UNIT 08 Chords and Arcs of a Circle

Students' Learning Outcomes

After completing this unit, the students will be able to:

- Solve problems by using the properties of a circle:
  - One and only one circle can pass through three non-collinear points.
  - A straight line, drawn from the centre of a circle to bisect a chord (Which is not a diameter) is perpendicular to the chord.
  - Perpendicular from the centre of a circle on a chord bisects it.
  - If two chords of a circle are congruent, then they will be equidistant from the centre.
  - Two chords of a circle which are equidistant from the centre are congruent.
  - If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are equal.
  - If two chords of a circle (or of congruent circles) are equal, then their corresponding arcs (minor, major or semi-circular) are congruent.
  - Equal chords of a circle (or of congruent circles) subtend equal angles at the centre (at the corresponding centres).
  - If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, then the chords are equal.
- Apply concepts of chords and arcs of a circle to real life world problems (such as decorative features, rainbow, bridges, and roller coaster track).

Circle and its Components

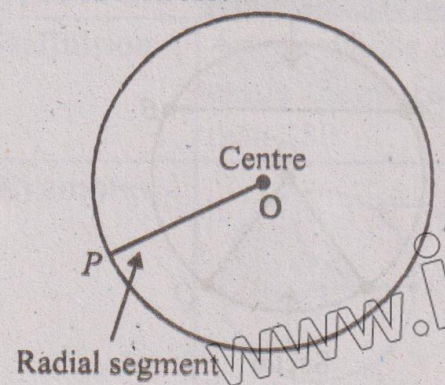
Circle: A circle is the set of all points that are at the same distance from a fixed point.

We also define circle as:

A circle is the locus of a point in a plane which moves in such a way that its distance from a fixed point is always constant.

Radial segment

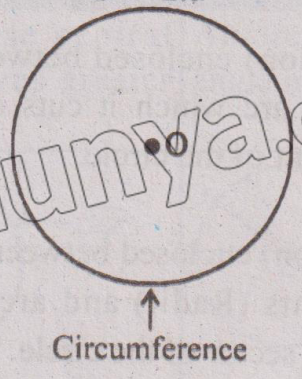
The line segment joining any point of the circle to its centre is called **radial segment** and its measure is called **radius** of the circle.



Circumference

The length of the boundary of a circle is called circumference.

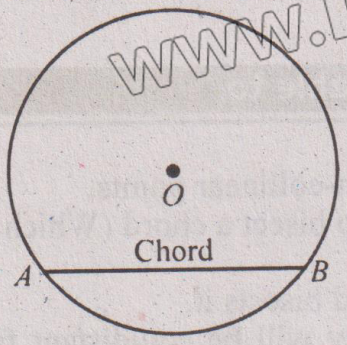
$C = 2\pi r$



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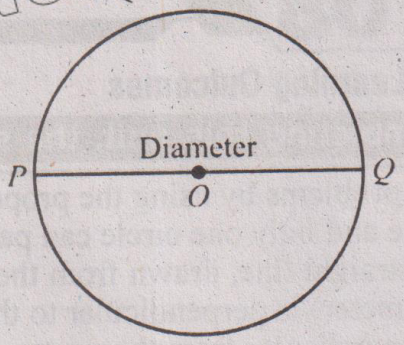
Chord

A line segment joining any two distinct points of a circle. In the below figure, $\overline{AB}$ is a chord of the circle.



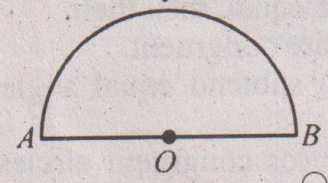
Diameter

A chord passing through the centre of circle is called diameter. $\overline{PQ}$ is a diameter.



Semi-Circle

A semi-circle is half of the circle, that is a figure bounded by a diameter and circumference cut by the diameter.



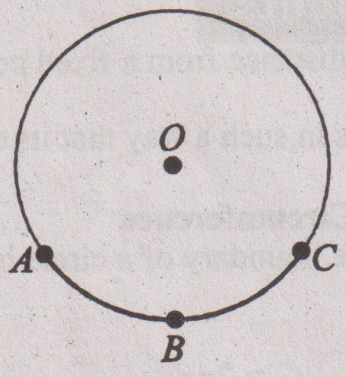
Arc

Any part of the circumference of a circle is called an arc. ABC is an arc of the circle. Similarly, AB and BC are also arcs.



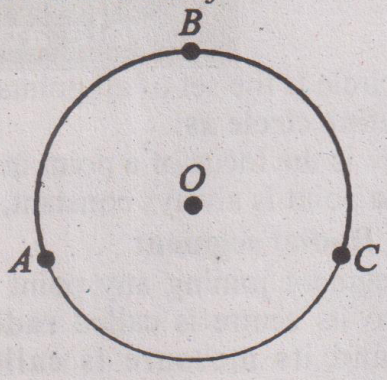
Minor Arc

A minor arc is an arc which is smaller than the arc of a semi-circle. ABC is a minor arc.



Major Arc

A major arc is an arc greater than the arc of a semi-circle. ABC is a major arc.

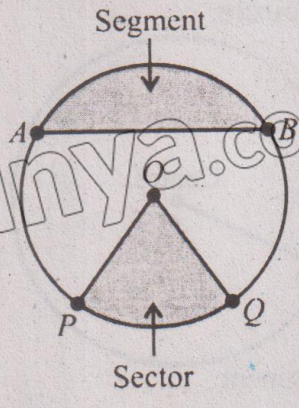


Segment:

The area (region) enclosed between a chord and the arc which it cuts off is called a segment of the circle.

Sector:

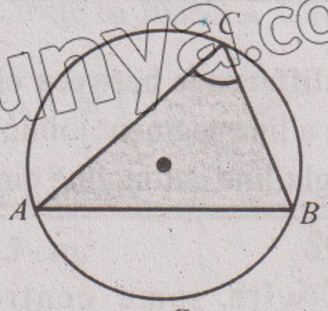
The area (region) enclosed between two radial segments (Radii) and arc of a circle is called sector of the circle.



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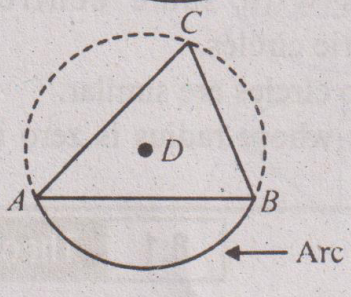
Angle in the Segment:

If a point on the arc of a segment is joined with the end points of a chord, the angle formed by the line segments so drawn is called angle in the segment. ∠ACB is an angle in the segment ACB. This is also called **circum angle.**



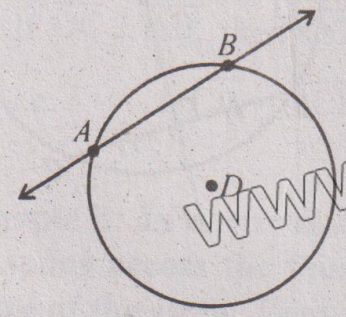
Angle Standing upon an Arc:

If a point on the circumference of a circle is joined to the extremities of an arc, the angle so formed is said to angle stand upon the arc.



Secant

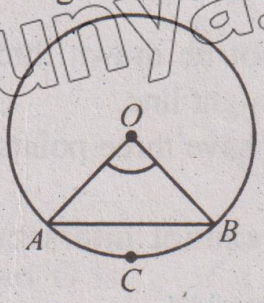
A straight line cutting the circumference of a circle at two distinct points is called a secant. $\overleftrightarrow{AB}$ is a secant.



Central Angle

If the extremities of an arc or of a chord are joined with the centre of a circle, angle so formed is called a **central angle.**

∠AOB is a central angle of chord AB or of arc ACB.

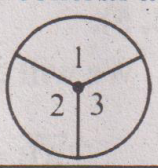


Key Concept

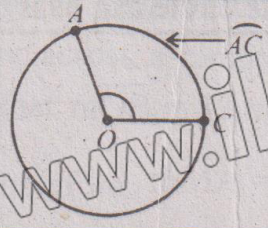
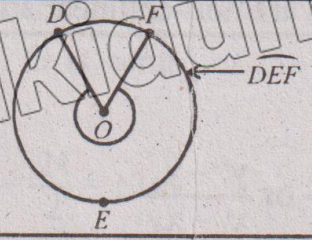
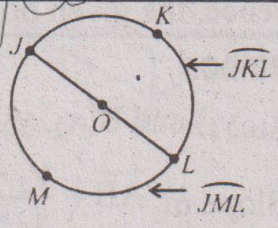
The sum of the measures of the central angles of a circle with no interior points in common is 360°.

i.e. $m\angle 1 + m\angle 2 + m\angle 3 = 360°$

Sum of central angles



Key Concept

Types of arc	Minor arc	Major arc	Semi-Circle
Definition	An arc whose central angle measures less than 180°.	An arc whose central angle measures greater than 180°.	An arc whose central angle measures 180°.
Examples			

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Challenge!

What is the difference between chord and secant?

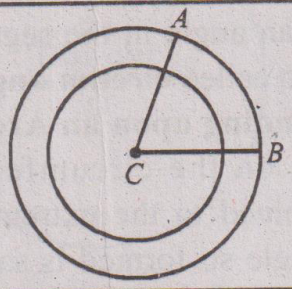
Ans. Chord is a line segment joining any two distinct points of a line circle. While secant is a straight line cutting the circumference of a circle at two distinct points.

Do you know?

(i) Circles with same centre are called concentric circles.

(ii) Any two circles are similar.

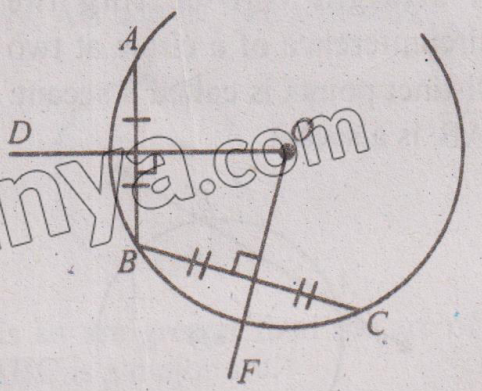
(iii) A circle whose radius is zero is called point circle.



8.1 Theorems (Properties) of a Circle

1. One and Only One Circle Can Pass Through Any Three Non-Collinear Points.

If A, B and C are three points not in the same straight line, then one and only one circle can pass through A, B and C.



Note

- A circle cannot be drawn through more than two points in a straight line.
- If two circles have three points common, then they coincide.
- Two circles cannot intersect in more than two points.

Example 1: Prove that a unique circle can pass through the points A(1, 1), B(4, 2) and C(3, 5).

Sol. We will check the points are non-collinear by using slope formula.

$$\text{Slope of } \overline{AB} = \frac{2-1}{4-1} = \frac{1}{3}$$

$$\text{Slope of } \overline{BC} = \frac{5-2}{3-4} = \frac{3}{-1} = -3$$

Slope of $\overline{AB} \neq$ Slope of $\overline{BC}$

Hence, the given points are non-collinear and unique circle can pass through them.

Remember!

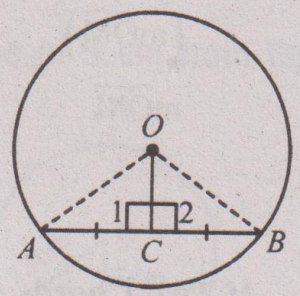
If $A(x_1, y_1)$ and $B(x_2, y_2)$,

then Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1}$ or $\frac{y_1 - y_2}{x_1 - x_2}$

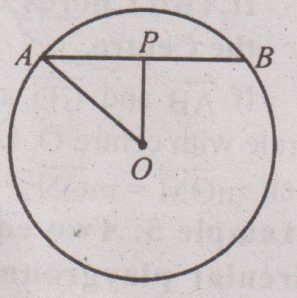
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2. A straight Line, Drawn from the Centre of a Circle to Bisect a Chord (which is not a diameter) is Perpendicular to the Chord

If $\overline{AB}$ is any chord (not a diameter) and $\overline{OC}$ meets $\overline{AB}$ at C such that $m\overline{CA} = m\overline{CB}$, then $\overline{OC} \perp \overline{AB}$



Example 2: In a circle, $m\overline{AB} = 8$ cm is bisected at point P by a line from the centre O. Find $m\overline{OP}$ if radius is 5 cm.



Sol. Since triangle APO is right angled triangle.

So, by Pythagoras theorem

$$\left(m\overline{OA}\right)^2 = \left(m\overline{OP}\right)^2 + \left(m\overline{AP}\right)^2$$

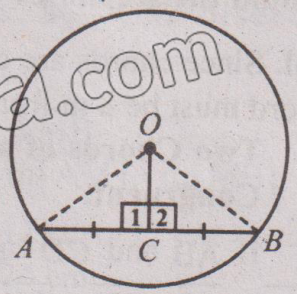
$$5^2 = \left(m\overline{OP}\right)^2 + 4^2$$

$$25 = \left(m\overline{OP}\right)^2 + 16$$

$$\left(m\overline{OP}\right)^2 = 25 - 16 = 9$$

$$m\overline{OP} = 3\text{cm}$$

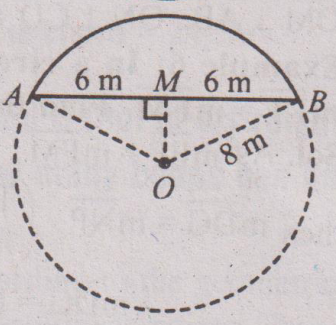
3. Perpendicular from the Centre of a Circle on a Chord Bisects It.



If $\overline{OC} \perp \overline{AB}$, then $\overline{OC}$ bisects $\overline{AB}$

i.e. $m\overline{CA} = m\overline{CB}$

Example 3: In a circular fountain of radius 8 m, a pipe AB spans across the fountain. If $m\overline{AB}$ = 12 m and O is centre of the circle. Find the distance from O to the chord AB.



Sol. Given that $m\overline{AB} = 12\,m$

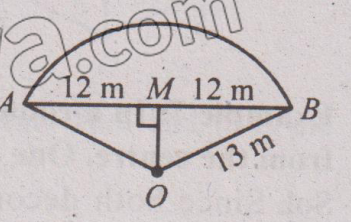
Distance from O to chord AB is perpendicular so by theorem, M is the mid-point.

$$m\overline{OM} = \sqrt{\left(m\overline{OB}\right)^2 - \left(m\overline{BM}\right)^2}$$ [By Pythagoras theorem]

$$= \sqrt{(8)^2 - (6)^2} = \sqrt{64 - 36} = \sqrt{28}$$

$$= 5.29\text{ m}$$

Example 4: Chord AB of 24 m is across a circular arch of radius 13 m. Find the height from the chord to the centre.



Sol. $m\overline{AB}$ = 24 m across a circular arch. O is centre of the circle of the arch that $m\overline{OB}$ = 13 m.

$\overline{OM} \perp \overline{AB}$, therefore, $m\overline{MA} = m\overline{MB} = \frac{24}{2} = 12\,m$

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Now, $\left(m\overline{OM}\right)^2 = \left(m\overline{OB}\right)^2 - \left(m\overline{MB}\right)^2$ [By Pythagoras theorem]

$\left(m\overline{OM}\right)^2 = (13)^2 - (12)^2$

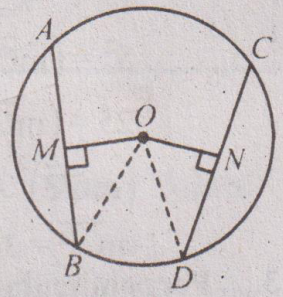
$m\overline{OM} = \sqrt{(13)^2 - (12)^2}$

$= \sqrt{169 - 144}$

$= \sqrt{25} = 5\,m$

4. If Two Chords of a Circle Are Congruent, then they will be Equidistant from the Centre.

If $\overline{AB}$ and $\overline{CD}$ are congruent (equal in measures) chords of a circle with centre O. i.e,. $m\overline{AB} = m\overline{CD}, \overline{OM} \perp \overline{AB}$ and $\overline{ON} \perp \overline{CD}$, then $m\overline{OM} = m\overline{ON}$

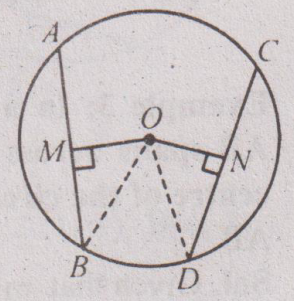


Example 5: Two equal-length ropes (chords) are tied in a circular playground. One is 8 m from the centre. Where should the second be placed?

Sol. Since chords are congruent, they must be equidistant from the centre. So, the second chord must be 8 m from the centre.

5. Two Chords of a Circle which are Equidistant from the Centre of a Circle, are Congruent.

If $\overline{AB}$ and $\overline{CD}$ are chords of a circle with centre O, $\overline{OM} \perp \overline{AB}$, $\overline{ON} \perp \overline{CD}$ and $m\overline{OM} = m\overline{ON}$, then $\overline{AB} \cong \overline{CD}$ i.e;



Example 6: In a circle with centre at F, $m\overline{NP}$ = 12 cm and $m\overline{EF} = m\overline{FM}$. Find the length of DE.

Sol. As $m\overline{EF} = m\overline{FM}$

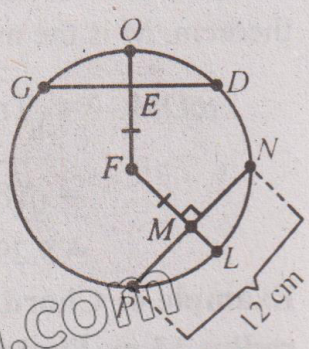
then $m\overline{DG} = m\overline{NP}$ (Two chord of a circle which are equidistant from the centre are congruent.)

$m\overline{DG} = 12\,cm$ $\because$ $m\overline{NP} = 12\,cm$

Since the radius bisects $\overline{DG}$, so

$\Rightarrow$ $m\overline{GE} = \frac{1}{2} m\overline{DG}$

$= \frac{1}{2}(12\,cm) = 6\,cm$



Example 7: In a round table design, two decorations are placed symmetrically 4 cm from the centre. One chord is 14 cm. What is the length of the other?

Sol. Since both decorations are equidistant from the centre, therefore chords are congruent.

So, the length of second chord is 14 cm.

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Solved Exercise 8.1

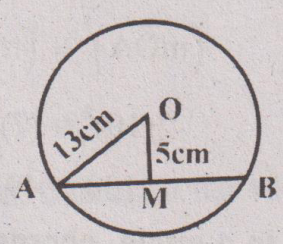
1. Calculate the length of a chord which stands at a distance of 5 cm from the centre of a circle whose radius is 13 cm.

Sol. Let

$\overline{AB}$ is the chord of the circle with O as centre.

Distance of chord from centre $\left(\overline{OM}\right) = 5\text{cm}$

Length of the radius $\left(\overline{OA}\right) = 13\text{ cm}$



Since triangle AMO is right angled triangle

So, by pythagoras theorem

$$\left(m\overline{OA}\right)^2 = \left(m\overline{AM}\right)^2 + \left(m\overline{OM}\right)^2$$

$$(13)^2 = \left(m\overline{AM}\right)^2 + (5)^2$$

$$169 = \left(m\overline{AM}\right)^2 + 25$$

$$169 - 25 = \left(m\overline{AM}\right)$$

$$144 = \left(m\overline{AM}\right)^2$$

$$\left(m\overline{AM}\right)^2 = 144$$

$$\sqrt{\left(m\overline{AM}\right)^2} = \sqrt{144}$$

$$m\overline{AM} = 12\text{ cm}$$

Since $\overline{OM} \perp AB$, So $\overline{OM}$ bisects $\overline{AB}$

i.e. $m\overline{AM} = m\overline{MB}$

$$m\left(\overline{AB}\right) = m\left(\overline{AM}\right) + m\left(\overline{MB}\right)$$

$$= 12\text{ cm} + 12\text{ cm}$$

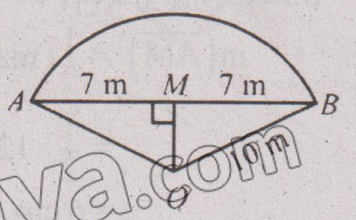
$$= 24\text{ cm}$$

So, Length of the chord = 24 cm

2. In construction, three steel rods are fixed at points A, B, and C (not in a straight line). A circular hoop needs to pass through all three. How many hoops can be used?

Ans. Since three points A, B and C are non-collinear points. According to the geometric theorem of circle, only one circle can pass through any three non-collinear points. Thus, only one hoop can be used.

3. In a park, lamp posts are 14 m apart on the edge of a circular part of radius 10 m as shown in figure. Find the distance of the chord from the centre of the park.



Sol. A and B are lamp posts on the edge of a circular part of radius 10m and O is the centre of the circle.

$m\left(\overline{OA}\right) = 10\text{ m}$

$m\left(\overline{AB}\right) = 14\text{ m}$

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$\overline{AM}$ is the half of $\overline{AB}$ as $\overline{OM} \perp \overline{AB}$

$m(\overline{AM}) = 7$ m

OMA is a right angled triangle

So, by Pythagoras theorem

$$m(\overline{OA})^2 = m(\overline{AM})^2 + m(\overline{OM})^2$$

$$(10)^2 = (7)^2 + m(\overline{OM})^2$$

$$100 = 49 + m(\overline{OM})^2$$

$$100 - 49 = m(\overline{OM})^2$$

$$51 = m(\overline{OM})^2$$

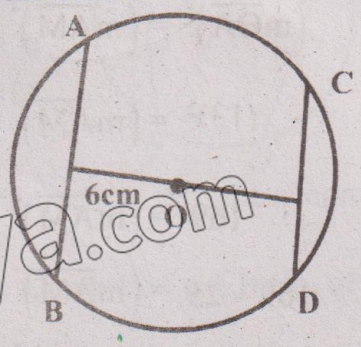
$$\sqrt{m(\overline{OM})^2} = \sqrt{51}$$

$$m\overline{OM} = 7.14 \text{ m}$$

So, the distance of the chord from the centre of the park = 7.14 m

4. In a circle, chords AB and CD both have length 10 cm. If the distance from the centre to AB is 6 cm, what is the distance from centre to CD?

Sol. In a circle $\overline{AB}$, and $\overline{CD}$ are two chords O is the centre of the circle.



$m(\overline{AB}) = 10$ cm

$m(\overline{CD}) = 10$ cm

$m(\overline{AB}) = m(\overline{CD})$

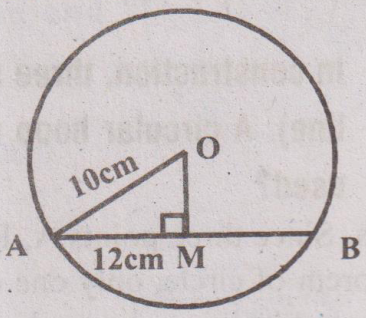
If two chords of a circle are congruent, then they will be equidistant from the centre.

Distance of $\overline{AB}$ from the centre = 6 cm

Distance of $\overline{CD}$ from the centre = 6 cm

5. Two holes A and B are drilled 12 cm apart on a circular tabletop of radius 10 cm. Find the perpendicular distance from the centre to AB.

Sol. Let $\overline{OM}$ is the perpendicular distance from the centre to $\overline{AB}$. A and B are two holes drilled on a circular table top.



$m(\overline{OA}) = 10$ cm (radius)

$m(\overline{AB}) = 12$ cm

Since $\overline{OM} \perp$ AB, So $\overline{OM}$ bisects $\overline{AB}$

$$m(\overline{AM}) = \frac{1}{2}(mAB)$$

$$= \frac{1}{2} \times 12 \text{ cm}$$

$$= 6 \text{ cm}$$

OMA is a right angled triangle So, by the Pythagoras theorem

$$m(\overline{AO})^2 = m(\overline{AM})^2 + m(\overline{OM})^2$$

$$(10)^2 = (6)^2 + m(\overline{OM})^2$$

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$$100 = 36 + \left(m\overline{OM}\right)^2$$

$$\left(m\overline{OM}\right)^2 = 100 - 36$$

$$\left(m\overline{OM}\right)^2 = 64$$

$$\sqrt{\left(m\overline{OM}\right)^2} = \sqrt{64}$$

$$m\overline{OM} = 8 \text{ cm}$$

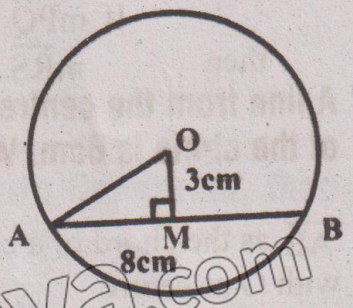
So, the perpendicular distance from centre to AB = 8 cm

6. A chord 8 cm long is at a distance of 3 cm from the centre. Calculate the radius of the circle.

Sol. Let $\overline{OA}$ is the radius of the circle with centre O.

$m\overline{OM}$ the distance of the chord from centre = 3cm

Since $\overline{OM}$ bisects the chord $\overline{AB}$ as $\overline{OM} \perp AB$



$$m\overline{AB} = 8 \text{ cm}$$

$$m\overline{AM} = \frac{1}{2}\left(m\overline{AB}\right)$$

$$= \frac{1}{2} \times 8\text{cm} \quad = 4 \text{ cm}$$

OMA is a right angled triangle

So, the pythagoras theorem

$$\left(m\overline{OA}\right)^2 = \left(m\overline{AM}\right)^2 + \left(m\overline{OM}\right)^2$$

$$\left(m\overline{OA}\right)^2 = (4)^2 + (3)^2$$

$$\left(m\overline{OA}\right)^2 = 16 + 9$$

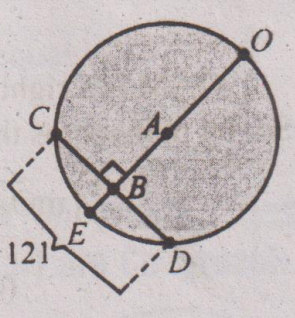
$$\left(m\overline{OA}\right)^2 = 25$$

$$\sqrt{\left(m\overline{OA}\right)^2} = \sqrt{25}$$

$$\left(m\overline{OA}\right) = 5$$

So, the radius of the circle = 5cm

7. In the given figure, $m\overline{CD}$ = 121 units and $m\overline{BC}$ = 3x units. Find the value of x.



Sol.

$$m\overline{CD} = 121 \text{ units}$$

$$m\overline{BC} = 3x \text{ units}$$

$\overline{AB} \perp \overline{CD}$, so $\overline{AB}$ bisects $\overline{CD}$

$$m\overline{CB} = \frac{1}{2}\left(m\overline{CD}\right)$$

$$3x = \frac{1}{2} \times 121$$

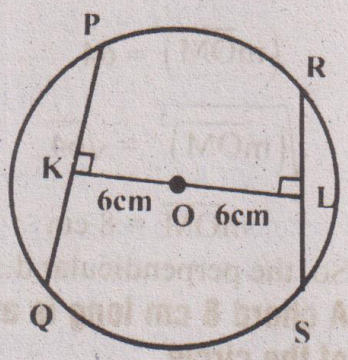
$$x = \frac{1}{2} \times 121 \times \frac{1}{3} = \frac{121}{6}$$

$$x = 20.17 \text{ units}$$

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8. In a circle with centre at O, the perpendicular distances of the each chords PQ and RS from the centre are 6 cm. If the length of chord PQ is 18 cm, find the length of the other chord.

Sol. Let O is the centre of the circle $\overline{OK}$ and $\overline{OL}$ are perpendicular distances of the chords PQ and RS from centre respectively



$$m\overline{OK} = m\overline{OL} = 6 \text{ cm}$$
$$m\overline{PQ} \text{ chord} = 18 \text{ cm}$$

Two chords of a circle which are equidistant from the centre of a circle, are congruent

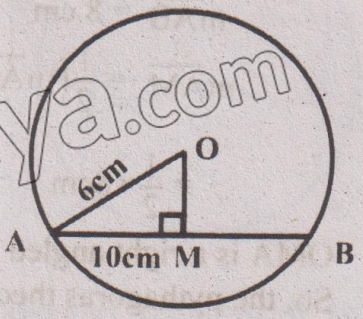
So $m\overline{PQ} = m\overline{RS}$

If $m\overline{PQ} = 18$ cm

then $m\overline{RS} = 18$ cm

9. A line from the centre of a circle cuts a 10 cm chord at right angle where radius of the circle is 6cm. What is the length from the centre to the chord?

Sol.



$\overline{AB}$ as the chord of a circle with O as centre

$m(\overline{AB})$ chord = 10 cm

$m(\overline{OA})$ radius = 6 cm

Since $\overline{OM} \perp \overline{AB}$, So OM bisects $\overline{AB}$ at point M then,

$$m\overline{AM} = \frac{1}{2}(m\overline{AB})$$
$$= \frac{1}{2} \times 10 \text{ cm}$$
$$= 5 \text{ cm}$$

OAM is a right angled triangle

So, by pythagoras theorem

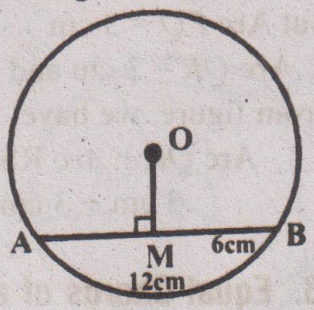
$$(m\overline{OA})^2 = (m\overline{AM})^2 + (m\overline{OM})^2$$
$$(6)^2 = (5)^2 + (m\overline{OM})^2$$
$$36 = 25 + m(OM)^2$$
$$(m\overline{OM})^2 = 36 - 25$$
$$(m\overline{OM})^2 = 11$$
$$\sqrt{(m\overline{OM})^2} = \sqrt{11}$$
$$\therefore m\overline{OM} = 3.32 \text{ cm}$$

So, thus, length from centre to the chord = 3.32 cm

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10. In a circle, a perpendicular is drawn from the centre to chord AB. If $m\overline{AB} = 12cm$, what is the length of each segment after bisecting?

Sol. Let O is the centre of a circle



$\overline{AB}$ is the chord and $\overline{OM}$ is the perpendicular to chord $\overline{AB}$

$m\overline{AB}$ (chord) = 12 cm

Since, is $\overline{OM} \perp AB$, So OM bisect chord $\overline{AB}$ at point M

Thus $m\overline{AM} = mMB \quad (\overline{OM} \perp \overline{AB})$

$$m\overline{AM} = \frac{1}{2}(mAB) = \frac{1}{2} \times 12cm$$

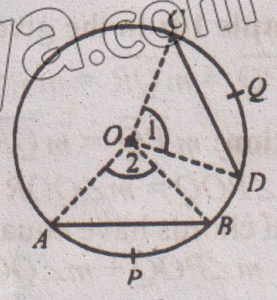
$$= 6 \text{ cm}$$

So $m\overline{MB} = 6 \text{ cm} \quad (\overline{AM} = \overline{MB})$

Thus, the length of each segment after bisecting = 6 cm

11. If Two Arcs of a Circle (or of Congruent Circles) are Congruent, then the Corresponding Chords are Equal.

In the circle with centre *O*, $\overset{\frown}{APB} \cong \overset{\frown}{COD}$ and $\overline{AB}$, $\overline{CD}$ are their corresponding chords, then $\overline{AB} \cong \overline{CD}$



Example 8:

In a circular garden, two pair of decorative lights are placed such that the arcs between each pair (*A* and *B*, *C* and *D*) are equal in length. If the straight-line distance between points *A* and *B* is 8 metres, what is the distance between points *C* and *D*?

Solution: Given that

$m\overline{AB}$ = 8 metres

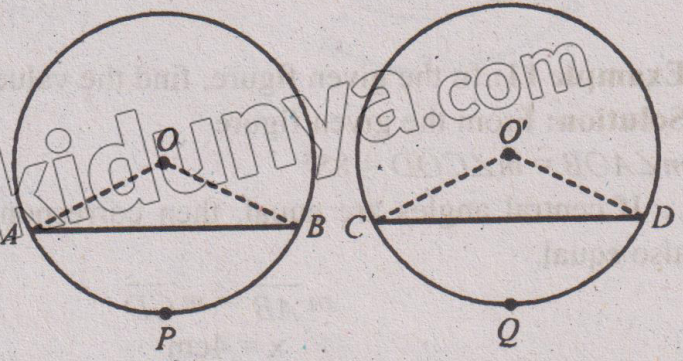
Since the arcs *AB* and *CD* are congruent, so their chords must be equal.

$m\overline{AB} = m\overline{CD}$

Hence, $m\overline{CD}$ = 8m

12. If Two Chords of a Circle (or of Congruent Circles) are Congruent (Equal in Measurement) then their Corresponding Arcs (Minor, Major) are Congruent.

If circles with centre *O* and *O′* are congruent and Chord *AB* ≅ Chord *CD*, then $\overset{\frown}{APB} \cong \overset{\frown}{CQD}$



Example 9: Find the value of *x* in the given figure.

Solution: Given that:

Chord *PQ* ≅ Chord *QR* ≅ Chord *RS*

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As, we know that equal chords have equal arcs,

$\therefore$ Arc $PQ \cong$ Arc $QR \cong$ Arc RS

But Arc $PQ = 3$ cm

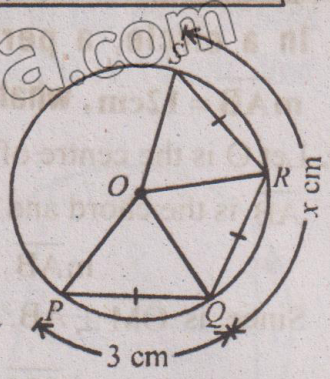
$\therefore$ Arc $QR = 3$ cm and Arc $RS = 3$ cm

From figure, we have

Arc QR + Arc $RS = x$

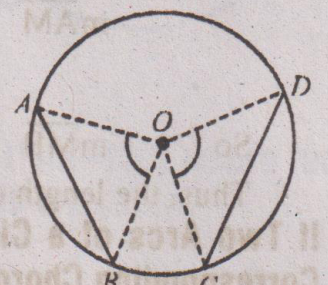
$3 \text{ cm} + 3 \text{ cm} = x$

$x = 6\text{cm}$

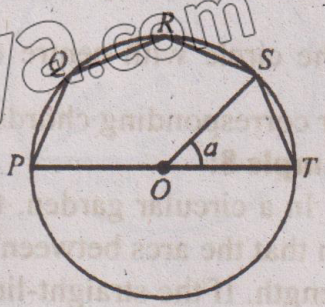


13. Equal Chords of a Circle (or of Congruent Circles) Subtend Equal Angles at the Centre (at the corresponding Centres)

In the circle with centre O, $m\overline{AB} = m\overline{CD}$, chord AB and chord CD subtended $\angle AOB$ and respectively at the centre O, then $m\angle AOB = m\angle COD$



Example 10: In the given figure, $\overline{PT}$ is a diameter of circle. If $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$, then find a.



Solution: $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$ (given)

$\therefore\ m\angle POQ = m\angle QOR = m\angle ROS = m\angle SOT = a$

(equal chords have equal angles at the centre)

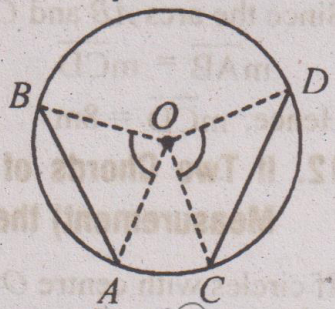
$m\angle POQ + m\angle QOR + m\angle ROS + m\angle SOT = 180°$

$4a = 180°$

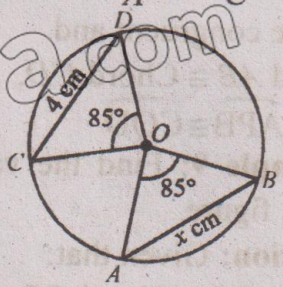
$a = 45°$

14. If the Angles Subtended by Two Chords of a Circle (or Congruent Circles) at the Centre (Corresponding Centres) are Equal, then the Chords are Equal

In the circle with centre O, $\overline{AB}$ and $\overline{CD}$ are its chords, they subtend $\angle BOA$ and $\angle COD$ respectively at the centre O, of the circle such that $m\angle BOA = m\angle COD$, then $m\overline{AB} = m\overline{CD}$



Example 11: In the given figure, find the value of x.



Solution: From the given figure

$m\angle AOB = m\angle COD = 85°$

$\therefore$ If central angles are equal, then corresponding chords are also equal.

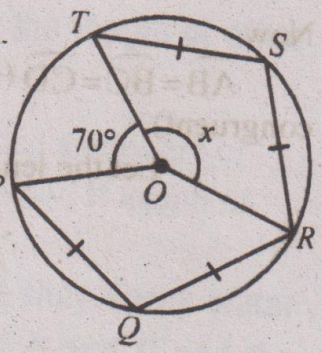
$m\overline{AB} = m\overline{CD}$

$x = 4\text{cm}$

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Solved Exercise 8.2

1. In the given figure, if $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$ and $m\angle POT = 70°$, then find x.



Sol. $m\overline{PQ} = m\overline{QR} = m\overline{RS} = m\overline{ST}$ (Givens)

$m\angle POQ = m\angle QOR = m\angle ROS = m\angle SOT$ (equal chords have equal angles at centre)

Let each equal angle $= \theta$

$m\angle TOP = 70°$

We know

$$m\angle POQ + m\angle QOR + m\angle ROS + m\angle TOP = 360°$$
$$\theta + \theta + \theta + \theta + 70° = 360°$$
$$4\theta + 70 = 360°$$
$$4\theta = 360° - 70°$$
$$4\theta = 290°$$
$$\theta = \frac{290°}{4}$$
$$\theta = 72.5°$$

Put

$\theta + \theta = x$ (give)

$2\theta = x$

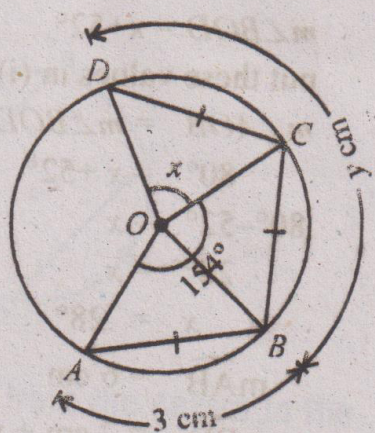
$\theta = 72.5°$

$2(72.5°) = x$

$145° = x$

$x = 145°$

2. In the given figure, find x and y



Sol. $m\overline{AB} = m\overline{BC} = m\overline{CD}$ (Given)

$m\angle AOB = m\angle BOC = m\angle COD$

(equal chords have equal angles at centre)

Let each equal angle $= \theta$

$m\angle COD = x$ (Given)

Add there three angle

$$m\angle AOB + m\angle BOC + m\angle COD = m\angle AOD$$
$$\theta + \theta + \theta = 3\theta$$
$$\theta + 2\theta = 3\theta$$
$$\theta + 154° = 3\theta$$
$$154° = 3\theta - \theta$$
$$154° = 2\theta$$
$$\theta = \frac{154°}{2}$$
$$\theta = 77°$$

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put θ $= x$ (Given)

$x = 77°$

Now

$\overset{\frown}{AB} = \overset{\frown}{BC} = \overset{\frown}{CD}$ (If chords of circle are congruent then corresponding arc are congruent)

Let the length of each arc $= x$ cm

Add there ares

$\overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{CD} = \overset{\frown}{AD}$

$z + z + z = 3z$

$z + 2z = 3z$

$2z = y$

$z + y = 3z$

$y = 3z - z$

$y = 2z$

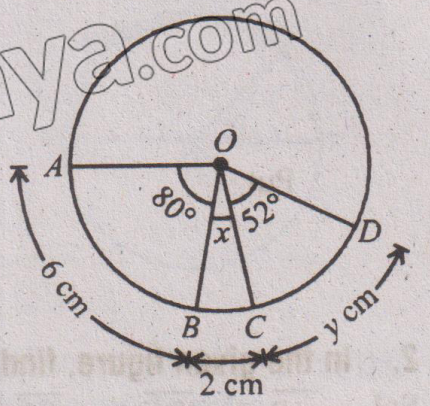
put z $= 3$cm

$y = 2(3\text{cm})$

$y = 6$cm

So, $x = 77°$, $y = 6$cm

3. Find the values of x and y in the given figure, such that $m\overset{\frown}{AB} = m\overset{\frown}{BD}$



Sol. $m\overset{\frown}{AB} = m\overset{\frown}{BD}$ (Given)

$m\angle AOB = m\angle BOD$ — (i) (Equal arcs of a circle subtened equal angls at center)

$m\angle AOB = 80°$ (Given)

$m\angle BOD = x + 52°$

put these values in (i)

$m\angle AOB = m\angle BOD$

$80° = x + 52°$

$80° - 52° = x$

$28° = x$

$x = 28°$

$m\overset{\frown}{AB} = 6$ cm (Given)

$m\overset{\frown}{BD} = 2$ cm + ycm (Given)

$m\overset{\frown}{AB} = m\overset{\frown}{BD}$ (Given)

6 cm $= 2$ cm $+ y$

6 cm $- 2$ cm $= y$

4 cm $= y$

$y = 4$cm

So, $x = 28°$, $y = 4$cm

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4. Two congruent arcs in a circular track subtend angles of 60° each at the centre. If the length of one chord of the circle is 10 metres, what is the length of other chord?

Ans. Since two arcs are congruent in a circular track and their subtended angles arc of 60° each. So, the length of their subtended chords will also be equal. Now, If length of one chord is 10 meter, then the length of other chord is also 10m.

5. In a circular fountain, two water jets are installed such that they spray water along arcs of equal length. If one jet sprays between points *P* and *Q* and the straight-line distance (chord *PQ*) is 12 metres, what is the straight-line distance (chord *RS*) covered by the second jet spraying along arc RS, which is congruent to arc *PQ*?

Sol. $m\overset{\frown}{PQ} = m\overset{\frown}{RS}$ (Given)

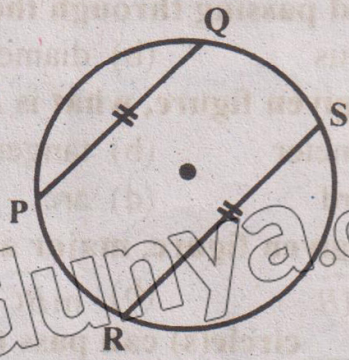
The chord subtened to $\overset{\frown}{PQ} = \overline{PQ}$

The chord subtended to $\overset{\frown}{RS} = \overline{RS}$

Since two arcs are congruent, So their subtened chords will also be equal.

Chord $m\overline{PQ} = 12$ m

Chord $m\overline{RS} = 12$ m



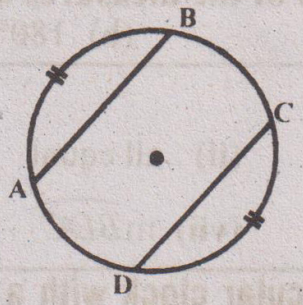
6. In a circular park, two walkways $\overline{AB}$ and $\overline{CD}$ are both straight paths (chords) of length 14 metres. What can be said about the minor arcs subtended by these walkways?

Sol. Length of chord $\overline{AB} = 14$m

Length of chord $\overline{CD} = 14$ m

$m\overline{AB} = m\overline{CD}$ (Given)

Since two chords of a circular park are congruent so, the minor arc subtended by chords $\overline{AB}$ and $\overline{CD}$ are also congruent.



7. In two congruent circular clocks, the minute hand points from the centre to the 3 on both. A decorative string connects 3 to 9 on both clocks. Are the arcs from 3 to 9 on both clocks congruent?

Ans. The size of circular clocks is same and they are congruent on clock face, the distance from 3 to 9 represents a straight line passing through the center (the diameter). Since the clocks are congruent distance (chord) from 3 to 9 is identical in both. Since the circles are congruent and chords are equal in lengths, the arcs the intercept must be congruent, Yes the arcs from 3 to 9 on both clocks are congruent.

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Solved Review Exercise 8

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) Distance of a point of the circumference to the center of the circle is called:
(a) radius (b) arc (c) chord (d) tangent

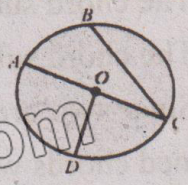
(ii) Radii of same circles are:
(a) all unequal (b) all equal
(c) half of any chord (d) double of the diameter

(iii) The boundary of the circle is called:
(a) chord (b) segment (c) circumference (d) diameter

(iv) Any part of a circumference is called:
(a) chord (b) diameter (c) radius (d) arc

(v) A chord passing through the centre of a circle is called:
(a) radius (b) diameter (c) secant (d) circmference

(vi) In the given figure, what is AB?
(a) diameter (b) tangent
(c) chord (d) arc



(vii) In the given figure, major arc is:
(a) $m\overset{\frown}{AB}$ (b) $m\overset{\frown}{BC}$ (c) $m\overset{\frown}{BDC}$ (d) $m\overset{\frown}{AD}$

(viii) _____ circle(s) can pass through the three non-collinear points?
(a) one (b) two (c) three (d) many

(ix) Perpendicular bisector of a chord always passes through the _____ of circle.
(a) arc (b) radius (c) centre (d) circumference

(x) The sum of the measures of central angles of a circle is:
(a) 90° (b) 180° (c) 270° (d) 360°

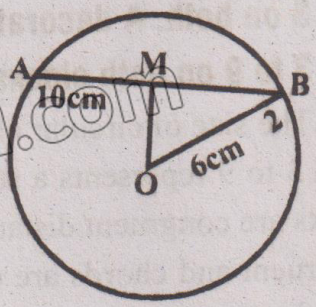
ANSWERS:

(i) radius (ii) all equal (iii) circumference (iv) arc (v) diameter

(vi) arc (vii) $m\overset{\frown}{BDC}$ (viii) one (ix) centre (x) 360°

---

2. On a circular clock with a radius of 6cm, the points from 2 to 10 form a chord that is 10cm long. Find the perpendicular distance from the centre of clock to the chord.



Sol: Let $\overline{OM}$ is the Perpendicular distance from the center 'O' to the chord $\overline{AB}$.

$m\overline{OB}$ the radius of clock = 6 cm

$m\overline{AB}$ the chord of clock = 10 cm

$\overline{OM} \perp \overline{AB}$, at point M. So, $\overline{OM}$ bisects the chord $\overline{AB}$ such that $m\overline{AM} = m\overline{MB}$

Tan $m\overline{MB} = \frac{1}{2}(m\overline{AB})$

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$$= \frac{1}{2}(10\,cm)$$

$$= 5\ cm$$

OMB is a right angled triangle by pythagoras theorem

$$\left(m\overline{OB}\right)^2 = \left(m\overline{MB}\right)^2 + \left(m\overline{OM}\right)^2$$

$$(6\ cm)^2 = (5cm)^2 + \left(m\overline{OM}\right)^2$$

$$36\ cm^2 = 25cm^2 + \left(m\overline{OM}\right)^2$$

$$36\ cm^2 - 25\ cm^2 = \left(m\overline{OM}\right)^2$$

$$11\ cm^2 = \left(m\overline{OM}\right)^2$$

$$\sqrt{\left(m\overline{OM}\right)^2} = \sqrt{11cm^2}$$

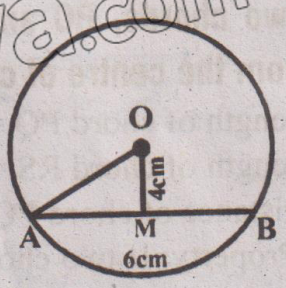
$$m\overline{OM} = 3.32\ cm$$

3. A chord 6 cm long is at a distance of 4 cm from the centre. Calculate the radius of the circle.

Sol: Let $\overline{OA}$ be the radius of a circle with O as centre.

$m\overline{AB}$ the chord of circle = 6 cm

$m\overline{OM}$ the perpendicular distance = 4 cm



Since, $\overline{OM} \perp \overline{AB}$, So $\overline{OM}$ bisects the chord $\overline{AB}$ such that $m\overline{AM} = m\overline{MB}$

Thus, $$m\overline{AM} = \frac{1}{2}(mAB)$$

$$= \frac{1}{2} \times 6cm = 3cm$$

OMA is the right angled triangle

By, Pythagoras theorm

$$\left(m\overline{OA}\right)^2 = \left(m\overline{AM}\right)^2 + \left(m\overline{OM}\right)^2$$

$$\left(m\overline{OA}\right)^2 = (3\ cm)^2 + (4\ cm)^2$$

$$\left(m\overline{OA}\right)^2 = 9\ cm^2 + 16\ cm^2$$

$$\left(m\overline{OA}\right)^2 = 25\ cm^2$$

$$\sqrt{\left(m\overline{OA}\right)^2} = \sqrt{25cm^2}$$

$$m\overline{OA} = 5\ cm$$

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4. In a circular park, two benches are placed so that the chords formed by their positions are both 10 m long. What can you say about the angles subtended at the centre by each bench?

Ans. Since both chords (benches) are 10 m long, they are congruent. Therefore, the angles subtended at the centre by each bench are equal.

(**Property:** in circles, equal chords subtended equal angles at the centre)

5. Jannat is designing a triangular garden with corners at point A(2, 1), B(4, 4) and C(1, 5). Can she install a circular fountain that touches all three corners?

Sol: Points of corners garden = A(2, 1), B(4, 4), C(1, 5) we will check the points are non-collinear by using slope formula

$$\text{Slope of } \overline{AB} = \frac{4-1}{4-2} = \frac{3}{2}$$

$$\text{Slope of } \overline{BC} = \frac{5-4}{1-4} = \frac{1}{-3} = -\frac{1}{3}$$

Slope $\overline{AB} \neq$ Slope $\overline{BC}$

Hence, the given point are non-collinear. Therefore, she can install a circular fountain that touches all three corners.

6. Two chords, PQ and RS, each measure 18cm in length. If the distance of PQ from the centre of circle is 7 cm. Find the distance of RS from the centre.

Sol: Length of chord PQ = 18 cm

Length of chord RS = 18 cm

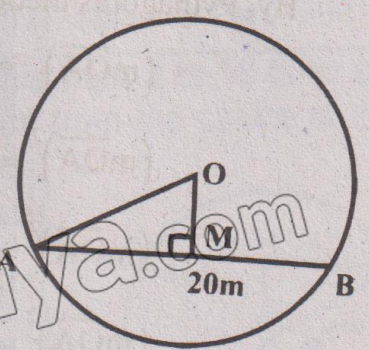
Distance of chord PQ from the centre = 7 cm

(Property: If two chords of a circle are congruent, then they will be equidistant from the centre.)

Since chords PQ and RS are congruent, the distance of RS from the centre is also 7 cm.

7. A tree branch lies across a circular pond, forming a 20 m chord. A measuring rod having length 10 m is perpendicular to chord from the centre. What is the radius of the pond?

Sol: Let $\overline{OA}$ is the radius of the circular pond with centre O. $\overline{AB}$ is the chord of pond $\overline{OM}$ is the length of perpendicular rod.



$$m\overline{AB} \text{ (chord)} = 20 \text{ m}$$

$$m\overline{OM} = 10 \text{ m}$$

$\overline{OM} \perp \overline{AB}$, So, $\overline{OM}$ bisects the chord $\overline{AB}$ such as $m\overline{AM} = m\overline{MB}$

$$m\overline{AM} = \frac{1}{2}(m\overline{AB})$$

$$= \frac{1}{2}(20\text{cm}) = 10 \text{ cm}$$

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OMA is a right angled triangle

[By pythagoras theorem]

$$\left(m\overline{OA}\right)^2 = \left(m\overline{AM}\right)^2 + \left(m\overline{OM}\right)^2$$

$$\left(m\overline{OA}\right)^2 = (10)^2 + (10)^2$$

$$\left(m\overline{OA}\right)^2 = 100 + 100$$

$$\left(m\overline{OA}\right)^2 = 200$$

$$\sqrt{\left(m\overline{OA}\right)^2} = \sqrt{200}$$

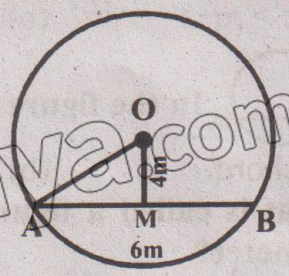
$$m\overline{OA} = 14.14m$$

8. A steel bar 6 m long lies inside a circular structure with ends on the circle. It is bisected by a perpendicular rod from the centre. Find the radius of the circular structure if the perpendicular distance from the centre to the bar is 4 m.

Sol: Let $\overline{OA}$ is the radius of a circular structure with O as centre.

$\overline{AB}$ is the steel bar with ends on the circle.

$\overline{OM}$ is the perpendicular distance from the centre to the bar.



$m\left(\overline{AB}\right)$ chord = 6 m

$m\left(\overline{OM}\right) = 4$ m

$\overline{OM} \perp AB$ from centre. So, $\overline{OM}$ bisects the chord $\overline{AB}$ such that $m\overline{AM} = mMB$

$$m\left(\overline{AM}\right) = \frac{1}{2}\left(m\overline{AB}\right)$$

$$= \frac{1}{2}(6cm)$$

$$= 3 \text{ cm}$$

Since, OAM is a right angled triangle

[By pythagoras theorem]

$$\left(m\overline{OA}\right)^2 = \left(m\overline{AM}\right)^2 + \left(m\overline{OM}\right)^2$$

$$\left(m\overline{OA}\right)^2 = (3)^2 + (4)^2$$

$$\left(m\overline{OA}\right)^2 = 9 + 16$$

$$\left(m\overline{OA}\right)^2 = 25$$

$$\sqrt{\left(m\overline{OA}\right)^2} = \sqrt{25}$$

$$m\overline{OA} = 5 \text{ m}$$

Radius of the circular structure = 5 m

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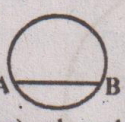
Objective Type Questions

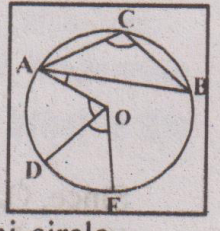
Additional MCQ's

Multiple Choice Questions prepared in the light of new Examination Techniques
(Knowledge, Understanding, Application, Analytical & Conceptual)

Circle and its Components

Encircle the correct option.

1. **Which term describes the set of all points in a plane that are at the same distance from a fixed point.**
   (a) diameter (b) chord (c) circle (d) radial segment
2. **Which is the mathematical formula for circumference (c) of a circle?**
   (a) $c = \pi r^2$ (b) $2\pi r$ (c) $c = \frac{2}{\pi}$ (d) πr^2
3. 

   In the figure $\overline{AB}$ is called:
   (a) chord (b) diameter (c) centre (d) radius
4. **What is called a figure bounder by a diameter and circumference cut by the diameter?**
   (a) arc (b) major arc (c) semi-circle (d) radius
5. **A major arc is an arc greater than the:**
   (a) semi-circle (b) segment (c) diameter (d) radius
6. **If an arc is smaller than a semi-circle, it is called:**
   (a) major arc (b) minor arc (c) sector (d) radius
7. **In the figure, what is a circumscribed angle?**
   

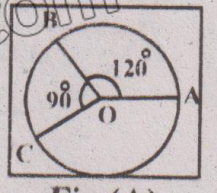
   (a) ∠ACB (b) ∠CBA
   (c) ∠AOD (d) ∠DOF
8. **An arc whose central angle exactly measures 180° is called:**
   (a) sector (b) minor arc (c) major arc (d) semi-circle
9. **An arc whose central angle measures less than 180° is called:**
   (a) major arc (b) minor arc (c) semi-circle (d) segment
10. **An arc whose central angle measures greater than 180° is called:**
    (a) major arc (b) minor arc (c) semi-circle (d) segment
11. **in the figure A, the m∠AOC is:**
    

    Fig (A)
    (a) 180° (b) 140°
    (c) 150° (d) 90°
12. **A secant cuts the circumference of a circle at:**
    (a) one point (b) two distinct points
    (c) three points (d) not any point

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13. **The vertex of central angle is always on:**
(a) circumference (b) chord (c) centre (d) secant

14. **Circles with same centre are called:**
(a) semi-circles (b) open circles (c) congruent circles (d) concentric circles

15. **A circle whose radius is zero is called:**
(a) semi-circle (b) point circle (c) major circle (d) minor circle

8.1 Theorems (properties) of a circle

16. **If two circles have three points in common, what can be concluded about them?**
(a) They are concentric (b) They are tangent
(c) They coincide (d) They are non-collinear

17. **What is the maximum number of points at which two distinct circles can intersect:**
(a) one (b) two (c) three (d) four

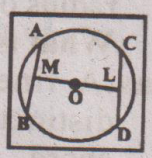
18. **If A (x_1, y_1) and B (x_2, y_2) the slope of AB is?**
(a) $\frac{y_2 - y_1}{x_2 - x_1}$ (b) $\frac{y_2 + y_1}{x_2 - x_1}$ (c) $\frac{y_2 - y_1}{x_2 + x_1}$ (d) $\frac{y_2 - y_3}{y_1 - y_2}$

19. **If A (4, 7), B (3, 9), then slope of AB is:**
(a) 2 (b) $\frac{7}{2}$ (c) -2 (d) $\frac{2}{3}$

20. **Chords equidistant from the centre of a circle are _____:**
(a) parallel (b) perpendicular (c) equal in length (d) bisecting each other

21. **Perpendicular from the centre of a circle on a chord _____ it:**
(a) bisect (b) equal (c) greater (d) less

22. **If the radius of a circle is 5 cm and the chord is 8 cm long. What is perpendicular distance from centre to the cord?**
(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

23. **In the figure $m\overline{AB} = m\overline{CD}$, if $m\overline{OM}$ is 5 cm, what is the $m\overline{OL}$.**


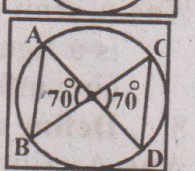
(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

24. **In a circle, if two central angle are congruent, then their corresponding arcs are:**
(a) supplementary (b) perpendicular (c) parallel (d) congruent

25. **In a circle if two chords are equal then corresponding arcs will be:**
(a) equal (b) not equal (c) greater (d) less than

26. **In the figure , the $m\overline{LM} = m\overline{PQ}$ the $m\angle POQ$ is:**


(a) 40° (b) 65°
(c) 90° (d) 60°

27. **In figure if the $m\overline{AB} = 10$ cm the $m\overline{CD}$ is:**


(a) 5 cm (b) 7 cm
(c) 9 cm (d) 10 cm

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ANSWERS:

1. circle 2. $2\pi r$ 3. chord 4. semi-circle 5. semi-circle 6. major arc
7. $\angle ACB$ 8. semi-circle 9. major arc 10. major arc 11. 150° 12. two distinct points
13. centre 14. concentric circles 15. point circle 16. they coincide 17. two 18. $\frac{y_2 - y_1}{x_2 - x_1}$
19. –2 20. equal in length 21. 22. 3 cm 23. 5 cm 24. congruent
25. equal 26. 60° 27. 10 cm

Additional Short Question

Short Answer Questions prepared in the light of new Examination Techniques (Knowledge, Understanding, Application, Analytical & Conceptual)

Circle and its components

Give short answers.

1. What is circle?

Ans. A circle is the set of all points that are at the same distance from a fixed point.

2. Define the circumference of a circle.

Ans. The length of the boundary of a circle is called circumference.

3. What is the difference between a radial segment and a radius.

Ans. The line segment joining any point of the circle to its centre is called radial segment and its measure is called radius of the circle.

4. What is the chord of a circle?

Ans. A line segment joining any two distinct points of a circle is called a chord.

5. Define the diameter of a circle.

Ans. A chord passing through the centre of the circle is called the diameter.

6. Define the term 'Arc'.

Ans. Any part of the circumference of a circle is called an arc.

7. Define a semi-circle.

Ans. A semi-circle is half of the circle, that is a figure bounded by a diameter and circumference cut by the diameter.

8. Define a minor arc.

Ans. A minor arc is a arc smaller than a semi-circle.

9. Define a major arc.

Ans. A major arc is an arc greater than the semi-circle.

10. Define a segment of a circle.

Ans. The area (region) enclosed between a chord and the arc which it cuts off is called a segment of the circle.

11. What is an angle in the segment?

Ans. If a point on the arc of a segment is joined with the end points of a chord, the angle formed by the line segments. So drawn is called angle in the segment.

12. What is the secent of a circle?

Ans. A straight line cutting the circumference of a circle at two distinct points is called a secant.

13. Define the central angle of a circle.

Ans. If the extremities of an arc or of a chord are joined with the centre of a circle, angle. So formed is called a central angle.

14. What is the sum of all the central angles of a circle?

Ans. The sum of the measures of all the central angles of a circle with no interior points in common is 360°.

15. Define concentric circles.

Ans. Circles with the same centre are called concentric circles.

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16. What is a point circle?

Ans. A circle whose radius is zero is called a point circle.

8.1 Theorems (properties) of a circle

17. What is the condition for three points to determine a unique circle?

Ans. The three points must be non-collinear, meaning they do not lie on the same straight line.

18. Determine if a unique circle can pass through the points P(0, 0), Q(2, 2) and R(4, 4).

Sol. Slope of $PQ = \frac{2-0}{2-0} = \frac{2}{2} = 1$ Slope of $QR = \frac{4-2}{4-2} = \frac{2}{2} = 1$

Since, the slopes are equal, the points are collinear. Therefore, a unique circle cannot pass through these three points.

19. What is the relationship between a line from the centre and a chord if the line bisects the chord?

Ans. If a straight line, drawn from the centre of a circle to bisect a chord is perpendicular to the chord. This means the angle formed between the line and the chord is 90°.

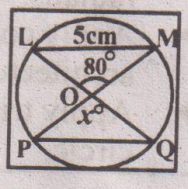
20. In a circle two chords are congruent of one chord is 15 cm from the centre. What is the distance of second chord from the centre.

Ans. Since, chords are congruent, they must be equidistant from the centre. So, the distance of second chord from the centre of the circle is also 15 cm.

21. In a circle the perpendicular distance of two chords from the centre of circle is equal. Find the length of other chord if the length of one chord is 10 cm.

Ans. Since, both chords are quidistant from the centre, therefore chords are equal. So, the length of other chord is also 10 cm.

22. In the given figure, find the value of x.



Sol. From the figure

$m\overline{LM} = m\overline{PQ} = 5$ cm

If the chords are equal, then corresponding central angles are also equal

$m \angle POQ = m \angle MOL$

$x = 80°$

23. In a circle with centre O, $\angle AOB = 60°$ and $\angle COD = 60°$. If the length of arc AB is 5 cm. What is the length of arc CD.

Sol. Since central angles are equal

$m\angle AOB = m\angle COD = 60°$

So, the corresponding arcs must be equal

$m\overset{\frown}{CD} = m\overset{\frown}{AB}$

$m\overset{\frown}{CD} = 5$ cm

24. In a circle with centre if arc AB is congruent to arc CD. If the measure of $\angle AOB$ is 80°. What is the measure of $\angle COD$.

Sol. Since the arcs are congruent.

$\overset{\frown}{AB} \cong \overset{\frown}{CD}$

So, the corresponding angles must be congruent

$m \angle COD \cong m \angle AOB$

$m \angle COD = 80°$

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