

EXERCISE 5.1

1- Determine the radicals and the radicands from the following.

(i) $\sqrt{3}$

(ii) $4 + 3\sqrt{a}$

(iii) $\sqrt{11}$

(iv) $8 - 2\sqrt{6}$

(v) $\frac{\sqrt{5}}{7}$

(vi) $\frac{9}{\sqrt{13}}$

Solution:-

	Radical	Radicands
(i) $\sqrt{3}$	$\sqrt{3}$	3
(iii) $\sqrt{11}$	$\sqrt{11}$	11
(v) $\frac{\sqrt{5}}{7}$	$\sqrt{5}$	5

	Radical	Radicands
(ii) $4 + 3\sqrt{a}$	$\sqrt{a}$	a
(iv) $8 - 2\sqrt{6}$	$\sqrt{6}$	6
(vi) $\frac{9}{\sqrt{13}}$	$\sqrt{13}$	13

2 Express the following in exponential form:

(i) $\sqrt{a^3}$

(ii) ${}^5\sqrt{a^3}$

(iii) $\frac{1}{{}^p\sqrt{a^k}}$

(iv) $\frac{1}{{}^b\sqrt{a^k}}$

Solution:-

$$\begin{aligned}
 \text{(i)} \quad \sqrt{a^3} &= (a^3)^{\frac{1}{2}} \\
 &= a^{3 \times \frac{1}{2}} \\
 &= a^{\frac{3}{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii)} \quad {}^5\sqrt{a^3} &= (a^3)^{\frac{1}{5}} \\
 &= a^{3 \times \frac{1}{5}} \\
 &= a^{\frac{3}{5}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii)} \quad \frac{1}{\sqrt[p]{a^k}} &= \frac{1}{(a^k)^{\frac{1}{p}}} \\
 &= \frac{1}{a^{k \times \frac{1}{p}}} \\
 &= a^{-\frac{k}{p}}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad \frac{1}{\sqrt[b]{a^k}} &= \frac{1}{(a^k)^{\frac{1}{b}}} \\
 &= \frac{1}{a^{k \times \frac{1}{b}}} \\
 &= \frac{1}{a^{\frac{k}{b}}} \\
 &= a^{-\frac{k}{b}}
 \end{aligned}$$

3. Write in the radical form and evaluate the result.

(i) $(25)^{1/2}$

(ii) $(64)^{1/3}$

(iii) $(81)^{1/4}$

(iv) $(27)^{1/3}$

(v) $(27)^{2/3}$

(vi) $8^{-1/3}$

(vii) $(1000)^{2/3}$

(viii) $(64)^{1/2}$

Solution:-(i)

$$\begin{aligned}
 (25)^{1/2} &= \sqrt{25} \\
 &= \sqrt{5^2} \\
 &= (5^2)^{\frac{1}{2}} \\
 &= 5^{2 \times \frac{1}{2}} = 5
 \end{aligned}$$

$$\sqrt{25} = 5 \text{ Ans.}$$

Solution:-(ii)

$$(64)^{1/3} = \sqrt[3]{64}$$

$$\begin{aligned}
 &= \sqrt[3]{4^3} \\
 &= (4^3)^{\frac{1}{3}} \\
 &= 4^{3 \times \frac{1}{3}} = 4
 \end{aligned}$$

$$\sqrt[3]{64} = 4 \text{ Ans.}$$

Solution:-(iii)

$$\begin{aligned}
 (81)^{3/4} &= \sqrt[4]{81} \\
 &= \sqrt[4]{3^4} \\
 &= (3^4)^{\frac{1}{4}} \\
 &= 3^{4 \times \frac{1}{4}} = 3
 \end{aligned}$$

$$\sqrt[4]{81} = 3 \text{ Ans.}$$

Solution:-(iv)

$$\begin{aligned}
 (27)^{1/3} &= \sqrt[3]{(27)} \\
 &= \sqrt[3]{(3^3)} \\
 &= 3^{3 \times \frac{1}{3}} = 3
 \end{aligned}$$

$$\sqrt[3]{27} = 3$$

Solution:-(v)

$$\begin{aligned}
 (27)^{2/3} &= \sqrt[3]{(27)^2} \\
 &= \sqrt[3]{(3^3)^2} \\
 &= \sqrt[3]{3^{3 \times 2}}
 \end{aligned}$$

$$= 3^{3 \times 2 \times \frac{1}{3}}$$

$$= 3^2 = 9$$

$$\sqrt[3]{(27)^2} = 9 \text{ Ans.}$$

Solution:-(vi)

$$8^{-1/3} = \frac{1}{8^{1/3}}$$

$$= \frac{1}{\sqrt[3]{8}}$$

$$= \frac{1}{\sqrt[3]{2^3}}$$

$$= \frac{1}{2^{3 \times \frac{1}{3}}}$$

$$= \frac{1}{2}$$

$$\frac{1}{\sqrt[3]{8}} = \frac{1}{2} \text{ Ans.}$$

Solution:-(vii)

$$(1000)^{2/3} = \sqrt[3]{(1000)^2}$$

$$= \sqrt[3]{(10^3)^2}$$

$$= \sqrt[3]{(10)^{3 \times 2}}$$

$$= (10)^{3 \times 2 \times \frac{1}{3}}$$

$$= (10)^2 = 100$$

$$\sqrt[3]{(1000)^2} = 100 \text{ Ans.}$$

$$\begin{aligned}
 (64)^{1/2} &= \sqrt{64} \\
 &= \sqrt{8^2} \\
 &= 8^{2 \times \frac{1}{2}} = 8 \\
 \sqrt{64} &= 8 \text{ Ans.}
 \end{aligned}$$

4. *Simplify and give answer in exponential form.*

- | | |
|---------------------------------|------------------------------------|
| (i) $\sqrt{a^{16}}$ | (ii) $\sqrt[3]{a^{15}}$ |
| (iii) $\sqrt[3]{27a^9}$ | (iv) $\sqrt[3]{8a^9}$ |
| (v) $\sqrt[4]{x^{32}}$ | (vi) $\sqrt[4]{81x^{20}}$ |
| (vii) $\sqrt[3]{125x^9y^{15}}$ | (viii) $\sqrt{(8+y)^7}$ |
| (ix) $\sqrt[4]{16x^2y^6}$ | (x) $\sqrt[4]{\frac{x^5y^6}{z^2}}$ |
| (xi) $\sqrt[3]{\frac{8x}{x+y}}$ | (xii) $\sqrt[p]{\frac{y^n}{a^m}}$ |

Solution:-(i)

$$\begin{aligned}
 \sqrt{a^{16}} &= (a^{16})^{\frac{1}{2}} \\
 &= a^{16 \times \frac{1}{2}} \\
 &= a^8 \text{ Ans.}
 \end{aligned}$$

Solution:-(ii)

$$\begin{aligned}
 \sqrt[3]{a^{15}} &= (a^{15})^{\frac{1}{3}} \\
 &= a^{15 \times \frac{1}{3}} \\
 &= a^5 \text{ Ans.}
 \end{aligned}$$

Solution:-(iii)

$$\begin{aligned}\sqrt[3]{27a^9} &= \sqrt[3]{3^3 a^9} \\ &= (3^3 a^9)^{\frac{1}{3}} \\ &= 3^{3 \times \frac{1}{3}} a^{9 \times \frac{1}{3}} \\ &= 3a^3 \text{ Ans.}\end{aligned}$$

Solution:-(iv)

$$\begin{aligned}\sqrt[3]{8a^9} &= \sqrt[3]{2^3 a^9} \\ &= (2^3 a^9)^{\frac{1}{3}} \\ &= 2^{3 \times \frac{1}{3}} a^{9 \times \frac{1}{3}} \\ &= 2a^3 \text{ Ans.}\end{aligned}$$

Solution:-(v)

$$\begin{aligned}\sqrt[4]{x^{32}} &= (x^{32})^{\frac{1}{4}} \\ &= x^{32 \times \frac{1}{4}} \\ &= x^8 \text{ Ans.}\end{aligned}$$

Solution:-(vi)

$$\begin{aligned}\sqrt[4]{81x^{20}} &= \sqrt[4]{3^4 x^{20}} \\ &= (3^4 x^{20})^{\frac{1}{4}} \\ &= 3^{4 \times \frac{1}{4}} x^{20 \times \frac{1}{4}} \text{ Ans.} \\ &= 3x^5\end{aligned}$$

Solution:-(vii)

$$\begin{aligned}
 \sqrt[3]{125x^9y^{15}} &= \sqrt[3]{5^3x^9y^{15}} \\
 &= (5^3x^9y^{15})^{\frac{1}{3}} \\
 &= 5^{3 \times \frac{1}{3}} x^{9 \times \frac{1}{3}} y^{15 \times \frac{1}{3}} \\
 &= 5x^3y^5 \text{ Ans.}
 \end{aligned}$$

Solution:-(viii)

$$\begin{aligned}
 \sqrt{(8+y)^7} &= \{(8+y)^7\}^{\frac{1}{2}} \\
 &= (8+y)^{7 \times \frac{1}{2}} \\
 &= (8+y)^{\frac{7}{2}}
 \end{aligned}$$

Solution:-(ix)

$$\begin{aligned}
 \sqrt[4]{16x^2y^6} &= \sqrt[4]{2^4x^2y^6} \text{ Ans.} \\
 &= (2^4x^2y^6)^{\frac{1}{4}} \\
 &= 2^{4 \times \frac{1}{4}} x^{2 \times \frac{1}{4}} y^{6 \times \frac{1}{4}} \\
 &= 2x^{\frac{1}{2}}y^{\frac{3}{2}} \text{ Ans.}
 \end{aligned}$$

Solution:-(x)

$$\begin{aligned}
 \sqrt[4]{\frac{x^5y^6}{z^2}} &= \left(\frac{x^5y^6}{z^2} \right)^{\frac{1}{4}} \\
 &= \frac{x^{5 \times \frac{1}{4}} y^{6 \times \frac{1}{4}}}{z^{2 \times \frac{1}{4}}} \\
 &= \frac{x^{\frac{5}{4}} y^{\frac{3}{2}}}{z^{\frac{1}{2}}} \\
 &= \frac{x^{\frac{5}{4}} y^{\frac{3}{2}}}{z^{\frac{1}{2}}} \text{ Ans.}
 \end{aligned}$$

Solution:-(xi)

$$\sqrt[3]{\frac{8x}{x+y}} = \sqrt[3]{\frac{2^3 x}{x+y}}$$

$$= \left(\frac{2^3 x}{x+y} \right)^{\frac{1}{3}}$$

$$= \frac{2^{3 \times \frac{1}{3}} x^{\frac{1}{3}}}{(x+y)^{\frac{1}{3}}}$$

Solution:-(xii)

$$= \frac{2x^{\frac{1}{3}}}{(x+y)^{\frac{1}{3}}} \text{ Ans.}$$

$$\sqrt[p]{\frac{y^m}{a^n}} = \left(\frac{y^m}{a^n} \right)^{\frac{1}{p}}$$

$$= \frac{y^{m \times \frac{1}{p}}}{a^{n \times \frac{1}{p}}}$$

$$= \frac{y^{\frac{m}{p}}}{a^{\frac{n}{p}}} \text{ Ans.}$$

5. **Simplify:**

(i) $\sqrt{3} \times \sqrt{7}$

(iii) $\sqrt[5]{81} \times \sqrt[5]{27}$

(v) $\sqrt[5]{118} \div \sqrt[5]{2}$

(vii) $a^{\frac{1}{4}} \times a^{\frac{2}{3}}$

(ii) $\sqrt[5]{4} \times \sqrt[5]{128}$

(iv) $\sqrt{2} \div \sqrt[3]{32}$

(vi) $\sqrt{27} \div \sqrt{81}$

(viii) $\frac{6}{7} \times \frac{1}{2}$

$$(ix) \left(x^{\frac{3}{4}} \times y^{\frac{1}{6}} \right)^6$$

$$(x) (x^3 y^2)^{\frac{1}{2}} \times (y^3 x^4)^{\frac{-1}{3}}$$

$$(xi) (x^3 y^2)^{\frac{1}{4}} \times \left(x^{\frac{1}{3}} y \right)^{\frac{3}{4}}$$

$$(xii) \left(a^{\frac{1}{4}} b^{\frac{1}{3}} \right)^{\frac{-1}{2}} \div \left(a^{\frac{1}{3}} b^{\frac{1}{4}} \right)^{-5}$$

$$(xiii) (x^2 y^3)^{\frac{1}{5}} \times \left(x^{\frac{1}{3}} y^2 \right)^{\frac{1}{4}}$$

Solution:-(i)

$$\begin{aligned} \sqrt{3} \times \sqrt{7} &= \sqrt{3 \times 7} \\ &= \sqrt{21} \text{ Ans.} \end{aligned}$$

Solution:-(ii)

$$\begin{aligned} \sqrt[5]{4} \times \sqrt[5]{128} &= \sqrt[5]{4 \times 128} \\ &= \sqrt[5]{512} \text{ Ans.} \end{aligned}$$

Solution:-(iii)

$$\begin{aligned} \sqrt[5]{81} \times \sqrt[5]{27} &= \sqrt[5]{81 \times 27} \\ &= \sqrt[5]{2187} \text{ Ans.} \end{aligned}$$

Solution:-(iv)

$$\begin{aligned} \sqrt{2} \div \sqrt[9]{32} &= \sqrt{2} \div \sqrt[9]{2^5} \text{ Ans.} \\ &= \sqrt{2} \div (2^{\frac{5}{9}})^{\frac{1}{9}} \\ &= (2)^{\frac{1}{2}} \div \left(2^{5 \times \frac{1}{9}} \right) \end{aligned}$$

$$= 2^{\frac{1}{2} - \frac{5}{9}}$$

$$= 2^{\frac{1}{2} - \frac{5}{9}}$$

$$= 2^{\frac{-1}{18}}$$

$$= \frac{1}{2^{\frac{1}{18}}}$$

$$= \frac{1}{2^{\frac{1}{9} \times \frac{1}{2}}}$$

Solution:-(v)

$$= \sqrt{2^9}^p \text{ Ans.}$$

$$\sqrt[5]{118} \div \sqrt[5]{2} = \frac{\sqrt[5]{118}}{\sqrt[5]{2}}$$

$$= \sqrt[5]{\frac{118}{2}}$$

Solution:-(vi)

$$\sqrt[3]{59} \text{ Ans.}$$

$$\sqrt{27} \div \sqrt{81} = \frac{\sqrt{27}}{\sqrt{81}}$$

$$= \sqrt{\frac{27}{81}}$$

$$= \sqrt{\frac{3}{9}}$$

$$= \frac{\sqrt{3}}{3} \text{ Ans.}$$

Solution:-(vii)

$$a^{\frac{1}{4}} \times a^{\frac{2}{3}} = a^{\frac{1}{4} + \frac{2}{3}}$$

$$\begin{aligned}
 &= a^{\frac{3+8}{12}} \\
 &= a^{\frac{11}{12}} \\
 &= a^{11 \times \frac{1}{12}} \\
 &= {}^{12}\sqrt{a^{11}} \text{ Ans.}
 \end{aligned}$$

Solution:-(viii)

$$\begin{aligned}
 x^{\frac{6}{7}} y^{\frac{1}{4}} &= x^{\frac{6 \times 4}{7 \times 4}} \times y^{\frac{1}{4}} \\
 &= \sqrt[4]{x^{\frac{24}{7}} y} \text{ Ans.}
 \end{aligned}$$

Solution:-(ix)

$$\begin{aligned}
 \left(x^{\frac{3}{4}} y^{\frac{1}{6}} \right)^6 &= \left(x^{\frac{3}{4}} y^{\frac{1}{6}} \right)^6 \\
 &= x^{\frac{3 \times 6}{4}} x^{\frac{1 \times 6}{6}} \\
 &= x^{\frac{9}{2}} y \\
 &= y \sqrt[2]{x^9} \\
 &= y \sqrt{x^3} \text{ Ans.}
 \end{aligned}$$

Solution:-(x)

$$\begin{aligned}
 (x^3 y^2)^{\frac{1}{2}} \times (y^3 x^4)^{-\frac{1}{3}} &= x^{3 \times \frac{1}{2}} y^{2 \times \frac{1}{2}} \times y^{\left(-\frac{1}{3}\right)} x^{\left(-\frac{4}{3}\right)} \\
 &= x^{\frac{3}{2}} y^{-1} x^{-\frac{4}{3}} \\
 &= x^{\frac{3}{2} - \frac{4}{3}} y^{-1} \times y^0 \\
 &= x^{\frac{9-8}{6}} \times y^{-1} y^0 = 1
 \end{aligned}$$

$$= x^{\frac{1}{6}}$$

$$= \sqrt[6]{x} \text{ Ans.}$$

Solution:-(xi)

$$(x^3 y^2)^{\frac{1}{4}} \times (x^{\frac{1}{3}} y)^{\frac{3}{4}} = \left(x^{3 \times \frac{1}{4}} y^{2 \times \frac{1}{4}} \right) \times \left(x^{\frac{1}{3} \times \frac{3}{4}} y^{1 \times \frac{3}{4}} \right)$$

$$= x^{\frac{3}{4}} y^{\frac{1}{2}} \times x^{\frac{1}{4}} y^{\frac{3}{4}}$$

$$= x^{\frac{3+1}{4}} y^{\frac{2+3}{4}}$$

$$= x^{\frac{4}{4}} y^{\frac{5}{4}}$$

$$= \sqrt[4]{x^4 y^5} \text{ Ans.}$$

Solution:-(xii)

$$\left(a^{\frac{1}{4}} b^{\frac{1}{3}} \right)^{\frac{1}{2}} \div \left(a^{\frac{1}{3}} b^{\frac{1}{4}} \right)^{-5} = \left(a^{\frac{1}{4} \left(\frac{1}{2} \right)} b^{\frac{1}{3} \left(\frac{1}{2} \right)} \right) \div \left(a^{\frac{1}{3} (-5)} b^{\frac{1}{4} (-5)} \right)$$

$$= \left(a^{\frac{1}{8}} b^{\frac{1}{6}} \right) \div \left(a^{-\frac{5}{3}} b^{-\frac{5}{4}} \right)$$

$$= \frac{a^{\frac{1}{8}} b^{\frac{1}{6}}}{a^{-\frac{5}{3}} b^{-\frac{5}{4}}}$$

$$= a^{\frac{1}{8} + \frac{5}{3}} \times b^{\frac{1}{6} + \frac{5}{4}}$$

$$= a^{\frac{-3+40}{24}} \times b^{\frac{-2+15}{12}}$$

$$= a^{\frac{37}{24}} b^{\frac{13}{12}}$$

$$= a^{\frac{37}{2 \times 12}} b^{\frac{13}{12}}$$

$$= \sqrt[12]{a^{\frac{37}{2}} b^{13}} \text{ Ans.}$$

Solution:-(xiii)

$$(x^2 y^3)^{\frac{1}{5}} \times (x^{\frac{1}{3}} y^2)^{\frac{1}{4}} = \left(x^{2 \times \frac{1}{5}} y^{3 \times \frac{1}{5}} \right) \times \left(x^{\frac{1}{3} \times \frac{1}{4}} y^{2 \times \frac{1}{4}} \right)$$

$$= x^{\frac{2}{5}} y^{\frac{3}{5}} \times x^{\frac{1}{12}} y^{\frac{1}{2}}$$

$$= x^{\frac{2}{5}} \times x^{\frac{1}{12}} \times y^{\frac{3}{5}} \times y^{\frac{1}{2}}$$

$$= x^{\frac{2}{5} + \frac{1}{12}} \times y^{\frac{3}{5} + \frac{1}{2}}$$

$$= x^{\frac{24+5}{60}} \times y^{\frac{6+5}{10}}$$

$$= x^{\frac{29}{60}} \times y^{\frac{11}{10}} \text{ Ans.}$$

Exercise 6.2

1- Write the base and exponent in the following.

(i) $16x^3$

(ii) x^9

(iii) $(4y)^3$

(iv) $(x-2)^3$

(v) $18x^5$

(vi) $5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$

Solution:-

(i) $16x^3$

Base = x ; = exponent 3

(ii) x^9

Base = x ; = exponent 9

(iii) $(4y)^3$

Base = $4y$; = exponent 3

(iv) $(x-2)^3$

Base = $(x-2)$; = exponent 3

(v) $18x^5$

Base = x ; = exponent 5

(vi) $5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$

$$= 5x^{\frac{3}{2} + \frac{1}{2}} = 5x^2$$

Base = x ; = exponent 2

Simplify and express with positive indices

2- $\sqrt{(a^2b^3)^6}$

3- $\sqrt[9]{(x^{-4}y^3)^{-3}}$

4- $(x^a y^{-b})^3 \times (x^3 y^2)^{-a}$

5- $\left(\frac{16x^2}{y^{-2}} \right)^{\frac{-1}{4}}$

=====

$$6- \left(\frac{27x^3}{8a^{-3}} \right)^{\frac{-2}{3}}$$

$$7- \left(\frac{a^{\frac{-1}{2}}}{4c^2} \right)^{-2}$$

$$8- \sqrt{a^{-2}b} \times 3\sqrt{ab^{-3}}$$

$$9- \left(\frac{a^{-3}}{b^{\frac{-2}{3}}c} \right)^{\frac{-3}{2}} \div \frac{ab^2c}{a^2c}$$

$$10- \frac{(a^4)^3(a^{-1}b)^{10}}{a^2b^7}$$

$$11- \frac{(x^3y)^3(2xy)^{-2}}{4x^{-4}y^{-5}}$$

$$12- \frac{(a^{-5})^3(ab)^{15}}{a^{-1}b^2}$$

$$13- a^5b^4c^2 \div abc$$

$$14- (2ab^2)^2(3abc^2)^{-2} \div (ab)^{-4}(bca)^5$$

$$15- \frac{2^3 \times 6^5}{3^{-3} \times 4^{-4}}$$

$$16- \frac{2^5 \times 9^{-1}}{27^{-3} \times 8^{-3}}$$

$$17- (2^{-3}a^4b)^{-1} \times (4^{-2}b^{-5})$$

$$18- (3^2)^5 \div 9^3 \times 27^{-1}$$

$$19- \left(\frac{3}{4} \right)^{-2} \div \left(\frac{4}{9} \right)^3 \times \left(\frac{27}{16} \right)^{-1}$$

$$20- \left(\frac{2}{3} \right)^{-1} \div \left(\frac{4}{9} \right)^{-2} \times 27$$

$$21- \frac{5^4}{3^7} \times \left(\frac{9}{15} \right)^3 \div \frac{27}{25}$$

$$22- a^{\frac{1}{2}}b^{\frac{2}{3}} \times a^{\frac{2}{3}}b^{\frac{1}{4}}$$

$$23- a^{\frac{2}{3}}b^{\frac{5}{6}} \times a^{\frac{1}{2}}b \div (ab)^{\frac{1}{3}}$$

$$24- (a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}})^6$$

$$25- (a^{\frac{1}{2}}b^{\frac{1}{3}})^{\frac{4}{3}} \div (a^{\frac{1}{3}}b^{\frac{1}{4}})^{\frac{1}{2}}$$

$$26- a^{\frac{2}{3}} \times a^{\frac{1}{2}} \div a^{\frac{1}{4}}$$

$$2- \sqrt{(a^3b^3)^6}$$

Solution:-

$$\begin{aligned}
 &= \sqrt{a^{2 \times 6} b^{3 \times 6}} \\
 &= \sqrt{a^{12} b^{18}} \\
 &= (a^{12} b^{18})^{\frac{1}{2}} = a^{12 \times \frac{1}{2}} b^{18 \times \frac{1}{2}} \\
 &= a^6 b^9
 \end{aligned}$$

3. $\sqrt[9]{(x^{-4} y^3)^{-3}}$ Ans.

Solution:-

$$\begin{aligned}
 &= \sqrt[9]{x^{(-4)(-3)} \times y^{3(-3)}} \\
 &= (x^{12} \times y^{-9})^{\frac{1}{9}} \\
 &= x^{12(\frac{1}{9})} \times y^{-9(\frac{1}{9})} \\
 &= x^{\frac{4}{3}} \times y^{-1} \\
 &= \frac{x^{\frac{4}{3}}}{y} \text{ Ans.}
 \end{aligned}$$

4. $(x^a y^{-b})^3 \times (x^3 y^2)^{-a}$

Solution:-

$$\begin{aligned}
 &= (x^{(a)(3)} y^{(-b)(3)}) \times (x^{(3)(-a)} y^{(2)(-a)}) \\
 &= (x^{3a} y^{-3b}) \times (x^{-3a} y^{-2a}) \\
 &= x^{3a} \times y^{-3b} \times x^{-3a} \times y^{-2a} \\
 &= x^{3a} \times x^{-3a} \times y^{-3b} \times y^{-2a} \\
 &= x^{3a-3a} \times y^{-3b-2a} \\
 &= x^0 \times y^{\frac{1}{3b+2a}}
 \end{aligned}$$

=====

$$= 1 \times y^{\frac{1}{3b+2a}} = y^{\frac{1}{2a+3b}} \text{ Ans.}$$

$$5. \left(\frac{16x^2}{y^{-2}} \right)^{-\frac{1}{4}}$$

Solution:-

$$\begin{aligned} &= \left(\frac{2^4 x^2}{y^{-2}} \right)^{-\frac{1}{4}} \\ &= \frac{2^{4(-\frac{1}{4})} x^{2(-\frac{1}{4})}}{y^{-2(-\frac{1}{4})}} \\ &= \frac{2^{-1} x^{-\frac{1}{2}}}{y^{\frac{1}{2}}} = \frac{1}{2x^{\frac{1}{2}} y^{\frac{1}{2}}} \text{ Ans.} \end{aligned}$$

$$6. \left(\frac{27x^3}{8a^{-3}} \right)^{\frac{2}{3}}$$

Solution:-

$$\begin{aligned} &= \left(\frac{3^3 x^3}{2^3 a^{-3}} \right)^{\frac{2}{3}} \\ &= \frac{3^{3(-\frac{2}{3})} x^{3(-\frac{2}{3})}}{2^{3(-\frac{2}{3})} a^{-3(-\frac{2}{3})}} \\ &= \frac{3^{-2} x^{-2}}{2^{-2} a^2} \end{aligned}$$

$$= \frac{2^2}{3^2 \times a^2 \times x^2}$$

$$= \frac{4}{9a^2x^2} \text{ Ans.}$$

7. $\left(\frac{a^{-\frac{1}{2}}}{4c^2} \right)^{-2}$

Solution:-

$$= \left(\frac{a^{-\frac{1}{2}}}{2^2 c^2} \right)^{-2}$$

$$= \frac{a^{(-\frac{1}{2})(-2)}}{2^{(2)(-2)} c^{(2)(-2)}}$$

$$= \frac{a}{2^{-4} c^{-4}} = 2^4 ac^4$$

$$= 16ac^4 \text{ Ans.}$$

8. $\sqrt{a^{-2}b} \times \sqrt[3]{ab^{-3}}$

Solution:-

$$= (a^{-2}b)^{\frac{1}{2}} \times 3 \times (ab^{-3})^{\frac{1}{3}}$$

$$= 3 \times (a^{(-2)(\frac{1}{2})} \times b^{\frac{1}{2}}) \times (a^{\frac{1}{3}} b^{-3(\frac{1}{3})})$$

$$= 3a^{-1} \times b^{\frac{1}{2}} \times a^{\frac{1}{3}} \times b^{-1}$$

$$= 3a^{-1} \times a^{\frac{1}{3}} \times b^{\frac{1}{2}} \times b^{-1}$$

$$\begin{aligned}
 &= 3a^{-1+\frac{1}{2}} \times b^{\frac{1}{2}-\frac{3}{2}} \\
 &= 3a^{-\frac{1}{2}} \times b^{-\frac{2}{2}} \\
 &= 3a^{-\frac{1}{2}} \times b^{-1} = \frac{3}{a^{\frac{1}{2}}b} \text{ Ans.}
 \end{aligned}$$

$$9. \left(\frac{a^{-3}}{b^{-\frac{2}{3}}c} \right)^{-\frac{3}{2}} \div \frac{ab^2c}{a^2c}$$

Solution:-

$$\begin{aligned}
 &= \frac{a^{(-3)(-\frac{3}{2})}}{b^{(-\frac{2}{3})(-\frac{3}{2})} \times c^{-\frac{3}{2}}} \times \frac{a^2c}{ab^2c} \\
 &= \frac{a^{\frac{9}{2}}}{b \times c^{-\frac{3}{2}}} \times \frac{a^2}{a^1b^2} \\
 &= \frac{a^{\frac{9}{2}+2-1} c^{\frac{3}{2}}}{b^{1+2}} \\
 &= \frac{a^{\frac{11}{2}} c^{\frac{3}{2}}}{b^3} \text{ Ans.}
 \end{aligned}$$

$$10. \frac{(a^4)^3(a^{-1}b)^{10}}{a^2b^7}$$

Solution:-

$$= \frac{(a^{4 \times 3})(a^{(-1)(10)}b^{(1)(10)})}{a^2b^7}$$

$$\begin{aligned}
 &= \frac{a^{12} \times a^{-10} \times b^{10}}{a^2 b^7} \\
 &= a^{12-10-2} \times b^{10-7} = a^0 b^3 \\
 &= 1 \times b^3 = b^3 \text{ Ans.}
 \end{aligned}$$

$$11. \quad \frac{(x^3 y)^3 (2xy)^{-2}}{4x^{-4} y^{-5}}$$

Solution:-

$$\begin{aligned}
 &= \frac{(x^{3 \times 3} y^{1 \times 3})(2^{-2} x^{-2} y^{-2})}{4x^{-4} y^{-5}} \\
 &= \frac{(x^9 y^3)(2^{-2} x^{-2} y^{-2})}{2^2 \times 4x^{-4} y^{-5}} \\
 &= \frac{x^{9-2+4} \times y^{3-2+5}}{4 \times 4} = \frac{x^{11} y^6}{16} \text{ Ans.}
 \end{aligned}$$

$$12. \quad \frac{(a^{-5})^3 \times (ab)^{15}}{a^{-1} b^2}$$

Solution:-

$$\begin{aligned}
 &= \frac{a^{(-5)(3)} \times a^{1 \times 15} b^{1 \times 15}}{a^{-1} \times b^2} \\
 &= \frac{a^{-15} \times a^{15} b^{15}}{a^{-1} \times b^2} \\
 &= a^{-15+15+1} \times b^{15-2} = ab^{13} \text{ Ans.}
 \end{aligned}$$

$$13. \quad a^5 b^4 c^2 \div abc$$

Solution:-

$$= a^5 b^4 c^2 \times \frac{1}{abc}$$

$$= a^{5-1} b^{4-1} c^{2-1} = a^4 b^3 c \text{ Ans.}$$

$$14. (2ab^2)^2 (3abc^2)^{-2} \div (ab)^{-4} (bca)^5$$

Solution:-

$$= (2^2 a^2 b^{2 \times 2}) (3^{-2} a^{-2} b^{-2} c^{2(-2)}) \div (a^{-4} b^{-4}) (b^5 c^5 a^5),$$

$$= \frac{(4a^2 b^4) \left(\frac{1}{3^2} a^{-2} b^{-2} c^{-4}\right)}{(a^{-4} b^{-4}) (b^5 c^5 a^5)}$$

$$= \frac{4a^2 b^4 a^{-2} b^{-2} c^{-4}}{9a^{-4} b^{-4} b^5 c^5 a^5}$$

$$= \frac{4a^2 a^{-2} b^4 b^{-2} c^{-4}}{9a^{-4} a^5 b^{-4} b^5 c^5}$$

$$= \frac{4a^0 b^2 c^{-4}}{9abc^5}$$

$$= \frac{4a^0 b^2 c^{-4}}{9abc^5} = \frac{4}{9} a^{0-1} b^{2-1} c^{-4-5} = \frac{4}{9} a^{-1} b c^{-9}$$

$$= \frac{4b}{9ac^9} \text{ Ans.}$$

$$15. \frac{2^3 \times 6^5}{3^{-3} \times 4^{-4}}$$

Solution:-

$$= \frac{2^3 \times (2 \times 3)^5}{3^{-3} \times (2 \times 2)^{-4}}$$

$$= \frac{2^3 \times 2^5 \times 3^5}{3^{-3} \times 2^{-4} \times 2^{-4}}$$

$$= \frac{2^3 \times 2^5 \times 3^5}{2^{-4} \times 2^{-4} \times 3^{-3}}$$

$$= 2^{3+5+4+4} \times 3^{5+3} = 2^{16} 3^8$$

$$= 65536 \times 6561 = 429981696 \text{ Ans.}$$

$$\text{also} = \frac{2^3 \times 6^5}{3^{-3} \times 4^{-4}} = 2^3 \times 6^5 \times 3^3 \times 4^4$$

$$= 2^3 \times 4^4 \times 3^3 \times 6^5 \text{ Ans.}$$

$$16. \frac{2^5 \times 9^{-1}}{(27)^{-3} \times (8)^{-3}}$$

Solution:-

$$= \frac{2^5 \times (3^2)^{-1}}{(3^3)^{-3} \times (2^3)^{-3}} = \frac{2^5 \times 3^{-2}}{3^{-9} \times 2^{-9}}$$

$$= 2^{5+9} \times 3^{-2+9} = 2^{14} \times 3^7 \text{ Ans.}$$

$$17. (2^{-3} a^4 b)^{-1} \times (4^{-2} b^{-5})$$

Solution:-

$$= 2^{(-3)(-1)} \times a^{4(-1)} \times b^{1(-1)} \times 4^{-2} b^{-5}$$

$$= 2^3 \times a^{-4} \times b^{-1} \times \frac{1}{4^2} \times b^{-5}$$

$$= 8a^{-4} b^{-1-5} \times \frac{1}{16}$$

$$= \frac{1}{2} \times \frac{1}{a^4} \times \frac{1}{b^6} \quad \text{as } -1-5=-6$$

$$= \frac{1}{2a^4 b^6} \text{ Ans.}$$

$$18. (3^2)^5 \div 9^3 \times (27)^{-1}$$

Solution:-

$$\begin{aligned} &= 3^{2 \times 5} \times \frac{1}{9^3 \times (27)^{-1}} = 3^{10} \times \frac{27}{9^3} \\ &= \frac{3^{10} \times 3^3}{(3^2)^3} = \frac{3^{10} \times 3^3}{3^6} = 3^{10-6} \times 3^3 = 3^{4+3} = 3^7 \\ &= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2187 \text{ Ans.} \end{aligned}$$

$$19. \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \left(\frac{27}{16}\right)^{-1}$$

Solution:-

$$\begin{aligned} &= \frac{3^{-2}}{4^{-2}} \div \left(\frac{2^2}{3^2}\right)^3 \times \left(\frac{3^3}{2^4}\right)^{-1} \\ &= \frac{4^2}{3^2} \div \left(\frac{2^{2 \times 3}}{3^{2 \times 3}}\right) \times \left(\frac{3^{3(-1)}}{2^{4(-1)}}\right) \\ &= \frac{(2^2)^2}{3^2} \div \left(\frac{2^6}{3^6}\right) \times \left(\frac{3^{-3}}{2^{-4}}\right) \\ &= \frac{2^{2(2)}}{3^2} \div (2^{6+4} \times 3^{-3-6}) \\ &= \frac{2^4}{3^2} \div (2^{10} \times 3^{-9}) \\ &= \frac{2^4}{3^2} \times \frac{1}{2^{10} \times 3^{-9}} = 2^{4-10} \times 3^{-2+9} \\ &= 2^{-6} \times 3^7 = \frac{3^7}{2^6} = \frac{2187}{64} \text{ Ans.} \end{aligned}$$

$$20. \left(\frac{2}{3}\right)^{-1} \div \left(\frac{4}{9}\right)^{-2} \times 27$$

Solution:-

$$\begin{aligned} &= \frac{2^{-1}}{3^{-1}} \div \left(\frac{2^2}{3^2}\right)^{-2} \times 3^3 \\ &= \frac{3}{2} \div \frac{2^{2(-2)}}{3^{2(-2)}} \times 3^3 = \frac{3}{2} \div \frac{2^{-4}}{3^{-4}} \times 3^3 \\ &= \frac{3}{2} \div \frac{1}{2^4} \times 3^{3+4} = \frac{3}{2} \div \frac{3^7}{2^4} \\ &= \frac{3}{2} \times \frac{2^4}{3^7} = \frac{2^{4-1}}{3^{7-1}} \\ &= \frac{2^3}{3^6} = \frac{8}{729} \text{ Ans.} \end{aligned}$$

$$21. \frac{5^4}{3^4} \times \left(\frac{9}{15}\right)^3 \div \frac{27}{25}$$

Solution:-

$$\begin{aligned} &= \frac{5^4}{3^4} \times \left(\frac{3}{5}\right)^3 \div \frac{3^3}{5^2} = \frac{5^4}{3^4} \times \frac{3^3}{5^3} \times \frac{5^2}{3^3} \\ &= \frac{5^{4+2-3}}{3^{4+3-3}} = \frac{5^3}{3^4} = \frac{125}{81} \text{ Ans.} \end{aligned}$$

$$22. a^{\frac{1}{3}} b^{\frac{2}{3}} \times a^{\frac{2}{3}} b^{\frac{1}{4}}$$

Solution:-

$$\begin{aligned} &= a^{\frac{1}{3}} \times a^{\frac{2}{3}} \times b^{\frac{2}{3}} \times b^{\frac{1}{4}} \\ &= a^{\frac{1}{3} + \frac{2}{3}} \times b^{\frac{2}{3} + \frac{1}{4}} = a^{\frac{1+2}{3}} \times b^{\frac{8+3}{12}} \end{aligned}$$

$$= a^{\frac{3}{3}} \times b^{\frac{11}{12}} = ab^{\frac{11}{12}} \text{ Ans.}$$

$$23. \quad a^{\frac{2}{3}} b^{\frac{5}{6}} \times a^{\frac{1}{2}} b \div (ab)^{\frac{1}{3}}$$

Solution:-

$$= a^{\frac{2}{3}} b^{\frac{5}{6}} \times a^{\frac{1}{2}} b \div (a^{\frac{1}{3}} b^{\frac{1}{3}})$$

$$= \frac{a^{\frac{2}{3}} b^{\frac{5}{6}} \times a^{\frac{1}{2}} b}{a^{\frac{1}{3}} b^{\frac{1}{3}}}$$

$$= a^{\frac{2}{3} + \frac{1}{2} - \frac{1}{3}} \times b^{\frac{5}{6} + 1 - \frac{1}{3}}$$

$$= a^{\frac{4+3-2}{6}} \times b^{\frac{5+6-2}{6}}$$

$$= a^{\frac{5}{6}} \times b^{\frac{9}{6}} = a^{\frac{5}{6}} \times b^{\frac{3}{2}} \text{ Ans.}$$

$$24. \quad (a^{\frac{1}{2}} b^{\frac{1}{3}} c^{\frac{1}{4}})^6$$

Solution:-

$$= a^{\frac{1}{2}(6)} b^{\frac{1}{3}(6)} c^{\frac{1}{4}(6)}$$

$$= a^3 b^2 c^{\frac{3}{2}} \text{ Ans.}$$

$$25. \quad (a^{\frac{1}{8}} b^{\frac{1}{3}})^{\frac{4}{3}} \div (a^{\frac{1}{3}} b^{\frac{1}{4}})^{\frac{1}{2}}$$

Solution:-

$$= \left(a^{(\frac{1}{8})(\frac{4}{3})} b^{(\frac{1}{3})(\frac{4}{3})} \right) \div \left(a^{(\frac{1}{3})(\frac{1}{2})} b^{(\frac{1}{4})(\frac{1}{2})} \right)$$

$$= (a^{\frac{1}{6}} b^{\frac{4}{9}}) \div (a^{\frac{1}{6}} b^{\frac{1}{8}})$$

$$\begin{aligned}
 &= \frac{a^{\frac{1}{6}} b^{\frac{4}{9}}}{a^{\frac{1}{6}} b^{\frac{1}{8}}} = a^{\frac{1}{6} - \frac{1}{6}} \times b^{\frac{4}{9} - \frac{1}{8}} \\
 &= a^0 \times b^{\frac{32-9}{72}} = 1 \times b^{\frac{32-9}{72}} = b^{\frac{23}{72}} \text{ Ans.}
 \end{aligned}$$

26. $a^{\frac{2}{3}} \times a^{\frac{1}{2}} \div a^{\frac{1}{4}}$

Solution:-

$$\begin{aligned}
 &= \frac{a^{\frac{2}{3}} \times a^{\frac{1}{2}}}{a^{\frac{1}{4}}} = a^{\frac{2}{3} + \frac{1}{2} - \frac{1}{4}} \text{ Ans.} \\
 &= a^{\frac{8+6-3}{12}} = a^{\frac{11}{12}}
 \end{aligned}$$

27. Simplify each of the following.

(i) $4^{\frac{3}{5}} \times 4^{\frac{1}{5}}$

(ii) $2^{\frac{1}{8}} \times 2^{\frac{3}{8}}$

(iii) $5x^{\frac{1}{3}} \times 2x^{\frac{1}{5}}$

(iv) $x^{\frac{3}{4}} \times x^{\frac{2}{5}}$

(v) $\frac{1}{2}y^{\frac{3}{7}} \times 4y^{\frac{2}{7}}$

(vi) $5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$

27 (i) $4^{\frac{3}{5}} \times 4^{\frac{1}{5}}$

Solution:-

$$= 4^{\frac{3}{5} + \frac{1}{5}} = 4^{\frac{3+1}{5}} = 4^{\frac{4}{5}} \text{ Ans.}$$

27 (ii) $2^{\frac{1}{8}} \times 2^{\frac{3}{8}}$

Solution:-

$$= 2^{\frac{1}{8} + \frac{3}{8}} = 2^{\frac{1+3}{8}} = 2^{\frac{4}{8}} = 2^{\frac{1}{2}} \text{ Ans.}$$

$$27 \text{ (iii)} \quad 5x^{\frac{1}{3}} \times 2x^{\frac{1}{5}}$$

Solution:-

$$= x^{\frac{3}{4} + \frac{2}{5}} = x^{\frac{15+8}{20}} = x^{\frac{23}{20}} \text{ Ans.}$$

$$27 \text{ (iv)} \quad x^{\frac{3}{4}} \times x^{\frac{2}{5}}$$

Solution:-

$$= 5 \times 2 \times x^{\frac{1}{3}} \times x^{\frac{1}{5}} = 10x^{\frac{1}{3} + \frac{1}{5}}$$

$$= 10x^{\frac{5+3}{15}} = 10x^{\frac{8}{15}} \text{ Ans.}$$

$$27 \text{ (v)} \quad \frac{1}{2}y^{\frac{3}{7}} \times 4y^{\frac{2}{7}}$$

Solution:-

$$= \frac{1}{2} \times 4 \times y^{\frac{3}{7}} \times y^{\frac{2}{7}}$$

$$= 2y^{\frac{3}{7} + \frac{2}{7}} = 2y^{\frac{3+2}{7}} = 2y^{\frac{5}{7}} \text{ Ans.}$$

$$27 \text{ (vi)} \quad 5x^{\frac{3}{2}} \times x^{\frac{1}{2}}$$

Solution:-

$$= 5x^{\frac{3}{2} + \frac{1}{2}} = 5x^{\frac{3+1}{2}}$$

$$= 5x^{\frac{4}{2}} = 5x^2 \text{ Ans.}$$

28- Simplify each of the following.

$$(i) a^{\frac{2}{3}} b^{\frac{3}{4}} \times a^{\frac{1}{3}} b^{\frac{3}{4}} \quad (ii) x^{\frac{3}{5}} y^{\frac{2}{9}} \times x^{\frac{1}{5}} y^{\frac{1}{3}}$$

$$(iii) 2ab^{\frac{1}{3}} \times 3a^{\frac{3}{5}} b^{\frac{4}{5}} \quad (iv) 6x^{\frac{3}{7}} \times \frac{1}{3} x^{\frac{1}{4}} y^{\frac{2}{5}}$$

$$(v) x^3 y^{\frac{1}{2}} z^{\frac{1}{3}} \times x^{\frac{1}{6}} y^{\frac{1}{3}} z^{\frac{1}{2}}$$

28 (i) $a^{\frac{2}{3}} b^{\frac{3}{4}} \times a^{\frac{1}{3}} b^{\frac{3}{4}}$

Solution:-

$$\begin{aligned} &= a^{\frac{2}{3}} \times a^{\frac{1}{3}} \times b^{\frac{3}{4}} \times b^{\frac{3}{4}} \\ &= a^{\frac{2}{3} + \frac{1}{3}} \times b^{\frac{3}{4} + \frac{3}{4}} = a^{\frac{2+1}{3}} \times b^{\frac{3+3}{4}} \\ &= a^{\frac{3}{3}} \times b^{\frac{6}{4}} = a \times b^{\frac{3}{2}} \text{ Ans.} \end{aligned}$$

28 (ii) $x^{\frac{3}{5}} y^{\frac{2}{9}} \times x^{\frac{1}{5}} y^{\frac{1}{3}}$

Solution:-

$$\begin{aligned} &= x^{\frac{3}{5}} \times x^{\frac{1}{5}} \times y^{\frac{2}{9}} \times y^{\frac{1}{3}} \\ &= x^{\frac{3}{5} + \frac{1}{5}} y^{\frac{2}{9} + \frac{1}{3}} = x^{\frac{3+1}{5}} \times y^{\frac{2+3}{9}} \\ &= x^{\frac{4}{5}} \times y^{\frac{5}{9}} = x^{\frac{4}{5}} y^{\frac{5}{9}} \text{ Ans.} \end{aligned}$$

28 (iii) $2ab^{\frac{1}{3}} \times 3a^{\frac{3}{5}} b^{\frac{4}{5}}$

Solution:-

$$\begin{aligned} &= 2 \times 3a \times a^{\frac{3}{5}} \times b^{\frac{1}{3}} \times b^{\frac{4}{5}} \\ &= 6a^{1 + \frac{3}{5}} \times b^{\frac{1}{3} + \frac{4}{5}} = 6a^{\frac{5+3}{5}} \times b^{\frac{5+12}{15}} \end{aligned}$$

=====

$$= 6a^{\frac{8}{5}} \times b^{\frac{17}{15}} = 6a^{\frac{8}{5}} b^{\frac{17}{15}} \text{ Ans.}$$

28 (iv)

$$6x^{\frac{3}{7}} \times \frac{1}{3} x^{\frac{1}{4}} y^{\frac{2}{5}}$$

Solution:-

$$= 6 \times \frac{1}{3} \times x^{\frac{3}{7}} \times x^{\frac{1}{4}} y^{\frac{2}{5}}$$

$$= 2x^{\frac{3}{7} + \frac{1}{4}} y^{\frac{2}{5}} = 2x^{\frac{12+7}{28}} \times y^{\frac{2}{5}}$$

$$= 2x^{\frac{19}{28}} y^{\frac{2}{5}} \text{ Ans.}$$

28 (v)

$$x^3 y^{\frac{1}{2}} z^{\frac{1}{3}} \times x^{\frac{1}{6}} y^{\frac{1}{3}} z^{\frac{1}{2}}$$

Solution:-

$$= x^3 \times x^{\frac{1}{6}} \times y^{\frac{1}{2}} \times y^{\frac{1}{3}} \times z^{\frac{1}{3}} \times z^{\frac{1}{2}}$$

$$= x^{3 + \frac{1}{6}} \times y^{\frac{1}{2} + \frac{1}{3}} z^{\frac{1}{3} + \frac{1}{2}}$$

$$= x^{\frac{18+1}{6}} \times y^{\frac{3+2}{6}} \times z^{\frac{2+3}{6}}$$

$$= x^{\frac{19}{6}} y^{\frac{5}{6}} z^{\frac{5}{6}} \text{ Ans.}$$

29- Simplify each of the following.

$$(i) 3^{\frac{1}{2}} \div 3^{\frac{1}{3}}$$

$$(ii) \frac{x^{\frac{4}{5}}}{x^{\frac{5}{9}}}$$

$$(iii) \frac{2x^{\frac{3}{5}}}{4x^{\frac{3}{5}}}$$

$$(iv) \frac{25y^{\frac{3}{5}}}{20y^{\frac{1}{4}}}$$

$$(v) x^3 y^2 \div x^{\frac{4}{3}} y^{\frac{3}{5}}$$

$$(vi) a^{\frac{5}{9}} b^{\frac{2}{3}} \div a^{\frac{2}{5}} b^{\frac{2}{5}}$$

$$(vii) 10x^{\frac{4}{5}} y \div 5x^{\frac{2}{3}} y^{\frac{1}{4}}$$

$$(viii) \frac{5a^{\frac{3}{4}} b^{\frac{3}{5}}}{20a^{\frac{1}{5}} b^{\frac{1}{4}}}$$

Solution:-29 (i)

$$\begin{aligned} 3^{\frac{1}{2}} \div 3^{\frac{1}{3}} &= \frac{3^{\frac{1}{2}}}{3^{\frac{1}{3}}} = 3^{\frac{1}{2} - \frac{1}{3}} \\ &= 3^{\frac{3-2}{6}} = 3^{\frac{1}{6}} \text{ Ans.} \end{aligned}$$

Solution:-29 (ii)

$$\begin{aligned} \frac{x^{\frac{4}{5}}}{x^{\frac{5}{9}}} &= x^{\frac{4}{5} - \frac{5}{9}} = x^{\frac{36-25}{45}} = x^{\frac{11}{45}} \text{ Ans.} \end{aligned}$$

Solution:-29 (iii)

$$\begin{aligned} \frac{2x^{\frac{3}{4}}}{4x^{\frac{3}{5}}} &= \frac{2}{4} x^{\frac{3}{4} - \frac{3}{5}} \\ &= \frac{1}{2} x^{\frac{15-12}{20}} = \frac{1}{2} x^{\frac{3}{20}} \text{ Ans.} \end{aligned}$$

Solution:-30 (i)

$$\begin{aligned}
 & \frac{25y^5}{70v^{\frac{1}{4}}} \\
 &= \frac{5}{4} \times y^{5\frac{3}{4}} = \frac{5}{4} \times y^{\frac{12-5}{20}} \\
 &= \frac{5}{4} y^{\frac{7}{20}} \text{ Ans.}
 \end{aligned}$$

Solution:-29(v)

$$\begin{aligned}
 & x^3 y^2 \div x^{\frac{4}{3}} y^{\frac{3}{5}} \\
 &= \frac{x^3 y^2}{x^{\frac{4}{3}} y^{\frac{3}{5}}} \\
 &= x^{3-\frac{4}{3}} \times y^{2-\frac{3}{5}} \\
 &= x^{\frac{9-4}{3}} \times y^{\frac{10-3}{5}} = x^{\frac{5}{3}} y^{\frac{7}{5}} \text{ Ans.}
 \end{aligned}$$

Solution:- 29(vi)

$$\begin{aligned}
 & a^{\frac{5}{9}} b^{\frac{2}{3}} \div a^{\frac{2}{5}} b^{\frac{2}{5}} \\
 &= \frac{a^{\frac{5}{9}} b^{\frac{2}{3}}}{a^{\frac{2}{5}} b^{\frac{2}{5}}} \\
 &= a^{\frac{5}{9}-\frac{2}{5}} \times b^{\frac{2}{3}-\frac{2}{5}}
 \end{aligned}$$

$$= a^{\frac{25-18}{45}} \times b^{\frac{10-6}{15}}$$

$$= a^{\frac{7}{45}} \times b^{\frac{4}{15}} = a^{\frac{7}{45}} b^{\frac{4}{15}} \text{ Ans.}$$

Solution:-29 (vii)

$$10x^{\frac{4}{5}}y \div 5x^{\frac{2}{3}}y^{\frac{1}{4}}$$

$$= \frac{10x^{\frac{4}{5}}y}{5x^{\frac{2}{3}}y^{\frac{1}{4}}}$$

$$= 2x^{\frac{4}{5}-\frac{2}{3}} \times y^{1-\frac{1}{4}} = 2x^{\frac{12-10}{15}} \times y^{\frac{4-1}{4}}$$

$$= 2x^{\frac{2}{15}} \times y^{\frac{3}{4}} = 2x^{\frac{2}{15}}y^{\frac{3}{4}} \text{ Ans.}$$

Solution:-29 (viii)

$$\frac{5a^{\frac{3}{4}}b^{\frac{3}{5}}}{20a^{\frac{1}{5}}b^{\frac{1}{4}}}$$

$$= \frac{1}{4} a^{\frac{3}{4}-\frac{1}{5}} \times b^{\frac{3}{5}-\frac{1}{4}}$$

$$= \frac{1}{4} a^{\frac{15-4}{20}} \times b^{\frac{12-5}{20}}$$

$$= \frac{1}{4} a^{\frac{11}{20}} \times b^{\frac{7}{20}} = \frac{1}{4} a^{\frac{11}{20}} b^{\frac{7}{20}} \text{ Ans.}$$

Exercise 6.3

Write the following in scientific notation:

1- 0.051

2- 89.99

3- 0.424

4- 2566324

5- 0.00000075

Write the following in the decimal form:

6- 0.86×10^4

7- 1.345×10^5

8- 5.1×10^9

9- 0.525×10^7

10- 636.5×10^6

Simplify and write your answer in scientific notation:

11- $\frac{0.96 \times 10^7}{2 \times 10^4}$

12- $\frac{2.61 \times 4 \times 10^8}{10^3}$

13- $\frac{521 \times 10^3 \times 12}{2 \times 10^2}$

Solution:-1.

$$\begin{aligned}
 &0.051 \\
 &= \frac{51}{1000} = \frac{51}{10^3} \\
 &= 51 \times 10^{-3} = 5.1 \times 10 \times 10^{-3} \\
 &= 51 \times 10^{-3+1} = 5.1 \times 10^{-2}
 \end{aligned}$$

Solution:-2.

$$89.99$$

$$\begin{aligned}
 &= \frac{8999}{100} = \frac{8999}{10^2} \\
 &= 8999 \times 10^{-2} \\
 &= (8.999 \times 1000) \times 10^{-2} \\
 &= 8.999 \times 10^3 \times 10^{-2} \\
 &= 8.999 \times 10^{3-2} \\
 &= 8.999 \times 10 \text{ Ans.}
 \end{aligned}$$

Solution:-3.

$$\begin{aligned}
 &0.424 \\
 &= \frac{424}{1000} = \frac{424}{10^3} \\
 &= 424 \times 10^{-3} = (4.24 \times 100) \times 10^{-3} \\
 &= 4.24 \times 10^2 \times 10^{-3} = 4.24 \times 10^{2-3} \\
 &= 4.24 \times 10^{-1} \text{ Ans.}
 \end{aligned}$$

Solution:-4.

$$\begin{aligned}
 &2566324 \\
 &= 2.566324 \times 10^6 \text{ Ans.}
 \end{aligned}$$

Solution:5.-

$$\begin{aligned}
 &0.00000075 \\
 &= \frac{75}{100000000} = \frac{75}{10^8} \\
 &= 75 \times 10^{-8} = (7.5 \times 10) \times 10^{-8} \\
 &= 7.5 \times 10 \times 10^{-8} = 7.5 \times 10^{1-8}
 \end{aligned}$$

$$= 7.5 \times 10^{-7} \text{ Ans.}$$

Solution:-6.

$$0.86 \times 10^4$$

$$= 0.86 \times 100000 = 8600 \text{ Ans.}$$

Solution:-7.

$$1.345 \times 10^{-5}$$

$$= 1.345 \times \frac{1}{10^5} = \frac{1.345}{100000} = .00001345 \text{ Ans.}$$

Solution:-8.

$$5.1 \times 10^{-9}$$

$$= 5.1 \times 10^{\frac{1}{9}} = 5.1 \times \frac{1}{1000000000}$$

$$= .0000000051 \text{ Ans.}$$

Solution:-9.

$$0.525 \times 10^{-7}$$

$$= 0.525 \times \frac{1}{10^7} = 0.525 \times \frac{1}{10000000}$$

$$= .0000000525 \text{ Ans.}$$

Solution:-10.

$$636.5 \times 10^{-6}$$

$$= 636.5 \times \frac{1}{10^6} = 636.5 \times \frac{1}{1000000}$$

$$= .0006365 \text{ Ans.}$$

Solution:-11.

=====

$$\begin{aligned}
 & \frac{0.96 \times 10^7}{2 \times 10^4} \\
 &= \left(\frac{0.96}{2} \right) \times \frac{10^7}{10^4} \\
 &= 0.48 \times 10^{7-4} = 0.48 \times 10^3 \text{ Ans.}
 \end{aligned}$$

Solution:-12.

$$\begin{aligned}
 & \frac{2.61 \times 4 \times 10^8}{10^3} \\
 &= 2.61 \times 4 \times \frac{10^8}{10^3} = 10.44 \times 10^{8-3} \\
 &= (1.044 \times 10) \times 10^5 = 1.044 \times 10^{5+1} \\
 &= 1.044 \times 10^6 \text{ Ans.}
 \end{aligned}$$

Solution:13.-

$$\begin{aligned}
 & \frac{521 \times 10^3 \times 12}{2 \times 10^2} \\
 &= \frac{521 \times 12}{2} \times \frac{10^3}{10^2} = 3126 \times 10 \\
 &= 3.126 \times 1000 \times 10 = 3.126 \times 10^3 \times 10 \\
 &= 3.126 \times 10^{3+1} = 3.126 \times 10^4 \text{ Ans.}
 \end{aligned}$$

14- Convert 4.5×10^5 cm into meters write the solution in decimal form.

Solution:-14. $= 4.5 \times 10^5 \text{ cm}$

$$\begin{aligned} &= \frac{4.5 \times 10^5}{100} = 4.5 \times \frac{10^5}{10^2} \text{ m} \\ &= 4.5 \times 10^{5-2} \text{ m} = 4.5 \times 10^3 \text{ Ans.} \end{aligned}$$

15- *The radius of earth is 6400 km. Convert it into meters and write the solution in scientific notation.*

Solution:-15.

Radius of earth = 6400

$$\begin{aligned} &= 6400 \times 1000 \text{ m} \\ &= 64 \times 100 \times 1000 \\ &= (6.4 \times 10) \times 100 \times 1000 \\ &= 6.4 \times 10^1 \times 10^2 \times 10^3 \\ &= 6.4 \times 10^{1+2+3} = 6.4 \times 10^6 \text{ Ans.} \end{aligned}$$

Exercise 6.4

1- Write down the characteristic of the logarithms of following numbers.

(i) 6350

(ii) 2035.6

(iii) 2.057

(iv) 0.8657

(v) 0.0732

(vi) 0.000721

Solution:-

(i) The characteristic of $\log 6350$ is 3, because
 $6350 = 6.350 \times 10^3$

(ii) The characteristic of $\log 2035.6$ is 3, because
 $2035.6 = 2.0356 \times 10^3$

(iii) The characteristic of $\log 2.057$ is 0, because
 $2.057 = 2.057 \times 10^0$

(iv) The characteristic of $\log 0.8657$ is -1, because
 $0.8657 = 8.757 \times 10^{-1}$

(v) The characteristic of $\log 0.732$ is -2, because
 $0.0732 = 7.32 \times 10^{-2}$

(vi) The characteristics of $\log 0.000721$ is -4 because
 $0.000721 = 7.21 \times 10^{-4}$

2- Write down the values of:

(i) $\log 52.13$

(ii) $\log 6.304$

(iii) $\log 0.6127$

(iv) $\log 0.0057$

(v) $\log 0.00003$

Solution:-

$$\log 52.13 = ?$$

- (i) The characteristic of $\log 52.13$ is 1, because

$$52.13 = 5.213 \times 10^1$$

- (ii) To find mantissa we search 52 in log table. The next digit of 52 in log table is '1' we search it on the top. We draw vertical line from '1' and horizontal lines from 52 these two lines meet at digit 7168.

	0	1	2	3	4	5	6	7	8	9	1 2 3 4 5 6 7 8 9
52		7168									2

- (iii) In number 5213 last digit is 3, we can see difference for 3 in the extreme right side of the table 3 to vertically and front of 52, lines meets with number 2 which we add in 7168, therefore the mantissa is
- $$7168 + 2 = 7170.$$

$$\text{Thus } \log 52.13 = 1.7170$$

2(ii) $\log 6.304 = ?$

Solution:-

The characteristics of $\log 6.304$ is 0 because

$$6.304 = 6.304 \times 10^0$$

	0	1	2	3	4	5	6	7	8	9	1 2 3 4 5 6 7 8 9
63	7993										3

The next digit of 63 is '0' the number 7993 is situated on the vertical side of '0' and horizontal side of 63.

The last digit of 6304 is '4'.

In the difference column the vertical side of 4 and horizontal side of 63 digit is 3.

Therefore mantissa is $7993 + 3 = 7996$.

Hence $\log 6.304 = 0.7996$.

2(iii) $\log 0.6127$

Solution:-

The characteristics of $\log 0.6127$ is -1 because

$$0.6127 = 6.127 \times 10^{-1}$$

	0	1	2	3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
61			7868		5

In 6127 the next digit of 61 is '2'. In log table vertical lines of 2 and horizontal lines of 61 are cut on 7868
Now in 6127 the last digit is 7. In difference column vertical of 7 and horizontal lines of 61 are cut on its digit '5'.

Therefore mantissa is $7868 + 5 = 7873$

Hence $\log 0.6127 = \bar{1}.7873$

2(iv) $\log 0.0057$

Solution:-

The characteristics of $\log 0.0057$ is -3 because

$$.0057 = 5.7 \times 10^{-3}$$

	0	1 2 3 4 5 6 7 8 9	2 3 4 5 6 7 8 9	
57	7559			

- 2 (iv) Now we draw horizontal line from 57 and vertical line from '0' which cut on the number 7559. Therefore the mantissa is 7559

$$\text{Thus } \log 0.0057 = \bar{3}.7559$$

2(v) $\log 0.00003$

In $\log 0.00003$ the characteristic is -5 because

$$0.00003 = 3 \times 10^{-5}.$$

	0	1 2 3 4 5 6 7 8 9	1,2,3,4,5,6 7 8 9
30	4771		

Now instead of 3, we take horizontal line from 30 and vertical line from 0, which cut with each other at the number 4771.

$$\text{Hence } \log 0.00003 = \bar{5}.4771$$

- 3- If $\log 6374 = 3.8044$, write down the values of:

(i) $\log 6.374$

(ii) $\log 0.6374$

(iii) $\log 0.00637$

3(i) $\log 6374 = 3.8044$

$$\text{Then } \log 6.374 = ?$$

Solution:- You can observe that in $\log 6374$ and $\log 6.374$ the mantissa will be one, which will be 8044.

Now in $\log 6.374$ the characteristic is 0 because

$$6.374 = 6.374 \times 10^0$$

$$\text{Thus } \log 6.374 = 0.8044$$

3:(ii) $\log 6374 = 3.8044$

Then $\log 0.6374 = ?$

Solution:- $\log 6374$ and $\log 0.6374$ the mantissa is same which is 8044. Now the characteristic in $\log 0.6374$ is -1 because $.6374 = 6.374 \times 10^{-1}$

Hence $\log 0.6374 = \overline{1}.8044$

3(iii) $\log 0.00637$

Solution:

$\log 6374$ and $\log 0.00637$. The mantissa is same.

That is 8044.

Now characteristic in $\log 0.00637$ is -3 because.

$0.00637 = 6.37 \times 10^{-3}$. Hence $\log 0.00637 = \overline{3}.8044$.

4- (i) If $\log x = 2.0374$, find x

(ii) If $\log x = 0.1597$, find x

(iii) If $\log x = 4.4236$, find x

Solution:(i)

$\log x = 2.0374$

Taking antilog

$\text{Antilog}(\log x) = \text{Antilog } 2.0374$

Now see antilog table

	1 2 3 4 5 6	7	8 9	12 3	4	5 6 7 8 9
.03		1089			1	

Characteristic -2 helpful for putting point.

With the help of mantissa 0374 we find required digits - and .0374 the next digit from .03 is '7'. We draw horizontal line from .03 and vertical line from '7' these are cut at 1089.

Thus in .0374 next digit of the 7 is '4'.

Now in different column draw vertical line from 4 and horizontal line from 0.3 these cut on '1'. So required digit is $1089 + 1 = 1090$. The characteristic -2 shows the right side of point is 0. The required digit is .01090.

Thus $\text{Antilog}(\log) = \text{Antilog } 2.0374$.

Hence $x = 0.01090$

Solution:(ii)

If $\log x = 0.1597$, find x

$\log x = 0.1597$

Taking $\text{Antilog}(\log x) = \text{Antilog } 0.1597$

Antilog Table

	1 2 3 4 5 6 7 8	9	1 2 3 4 5 6 7 8 9
.15		1442	2

The characteristic '0' is helpful for putting point

In .1597 the next digit from .15 is 9.

From .15 we draw horizontal line from 9 draw vertical line these lines cut digit 1442. In .1597 the next digit of 9 is '7'. Now in difference column vertical lines

from 7 and horizontal line from .15 which cut with each other at '2'.

Hence required digit is $1442 + 2 = 1444$

Thus $\text{Antilog}(\log x) = \text{Antilog} .1597$

$$x = 1.444$$

Characteristic 0, show there is '1' digit at left side of point.

Solution: (iii)

If $\log x = 4.4236$, find x

$\text{Antilog}(\log x) = \text{Antilog} 4.4236$

	12	3	4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
.42		2649		4

Characteristic = 4

(i) In number .4236 next digit of .42 is 3 search .42 in the log table vertical line from 3 and horizontal line from .42 meet each other at 2649.

(ii) For the difference of 6 .42 search in 6 column that is $2649 + 4 = 2653$. Characteristic is 4, therefore required digit will be 5 digit number.

Hence 26530

$$x = 26530 \text{ (0 will be added at the right side.)}$$

Laws of Logarithms

(i) $\log_a^{mn} = \log_a^m + \log_a^n$

$$(ii) \quad \log_a \left(\frac{m}{n} \right) = \log_a^m - \log_a^n$$

$$(iii) \quad \log_a^{mn} = n \log_a^m$$

1st Law:

Proof: Let m and n are positive integers and ' a ' is any admissible base, then taking

$$x = \log_a m \dots \dots \dots (i)$$

$$y = \log_a n \dots \dots \dots (ii)$$

$$\text{Then } a^x = m \quad a^y = n$$

$$\text{Thus } mn = a^x \times a^y = a^{x+y}$$

$$\log_a mn = x + y$$

$$= x + y$$

Putting values of x and y from (i) and (ii)

$$\log_a mn = \log_a m + \log_a n$$

2nd Law

Proof: ii. Let m and n are positive integers and ' a ' is any admissible base, (i.e. $a > 1$), then

$$x = \log_a m \dots \dots \dots (i)$$

$$y = \log_a n \dots \dots \dots (ii)$$

$$\text{Therefore } a^x = m, a^y = n$$

$$\frac{m}{n} = \frac{a^x}{a^y}$$

$$=a^x \cdot a^y$$

$$=a^{x+y}$$

$$\log_a \left(\frac{m}{n} \right) = x - y$$

Putting values of x and y from (i) and (ii)

$$\log_a \left(\frac{m}{n} \right) = \log_a m - \log_a n$$

3rd Law

Proof: iii. Let $x = \log_a m$, then as before

$$x = \log_a m$$

$$a^x = m$$

$$(a^x)^n = m^n$$

$$m^n = (a^x)^n$$

$$= a^{nx}$$

Therefore $\log_a m^n = nx$

Putting value of x

$$\log_a m^n = n \log_a m$$

Exercise 6.5

1- Solve

$$(i) \frac{\log 81}{\log 9} \quad (ii) \frac{\log 36}{\log 6} \quad (iii) \frac{\log 243}{\log 9}$$

Solution:-(i)

$$\begin{aligned} \frac{\log 81}{\log 9} &= \frac{\log 9^2}{\log 9} \\ &= \frac{2 \log 9}{\log 9} = 2 \text{ Ans.} \end{aligned}$$

Solution:- (ii)

$$\begin{aligned} \frac{\log 36}{\log 6} &= \frac{\log 6^2}{\log 6} \\ &= \frac{2 \log 6}{\log 6} = 2 \text{ Ans.} \end{aligned}$$

Solution:-(iii)

$$\begin{aligned} \frac{\log 243}{\log 9} &= \frac{\log 3^5}{\log 3^2} \\ &= \frac{5 \log 3}{2 \log 3} = \frac{5}{2} \text{ Ans.} \end{aligned}$$

2- Evaluate

$$(i) \log 5 + \log 4 + \log 3 - \log 6$$

$$(ii) \log 5 + \log 20 + \log 24 + \log 25 - \log 60$$

$$(iii) 2\log 3 + 3\log 4 + 4\log 5 - 2\log 6$$

$$(iv) 2\log 5 + \log 8 - \frac{1}{2}\log 4$$

$$(v) \log 200 + \log 5$$

$$\text{Hint } \left(\log 5 = \log \left(\frac{10}{2} \right) = \log 10 - \log 2 = 1 - \log 2 \right)$$

Solution:-2(i)

$$\begin{aligned} & \log 5 + \log 4 + \log 3 - \log 6 \\ &= \log \frac{5 \times 4 \times 3}{6} \\ &= \log 10 = 1.000 \end{aligned}$$

Solution:-2(ii)

$$\begin{aligned} & \log 5 + \log 20 + \log 24 + \log 25 - \log 60 \\ &= \log \frac{5 \times 20 \times 24 \times 25}{60} \\ &= \log 5 \times 8 \times 25 = \log 5 \times 2^3 \times 5^2 \\ &= \log 5^3 \times 2^3 = \log 5^3 + \log 2^3 \\ &= 3\log 5 + 3\log 2 = 3[\log 5 + \log 2] \\ &= 3 \log 5 \times 2 \\ &= 3\log 10 = 3[1.0000] = 3.000 \text{ Ans.} \end{aligned}$$

Solution:-2(iii)

$$\begin{aligned}
 & 2 \log 3 + 3 \log 4 + 4 \log 5 - 2 \log 6 \\
 &= \log 3^2 + \log 4^3 + \log 5^4 - \log 6^2 \\
 &= \log \frac{3^2 \times 4^3 \times 5^4}{6^2} \\
 &= \log \frac{\cancel{3} \times \cancel{3} \times \cancel{4} \times 4 \times 4 \times 5^4}{\cancel{6}_2 \times \cancel{6}_2} \\
 &= \log 4 \times 4 \times 5^4 = \log 2^2 \times 2^2 \times 5^4 \\
 &= \log 2^4 \times 5^4 = 4 \log 2 + 4 \log 5 \\
 &= 4[\log 2 + \log 5] = 4[\log 2 \times 5] \\
 &= 4 \log 10 = 4(1.0000) = 4.0000 \text{ Ans.}
 \end{aligned}$$

Solution:-2(iv)

$$\begin{aligned}
 & 2 \log 5 + \log 8 - \frac{1}{2} \log 4 \\
 &= \log 5^2 + \log 2^3 - \frac{1}{2} \log 2^2 \\
 &= \log 5^2 + \log 2^3 - \frac{1}{2} \times 2 \log 2 \\
 &= \log 5^2 + \log 2^3 - \log 2 \\
 &= \log \frac{5^2 \times 2^3}{2} = \log 5^2 \times \log 2^2 \\
 &= \log (5 \times 2)^2 = 2 \log 10 \\
 &= 2(1.0000) = 2.0000 \text{ Ans.}
 \end{aligned}$$

Solution:-2(v)

$$\begin{aligned}
 & \log 200 + \log 5 \\
 &= \log 200 \times 5 \\
 &= \log 5^2 \times 2^3 \times 5 = \log 5^3 \times 2^3 \\
 &= \log (5 \times 2)^3 = 3 \log 10 \\
 &= 3 \times 1.0000 = 3.000 \text{ Ans.}
 \end{aligned}$$

3- Simplify without using logarithm table.

(i) $\log 1.3472 + \log 22.79 - \log 5$

(ii) $\log 22.13 + \log 0.354 + \log 7 - \log 3$

(iii) $\log 57.86 + \log 4.385 - \log 2.391 - \log 3.072$

Solution:-3(i)

$$\begin{aligned}
 & \log 1.3472 + \log 22.79 - \log 5 \\
 &= \log \frac{1.3472 \times 22.79}{5} \text{ Ans.}
 \end{aligned}$$

Solution:-3(ii)

$$\begin{aligned}
 & \log 22.13 + \log 0.354 + \log 7 - \log 3 \\
 &= \log \frac{22.13 \times 0.354 \times 7}{3} \text{ Ans.}
 \end{aligned}$$

Solution:-3(iii)

$$\begin{aligned}
 & \log 57.86 + \log 4.385 - \log 2.391 - \log 3.072 \\
 &= \log \frac{57.86 \times 4.385}{2.391 \times 3.072} \text{ Ans.}
 \end{aligned}$$

4 Solve with the help of logarithm table.

$$(i) \frac{2.38 \times 3.901}{4.83} \quad (ii) \frac{8.67 \times 3.94}{1.78}$$

$$(iii) \frac{25.36 \times 3.4569}{9.87 \times 8.93}$$

Solution:-4(i)

$$x = \frac{2.38 \times 3.901}{4.83}$$

taking log

$$\begin{aligned} \log x &= \log \frac{2.38 \times 3.901}{4.83} \\ &= \log 2.38 + \log 3.901 - \log 4.83 \\ &= 0.3766 + 0.5911 - 0.6839 \end{aligned}$$

$$\log x = 0.2838$$

0

$$\text{Antilog}(\log x) = \text{Antilog} 0.2838$$

$$x = 1.923 \text{ Ans.}$$

Solution:-4(ii)

$$\text{Let } x = \frac{8.67 \times 3.94}{1.78}$$

taking log

$$\begin{aligned} \log x &= \log \frac{8.67 \times 3.94}{1.78} \\ &= \log 8.67 + \log 3.94 - \log 1.78 \end{aligned}$$

$$= 0.9380 + 0.5955 - 0.2504$$

$$\log x = 1.2831$$

taking antilog

$$\text{Antilog}(\log x) = \text{Antilog } 1.2831$$

$$x = 19.19 \text{ characteristic is 1.}$$

Solution:-4(iii)

$$x = \frac{25.36 \times 3.4569}{9.87 \times 8.93}$$

$$\text{Let } \log x = \log \frac{25.36 \times 3.4569}{9.87 \times 8.93}$$

$$\log x = \log 25.36 + \log 3.4569 - \log 9.87 - \log 8.93$$

$$= 1.4041 + 0.5386 - 0.9943 - 0.9509$$

$$= 1.9427 - 1.9452$$

$$\log x = -.0025$$

taking antilog

$$\text{Antilog}(\log x) = \text{Antilog} (-.0025)$$

$$= \text{Antilog} (-1 + 1 - .0025)$$

$$= \text{Antilog} (1.9975)$$

$$= .9942 \text{ Ans.}$$

5- Prove that

$$(i) \log \left(\frac{a^2}{bc} \right) + \log \left(\frac{b^2}{ca} \right) + \log \left(\frac{c^2}{ab} \right) = 0$$

$$(ii) \quad 3\log 2 + 2\log 3 + \log 5 = \log 360$$

$$(iii) \quad 5\log 2 - \log 9 = \log 27$$

$$(iv) \quad \log\left(\frac{75}{15}\right) + \log\left(\frac{32}{243}\right) - 2\log\left(\frac{5}{9}\right) = \log 2$$

$$(v) \quad 2\log\left(\frac{11}{3}\right) + \log\left(\frac{130}{77}\right) - \log\left(\frac{55}{91}\right) = \log 2$$

Proof:

$$\begin{aligned} 5(i) \quad & \log\left(\frac{a^2}{bc}\right) + \log\left(\frac{b^2}{ca}\right) + \log\left(\frac{c^2}{ab}\right) \\ &= \log\left(\frac{a^2}{bc} \times \frac{b^2}{ca} \times \frac{c^2}{ab}\right) \\ &= \log 1 = 0 = R.H.S \end{aligned}$$

Proof:

$$5(ii) \quad 3\log 2 + 2\log 3 + \log 5 = \log 360$$

$$L.H.S$$

$$= 3\log 2 + 2\log 3 + \log 5$$

$$= \log 2^3 + \log 3^2 + \log 5$$

$$= \log 2^3 \times 3^2 \times 5 = \log 8 \times 9 \times 5$$

$$= \log 360 = R.H.S$$

Proof:

$$5(iii) \quad 5\log 3 - \log 9 = \log 27$$

$$\begin{aligned}
 & 5\log 3 - \log 9 \\
 &= \log 3^5 - \log 9 \\
 &= \log \frac{3^5}{9} = \log \frac{3^5}{3^2} = \log 3^{5-2} \\
 &= \log 3^3 = \log 27 = \text{R.H.S}
 \end{aligned}$$

Proof:

$$5(\text{iv}) \quad \log \frac{75}{16} + \log \frac{32}{243} - 2\log \frac{5}{9} = \log 2$$

L.H.S

$$\begin{aligned}
 &= \log \frac{75}{16} + \log \frac{32}{243} - 2\log \frac{5}{9} \\
 &= \log \frac{75}{16} + \log \frac{32}{243} - \log \left(\frac{5}{9} \right)^2 \\
 &= \log \frac{75}{16} \times \frac{32}{243} \times \frac{1}{\left(\frac{5}{9} \right)^2} \\
 &= \log \frac{75}{16} \times \frac{32}{243} \times \frac{9^2}{5^2} \\
 &= \log \frac{\cancel{7} \cancel{5}^3}{\cancel{1} \cancel{6}_1} \times \frac{\cancel{3} \cancel{2}^2}{\cancel{2} \cancel{4} \cancel{3}_3} \times \frac{\cancel{8} \cancel{1}^1}{\cancel{2} \cancel{5}} \\
 &= \log 2 = \text{R.H.S}
 \end{aligned}$$

Proof:

$$5(v) \quad 2 \log \left(\frac{11}{13} \right) + \log \frac{130}{77} - \log \frac{55}{91} = \log 2$$

L.H.S

$$2 \log \left(\frac{11}{13} \right) + \log \frac{130}{77} - \log \frac{55}{91}$$

$$= \log \left(\frac{11}{13} \right)^2 + \log \frac{130}{77} - \log \frac{55}{91}$$

$$= \log \left(\frac{11}{13} \right)^2 \times \frac{130}{77} \times \frac{55}{91}$$

$$= \log \left(\frac{11}{13} \right)^2 \times \frac{130}{77} \times \frac{1}{\frac{55}{91}}$$

$$= \log \frac{11 \times 11}{13 \times 13} \times \frac{130^{10^2}}{77_7} \times \frac{91^{10^2}}{55_5}$$

$$\log 2 = R.H.S.$$

6- **Show that:** $3 \log 4 + 2 \log 5 - \frac{1}{3} \log 64 - \frac{1}{2} \log 16 = 2$

$$3 \log 4 + 2 \log 5 - \frac{1}{3} \log 64 - \frac{1}{2} \log 16 = 2$$

L.H.S

$$= 3 \log 4 + 2 \log 5 - \frac{1}{3} \log 64 - \frac{1}{2} \log 16 = 2$$

$$\begin{aligned}
&= \log 4^3 + \log 5^2 - \frac{1}{3} \log 4^3 - \frac{1}{2} \log 4^2 \\
&= \log 4^3 + \log 5^2 - \log(4^3)^{\frac{1}{3}} - \log(4^2)^{\frac{1}{2}} \\
&= \log 4^3 + \log 5^2 - \log 4 - \log 4 \\
&= \log 4^3 \times 5^2 - 2 \log 4 \\
&= \log 4^3 \times 5^2 - \log 4^2 = \log \frac{4^3 \times 5^2}{4^2} \\
&= \log 4 \times 25 = \log 100 = \log 10^2 \\
&= 2 \log 10 = 2 = R.H.S
\end{aligned}$$

7. Show that: $\log(1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$

$$\left. \begin{aligned} x &= \log_{10} 1 \\ y &= \log_{10} 2 \\ z &= \log_{10} 3 \end{aligned} \right\} A$$

Therefore $10^x = 1$ (i)

$$10^y = 2 \quad (\text{ii})$$

$$10^z = 3 \quad (\text{iii})$$

$$10^x \times 10^y \times 10^z = 1 \times 2 \times 3$$

$$10^{x+y+z} = 1 \times 2 \times 3$$

$$1 \times 2 \times 3 = 10^{x+y+z}$$

$$\log_{10}(1 \times 2 \times 3) = x + y + z \quad \text{logarithmic form}$$

$$= \log 1 + \log 2 + \log 3$$

From (A)

8- Using logarithmic table evaluate the following:

$$(i) \ 69.13 \times 0.34 \times 0.014 \quad (ii) \ \frac{8.67 \times 3.94}{1.78}$$

$$(iii) \ \frac{4}{3} \times 3.142 \times (1.5)^3 \quad (iv) \ \frac{(25.36)^2 \times (0.4569)}{847.5}$$

$$(v) \ \frac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}} \quad (vi) \ \sqrt{\frac{3\sqrt{0.0125} \times \sqrt{31.15}}{0.00081}}$$

$$(vii) \ \frac{(6.45)^3 \times (0.00034)^{\frac{1}{5}} \times (981.9)}{(9.37)^2 \times (8.93)^{\frac{1}{4}} \times (0.0617)}$$

$$(viii) \ \frac{(0.0437)^{\frac{2}{3}} \times (1.407)^2}{(0.0015)^{\frac{1}{3}} \times (1.235)^{\frac{1}{7}}}$$

Solution:- 8(i)

$$x = 69.13 \times 0.34 \times 0.014$$

Taking log

$$\log x = \log(69.13 \times 0.34 \times 0.014)$$

$$= \log 69.13 + \log 0.34 + \log 0.014$$

$$= 1.8397 + (-1.5315) + (-2.1461)$$

$$= 1.8397 + (-4685) + (-1.8539)$$

$$= 1.8397 - 4685 - 1.8539$$

$$\log x = -0.4827$$

$$= +1 - 1 - 0.4827$$

$$\log x = 1.5173$$

Taking anti-log

$$\text{Antilog}(\log x) = \text{Antilog } 1.5173$$

$$x = .3291 \text{ Ans.}$$

Solution:- 8(ii)

$$\text{Let } x = \frac{8.67 \times 3.94}{1.78}$$

$$\log x = \log 8.67 + \log 3.94 - \log 1.78 \quad \text{Taking log}$$

$$= 0.9380 + 0.5955 - 0.2504$$

$$\log x = 1.2831$$

$$\text{Antilog}(\log x) = \text{Antilog } 1.2831 \text{ Taking anti-log}$$

$$= 19.19 \text{ Ans.}$$

Solution:- 8(iii)

$$\text{Let } x = \frac{4}{3} \times 3.142 \times (1.5)^3$$

$$\log x = \log \frac{4}{3} + \log 3.142 + 3 \log 1.5 \quad \text{Taking log}$$

$$= \log 4 - \log 3 + \log 3.142 + 3 \log 1.5$$

$$= 0.6021 - 0.4771 + 0.4972 + 3(0.1761)$$

$$= 0.6021 - 0.4771 + 0.4972 + 0.5283$$

$$\log x = 1.1505$$

Taking anti-log

$$\text{Antilog}(\log x) = \text{Antilog } 1.1505$$

$$x = 14.15 \text{ Ans.}$$

Solution:-8(iv)

$$\text{Let } x = \frac{(25.36)^2 \times (0.4569)}{847.5}$$

$$\log x = \log \frac{(25.36)^2 \times (0.4569)}{847.5} \text{ taking log}$$

$$= \log(25.36)^2 + \log(0.4569) - \log 847.5$$

$$= 2\log 25.36 + \log 0.4569 - \log 847.5$$

$$= 2(1.4041) + (1.6599) - (2.928)$$

$$= 2.8082 + 1.6599 - 2.928$$

$$= -0.4599$$

$$= 1 - 1 - 0.4599$$

$$\log x = 1.5401$$

taking anti-log

$$\text{Antilog}(\log x) = \text{Antilog } 1.5401$$

$$x = .3468$$

Solution:-8(v)

$$\text{Let } x = \frac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}}$$

Taking log

$$\begin{aligned}
 \log x &= \log \frac{0.9876 \times (16.42)^2}{(4.567)^{\frac{1}{3}}} \\
 &= \log 0.9876 + \log (16.42)^2 - \log (4.567)^{\frac{1}{3}} \\
 &= \log 0.9876 + 2 \log 16.42 - \frac{1}{3} \log 4.567 \\
 &= \bar{1}.9946 + 2(1.2153) - \frac{1}{3}(0.6596) \\
 &= \bar{1}.9946 + 2.4306 - .2199 \\
 &= -1 + .9946 + 2.4306 - .2199 \\
 \log x &= 2.2053
 \end{aligned}$$

Antilog(log x) = Antilog 2.2053 Taking anti-log

$$x = 160.4 \text{ Ans.}$$

Solution:- 8(vi)

$$\begin{aligned}
 \text{Let } x &= \sqrt{\frac{{}^3\sqrt{0.0125} \times \sqrt{31.15}}{0.00081}} \\
 &= \left(\frac{3 \times \sqrt{0.0125} \times \sqrt{31.15}}{0.00081} \right)^{\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\frac{3 \times (0.0125)^{\frac{1}{2}} \times (31.15)^{\frac{1}{2}}}{0.00081} \right)^{\frac{1}{2}} \\
 &= \frac{3^{\frac{1}{2}} \times (0.0125)^{\frac{1}{2} \times \frac{1}{2}} \times (31.15)^{\frac{1}{2} \times \frac{1}{2}}}{(0.00081)^{\frac{1}{2}}} \\
 &= \frac{3^{\frac{1}{2}} \times (0.0125)^{\frac{1}{4}} \times (31.15)^{\frac{1}{4}}}{(0.00081)^{\frac{1}{2}}}
 \end{aligned}$$

taking log

$$\begin{aligned}
 \log x &= \log \frac{3^{\frac{1}{2}} \times (0.0125)^{\frac{1}{4}} \times (31.15)^{\frac{1}{4}}}{(0.00081)^{\frac{1}{2}}} \\
 &= \frac{1}{2} \log 3 + \log (0.0125)^{\frac{1}{4}} + \log (31.15)^{\frac{1}{4}} - \log (0.00081)^{\frac{1}{2}} \\
 &= \frac{1}{2} \log 3 + \frac{1}{4} \log .0125 + \frac{1}{4} \log 31.15 - \frac{1}{2} \log (.00081) \\
 &= \frac{0.4771}{2} \times \frac{1}{4} (2.0969) + \frac{1}{4} (1.4935) - \frac{1}{2} (4.9085) \\
 &= \frac{0.4771}{2} + \frac{1}{4} (-2 + .0969) + .3734 - \frac{1}{2} (-4 + .9085) \\
 \log x &= 0.2386 + \frac{1}{4} (-1.9031) \\
 &\quad + .3734 - \frac{1}{2} (-3.0915)
 \end{aligned}$$

$$= 0.2386 - .4758 + .3734 + 1.5458$$

$$\log x = 1.6820$$

taking anti-log

$$\text{Antilog}(\log x) = \text{Antilog } 1.6820$$

$$x = 48.08 \text{ Ans.}$$

Solution:-8(vii)

$$\text{Let } x = \frac{(6.45)^3 \times (.00034)^{\frac{1}{3}} \times (981.9)}{(9.37)^2 \times (8.93)^{\frac{1}{4}} \times (.0617)}$$

taking log

$$\begin{aligned} \log x &= \log(6.45)^3 + \log(.00034)^{\frac{1}{3}} \\ &\quad + \log 981.9 - \log(9.37)^2 - \\ &\quad \log(8.93)^{\frac{1}{4}} - \log(.0617) \\ &= 3 \log 6.45 + \frac{1}{3} \log(.00034) \\ &\quad + \log(981.9) - 2 \log(9.37) \\ &\quad - \frac{1}{4} \log 8.93 - \log .0617 \end{aligned}$$

$$\begin{aligned}
&= 3(0.8096) + \frac{1}{3}(\bar{4}.5315) \\
&\quad + 2.9921 - 2(0.9717) - \\
&\quad \frac{1}{4}(0.9509) - (\bar{2}.7903) \\
&= 3(0.8096) + \frac{1}{3}(-3.4685) \\
&\quad + 2.9921 - 2(0.9717) - \\
&\quad \frac{1}{4}(0.9509) - (-1.2097) \\
&= 2.4288 - 1.1562 + 2.9921 \\
&\quad - 1.9434 - .2377 + 1.2097
\end{aligned}$$

$$\log x = 3.2933$$

taking antilog

$$\text{Antilog}(\log x) = \text{Antilog} 3.2933$$

$$= 19.64$$

Solution:- 8(viii)

$$\text{Lets } x = \frac{(0.0437)^{\frac{2}{3}} \times (1.407)^2}{(0.0015)^{\frac{1}{3}} \times (1.235)^{\frac{1}{7}}}$$

$$\begin{aligned}
\log x &= \log(.0437)^{\frac{2}{3}} + \log(1.407)^2 \\
&\quad - \log(.0015)^{\frac{1}{3}} - \log(1.235)^{\frac{1}{7}} \\
&= \frac{2}{3} \log(.0437) + 2 \log(1.407) \\
&\quad - \frac{1}{3} \log(.0015) - \frac{1}{7} \log(1.235) \\
&= \frac{2}{3}(\bar{2}.6405) + 2(0.1483) \\
&\quad - \frac{1}{3}(\bar{3}.3761) - \frac{1}{7}(0.0917) \\
&= \frac{2}{3}(-1.3595) + 2(0.1483) \\
&\quad - \frac{1}{3}(-2.8239) - \frac{1}{7}(0.0917) \\
&= \frac{-2.7190}{3} + .2966 + 0.9413 \\
&\quad - 0.0131 \\
&= -.9063 + .2966 + 0.9413 - 0.0131
\end{aligned}$$

$$\log x = 0.3185$$

Taking antilog

$$\text{Antilog}(\log x) = \text{Antilog}(0.3185)$$

$$= 2.082 \text{ Ans.}$$

9. If $v = \sqrt{\frac{gl}{2\pi}}$ find v when

$$l = 150, g = 32.16, \pi = 3.142$$

Solution:-

$$v = \sqrt{\frac{gl}{2\pi}}$$

$$g = 32.16 \text{ and } l = 150 \pi = 3.142$$

$$v = \sqrt{\frac{(32.16)(150)}{2(3.142)}}$$

taking log

$$\log v = \log \sqrt{\frac{(32.16)(150)}{2(3.142)}}$$

$$= \log \left(\frac{32.16 \times 150}{2 \times (3.142)} \right)^{\frac{1}{2}}$$

$$= \frac{1}{2} \log \frac{32.16 \times 150}{2 \times (3.142)}$$

$$= \frac{1}{2} [\log 32.16 + \log 150 - \log 2 - \log 3.142]$$

$$= \frac{1}{2} [1.5073 + 2.1761 - 0.3010 - 0.4972]$$

$$= \frac{1}{2} [2.8851]$$

$$\log v = 1.4426$$

taking antilog

$$\text{Antilog}(\log v) = \text{Antilog } 1.4426$$

- 10- If $H = \frac{I^2 R t}{4.2}$ find H . When $I = 1.3, R = 6.7$
and $t = 25$

Solution:-

$$H = \frac{I^2 R t}{4.2}$$

Putting values $I = 1.3 \quad R = 6.7 \quad t = 25$

$$H = \frac{(1.3)^2 (6.7) (25)}{4.2}$$

$$\begin{aligned} \log H &= \log \frac{(1.3)^2 (6.7) (25)}{4.2} \\ &= \log \frac{(1.3)^2 (6.7) (25)}{4.2} \\ &= \log (1.3)^2 + \log 6.7 \\ &\quad + \log 25 - \log 4.2 \\ &= 2 \log 1.3 + \log 6.7 \\ &\quad + \log 25 - \log 4.2 \\ &= 2(0.1139) + 0.8261 \\ &\quad + 1.3979 - 0.6232 \\ &= .2278 + 0.8261 + 1.3979 - 0.6232 \end{aligned}$$

$$\log H = 1.8286$$

taking antilog

$$\text{Antilog}(\log H) = \text{Antilog } 1.8286$$

$$H = 67.39 \text{ Ans.}$$

11- Find h , if $h = \frac{v}{\pi(R^2 - r^2)}$, when

$$v = 1190, R = 83.6, r = 62.4 \text{ and } \pi = 3.14$$

Solution:-

$$h = \frac{v}{\pi(R + r)(R - r)}$$

Putting values r, R, π, v

$$h = \frac{1190}{3.14(83.6 + 62.4)(83.6 - 62.4)}$$

$$h = \frac{1190}{3.14(146)(21.2)}$$

taking log

$$\log h = \log \frac{1190}{3.14 \times 146 \times 21.2}$$

$$\log 1190 - \log 3.14 - \log 146$$

$$= -\log 21.2$$

$$= 3.0755 - 0.4969 - 2.1644 - 1.3263$$

$$= -0.9121$$

$$= \underline{1} - 1 - 0.9121$$

$$\log h = 1.0879$$

taking antilog

$$\text{Antilog } h = \text{Antilog } 1.0879$$

$$h = .1224 \text{ Ans.}$$