 8 = r^5$$

$$1024 = r^5$$

$$4^5 = r^5$$

$$r = 4$$

$$G_1 = ar$$

$$= \frac{1}{8} \times 4 = \frac{1}{2}$$

$$G_2 = ar^2$$

$$= \frac{1}{8} \times 4^2$$

$$= \frac{1}{8} \times 16 = 2$$

$$G_3 = ar^3$$

$$= \frac{1}{8} \times 4^3$$

$$= \frac{1}{8} \times 64 = 8$$

$$G_4 = \frac{1}{8} r^4$$

$$= \frac{1}{8} \times 4^4 = 32$$

$\frac{1}{2}, 2, 8, 32$ are required G.Ms.

11. Insert six geometric means between 56 and $-\frac{7}{16}$

Solution:- 56, $-\frac{7}{16}$

Let $G_1, G_2, G_3, G_4, G_5, G_6$ be the six G.Ms between 56

and $-\frac{7}{16}$

Here $n = 8$

$a = 56$

$a_6 = -\frac{7}{16}$

Since $a_n = ar^{n-1}$

$a_6 = ar^{6-1}$

$$-\frac{7}{16} = 56r^7$$

$$r^7 = -\frac{7}{16 \times 56}$$

$$= -\frac{1}{16 \times 8}$$

$$= -\frac{1}{128}$$

$$r^7 = \left(-\frac{1}{2}\right)^7$$

$$r = -\frac{1}{2}$$

$$G_1 = ar$$

$$= 56 \left(-\frac{1}{2}\right) = -28$$

$$G_2 = ar^2$$

$$= 56 \left(-\frac{1}{2}\right)^2$$

$$= 56 \times \frac{1}{4} = 14$$

$$G_3 = ar^3$$

$$= 56 \times \left(-\frac{1}{2}\right)^3$$

$$= 56 \times \left(-\frac{1}{8}\right) = -7$$

$$G_4 = ar^4$$

$$= 56 \left(-\frac{1}{2} \right)^4$$

$$= 56 \times \frac{1}{16} = \frac{7}{2}$$

$$G_5 = ar^5$$

$$= a \left(-\frac{1}{2} \right)^5$$

$$= 56 \times \left(-\frac{1}{32} \right) = -\frac{7}{4}$$

$$G_6 = ar^6$$

$$= 56 \times \left(-\frac{1}{2} \right)^6$$

$$= 56 \times \frac{1}{64} = \frac{7}{8}$$

Six required G. Ms are $-28, 14, -7, \frac{7}{2}, -\frac{7}{4}, \frac{7}{8}$.

12- Insert five geometric means between $\frac{32}{81}$ and $\frac{9}{2}$

Solution:-

Numbers are $\frac{32}{81}, \frac{9}{2}$

Let G_1, G_2, G_3, G_4, G_5 be the five G. Ms between $\frac{32}{81}$

and $\frac{9}{2}$.

Here $n = 7$

$$a = \frac{32}{81}$$

$$a_6 = +\frac{9}{2}$$

Since $a_n = ar^{n-1}$

$$a_n = ar^{6-1}$$

$$\frac{9}{2} = \frac{32}{81} r^5$$

$$r^5 = \frac{9}{2} \times \frac{32}{81}$$

$$r^5 = \frac{729}{64} = \left(\frac{3}{2}\right)^5$$

Therefore $r = \frac{3}{2}$

$$G_1 = ar$$

$$= \frac{32}{81} \times \frac{3}{2} = \frac{16}{27}$$

$$G_2 = ar^2$$

$$= \frac{32}{81} \times \left(\frac{3}{2}\right)^2$$

$$= \frac{32}{81} \times \frac{9}{4} = \frac{8}{9}$$

$$G_3 = ar^3$$

$$\begin{aligned} &= \frac{32}{81} \times \left(\frac{3}{2}\right)^3 \\ &= \frac{32}{81} \times \frac{27}{8} = \frac{4}{3} \end{aligned}$$

$$G_4 = ar^4$$

$$\begin{aligned} &= \frac{32}{81} \times \left(\frac{3}{2}\right)^4 \\ &= \frac{32}{81} \times \frac{81}{16} = 2 \end{aligned}$$

$$G_5 = ar^5$$

$$\begin{aligned} &= \frac{32}{81} \times \left(\frac{3}{2}\right)^5 \\ &= \frac{32}{81} \times \frac{243}{32} = 3 \end{aligned}$$

Five required G.Ms are $\frac{16}{27}, \frac{8}{9}, \frac{4}{3}, 2, 3$