

CIRCULAR MOTION

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

Describe angular motion.

Define angular displacement, angular velocity and angular acceleration.

Define radian and convert an angle from radian measure to degree and vice versa.

Derive the equation $a_c = r\omega^2 = v^2/r$ and $F_c = m\omega^2 r = mv^2/r$.

Understand and describe moment of inertia of a body.

Understand the concept of angular momentum.

Describe examples of conservation of angular momentum.

Describe the motion of artificial satellites.

Understand that the objects in satellites appear to be weightless.

Understand that how and why artificial gravity is produced.

Calculate the radius of geo-stationary orbits and orbital velocity of satellites.

Describe Newton's and Einstein's views of gravitation.

Q.1 Define circular motion.

Ans. **CIRCULAR MOTION**

When a body moves in such a way that its distance from the fixed point remains constant then such a motion is called circular motion.

For Example:

(i) Motion of earth around the sun.

(ii) Motion of moon around the earth.

Q.2 Explain angular displacement with its units.

Ans. **ANGULAR DISPLACEMENT**

The angle through which a particle moves in an interval of time while moving along a circle is called angular displacement.

Consider the motion of a single particle P of mass m in a circular path of radius r. Suppose this motion is taking place by attaching the particle P at the end of a mass less rigid rod of length r whose other end is pivoted at the centre O of the circular path, as shown in Fig. (i). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. (ii). The z–axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t, its position is OP_1, making angle θ with x-axis. At later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis Fig. (iii).

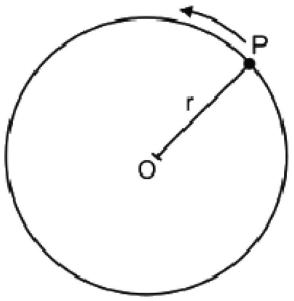


Fig. (i)

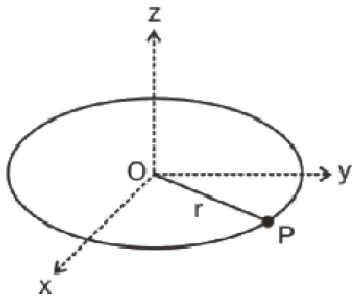


Fig. (ii)

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt.

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is counter clockwise.

For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

The direction associated with $\Delta\theta$ is along the axis rotation and is given by right hand rule which states that

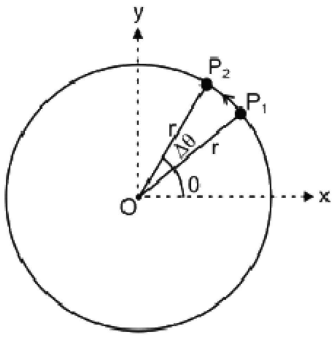


Fig. (iii)

Right Hand Rule

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement, as shown in Fig. (iv).

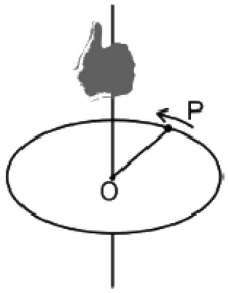


Fig. (iv)

Unit

There are three units of angular displacement

(i) degree

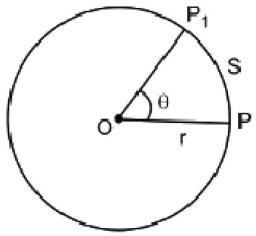
(ii) revolution

(iii) radian

But radian is SI unit. The angular displacement is one radian if the angle between two radii of a circle which cut off on the circumference an arc equal in length to the radius.

Q.3 Show that $S = r\theta$

Ans. Consider an arc of length S of a circle of radius r which subtends an angle θ at the centre of the circle. Its value in radian is given by



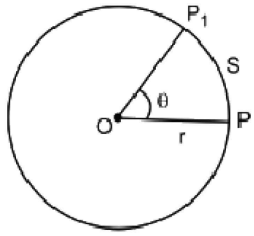
$$\theta = \frac{\text{Arc length}}{\text{Radius}} \text{ (rad)}$$

$$\theta = \frac{S}{r}$$

$$S = r\theta$$

Q.4 Show that 1 radian = 57.3°.

Ans. If 'OP' is rotating, the point 'P' covers a distance $S = 2\pi r$ in one revolution of P. In radian, it is



$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

$$1 \text{ revolution} = 2\pi \text{ rad.} = 360°$$

$$1 \text{ rad.} = \frac{360°}{2\pi}$$

$$1 \text{ rad.} = 57.3°$$

$$\therefore \quad 2\pi \text{ radian} = 360^o$$

$$1 \text{ radian} = \frac{360^o}{2\pi}$$

$$= \frac{180^o}{3.14}$$

$$1 \text{ radian} = 57.3^o$$

Q.5 Define angular velocity and instantaneous angular velocity with its units.

Ans. **ANGULAR VELOCITY**

"The rate of change of angular displacement is called angular velocity". **(OR)** The angular velocity is also defined as the rate at which the angular displacement is changing with time. It is denoted by ω. If Δθ is the angular displacement during the time interval Δt, the average angular velocity during the interval is

$$\omega_{ave} = \frac{\Delta\theta}{\Delta t}$$

It is a vector quantity. Its direction is along the axis of rotation and given by right hand rule.

Dimensions

$$[\omega] = [T^{-1}]$$

The instantaneous angular velocity ω is the limit of the ratio $\frac{\Delta\theta}{\Delta t}$ as Δt following instant t, approaches to zero. Thus

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Units

Angular velocity measured in radians per second which is SI units. Sometime it is also measures in terms of **revolution per minute**.

Q.6 Define angular acceleration and instantaneous angular acceleration with its units.

Ans. **ANGULAR ACCELERATION**

The rate of change of angular velocity is called angular acceleration.

It is denoted by α.

It is a vector quantity. Its direction is along axis of rotation and given by right hand rule. If ω_i and ω_f are the value of instantaneous velocity of a rotating body at time t_i and t_f then, the average angular acceleration during the intervals $t_f - t_i$ is given by

$$\alpha_{ave} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$\therefore \quad \alpha = \frac{\Delta\omega}{\Delta t}$$

Unit

Its SI unit is radian per sec^2.

Dimensions

$$[\alpha] = [T^{-2}]$$

Instantaneous Angular Acceleration

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches to zero then mathematically

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

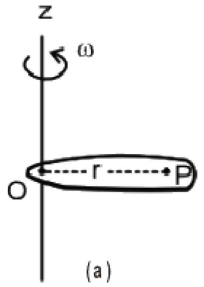
Explanation

Till now we have been considering the motion of a particle P on a circular path. The point P was fixed at the end of a rotating massless rigid rod. Now we consider the rotation of a rigid body as shown in figure. Imagine a point P on the rigid body. Line OP is the perpendicular dropped from P on the axis of rotation. It is usually referred as reference line. As the body rotates, line OP also rotates with it with the same angular velocity and angular acceleration. Thus the rotation of a rigid body can be described by

Q.7 Show that $V = r\omega$ (relation between linear and angular velocity).

Ans. Consider a rigid body rotating about z-axis with an angular velocity ω as shown in Fig. (a).

Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity v whereas the line OP rotates with angular velocity ω as shown in Fig. (b). We are interested in finding out the relation between ω and v. As the axis of rotation is fixed, so the direction of ω always remains the same and ω can be manipulated as a scalar. As regards the linear velocity of the point P, we consider its magnitude only which can also be treated as a scalar.



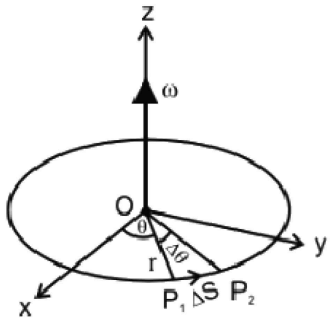
(a)

Suppose during the course of its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP has an angular displacement $\Delta\theta$ radian during this interval. ΔS and $\Delta\theta$ are related by equation.

$$\Delta S = r\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \quad \ldots\ldots (1)$$



(b)

In the limit when $\Delta t \to 0$ the ratio $\frac{\Delta S}{\Delta t}$ represents v, the magnitude of the velocity with which point P is moving on the circumference of the circle. Similarly $\Delta\theta/\Delta t$ represents the angular velocity ω of the reference line OP. So Eq. (1) becomes

$$v = r\omega \quad \ldots\ldots (2)$$

In Fig. (b), it can be seen that the point P is moving along the arc P_1P_2. In the limit when $\Delta t \to 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why the linear velocity of the point P is also known as tangential velocity.

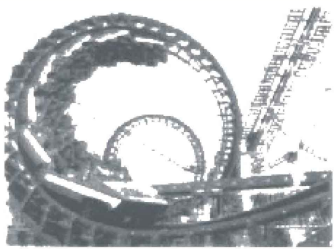
Q.8 Show that $a_t = r\alpha$ (relation between linear and angular acceleration).

Ans. If the reference line OP is rotating with an angular acceleration α, the point P will also have a linear or tangential acceleration a_t. Using Eq. (1) it can be shown that the two accelerations are related by

$$a_t = r\alpha \quad \ldots\ldots (1)$$

Eqs. (1) and (2) show that on a rotating body, points that are at different distances from the axis do not have the same speed or acceleration, but all points on a rigid body displacement, angular speed and angular acceleration at any instant. Thus by the use of angular variables we can describe the motion of the entire body in a simple way.

Point to Ponder

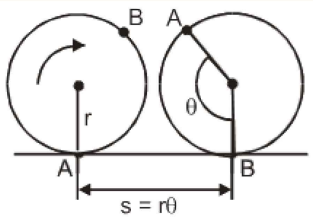


You may feel scared at the top of

in linear motion except that θ, ω and α have replaced S, v and a, respectively. As the other equations of linear motion were obtained by algebraic manipulation of these equations, it follows that analogous equations will also apply to angular motion. Given below are angular equations together with their linear counterparts.

Linear		Angular		
$v_f = v_i + at$		$\omega_f = \omega_1 + \alpha t$		..... (1)
$2aS = v_f^2 - v_i^2$		$2\alpha\theta = \omega_f^2 - \omega_i^2$		..... (2)
$S = v_i t + \frac{1}{2} at^2$		$\theta = \omega_i t + \frac{1}{2}\alpha t^2$		..... (3)

Do You Know?



As the wheel turns through an angle, it lays out a tangential distance S = rθ.

The angular equations (1) to (3) hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence they can be manipulated as scalars.

Q.9 What is centripetal force? Also derive the expression for centripetal force.

Ans. **CENTRIPETAL FORCE**

The force needed to bend the normally straight path of the particle into a circular path is called the centripetal force.

It is denoted by F_C and mathematically

$$F_C = \frac{m V^2}{r}$$

Explanation

If the particle moves from A to B with uniform speed V as shown in Fig. (i). The velocity of the particle changes its direction but not its magnitude. The change in velocity is shown in Fig. (ii). Hence acceleration of the particle is

$$a = \frac{\Delta V}{\Delta t} \qquad \text{..... (1)}$$

Where Δt is the time taken by the particle to travel from A to B. Let the velocities at A and B are $\vec{V}_1$ and $\vec{V}_2$. Since speed of the particle is V, so the time taken to travel a distance S, as shown in Fig. (i) is

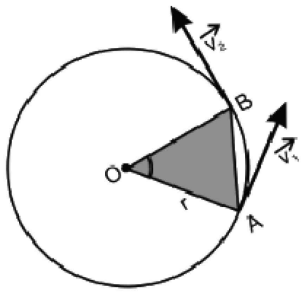


Fig. (i)

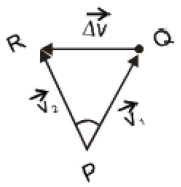


Fig. (ii)

$$\Delta t = \frac{S}{V} \qquad (\because S = V t)$$

Put in equation (1)

So $$a = V\frac{\Delta V}{S} \qquad \text{..... (2)}$$

Let us now draw a triangle PQR such that PQ is parallel and equal to $\vec{V}_1$ and PR is parallel and equal to $\vec{V}_2$, as shown in Fig. (ii). We know that the radius of a circle is perpendicular to its tangent, so OA is perpendicular to $\vec{V}_1$ and OB is perpendicular to $\vec{V}_2$ Fig. (i). Therefore, angle AOB

between their equal arms are equal. Hence, the triangle OAB Fig. (i) is similar to triangle PQR Fig. (ii). Hence, we can write

$$\frac{\Delta V}{AB} = \frac{V}{r}$$

$$\frac{\Delta V}{V} = \frac{AB}{r}$$

If the point B is close to the point A on the circle, as will be the case when $\Delta t \to 0$, the arc AB is of nearly the same length as the line AB. To that approximation,

$$AB = S$$

$$\therefore \quad \frac{\Delta V}{V} = \frac{S}{r}$$

$$\therefore \quad \Delta V = S\frac{V}{r}$$

Putting this value in the equation (2), we get

$$a = \frac{V}{S} S\frac{V}{r}$$

$$\therefore \quad a = \frac{V^2}{r} \qquad \ldots\ldots\ldots (3)$$

Where a is the instantaneous acceleration. As this acceleration is caused by the centripetal force, it is called **centripetal acceleration** denoted by a_c. This acceleration is directed along the radius towards the centre of the circle. In Fig. (i) and (ii), since PQ is perpendicular to OA and PR is perpendicular to OB, so QR is perpendicular to AB. It may be noted that QR is parallel to the perpendicular bisector of AB. As the acceleration of the object moving in the circle parallel to ΔV when $AB \to 0$, so centripetal acceleration is directed along radius towards the centre of the circle. It can, concluded that

The instantaneous acceleration of an object traveling with uniform speed in a circle is directed towards the centre of the circle and is called centripetal acceleration.

The centripetal force has the same direction as the centripetal acceleration and its value is given by

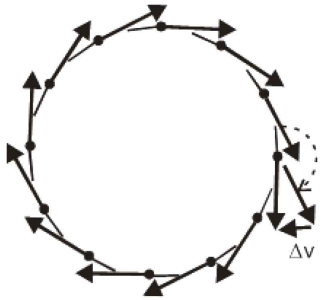
$$F_C = m\,a_c = \frac{m\,V^2}{r}$$

In angular motion.

Put $V = r\,\omega$

then $F_C = \frac{mr\omega^2}{r}$

Do You Know?



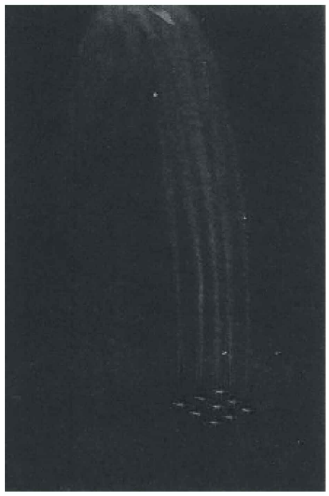
Direction of motion changes continuously in circular motion.

Tid-bits



Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

Do You Know?



Curved flight at high speed requires a large centripetal force that makes the stunt dangerous even if the air planes are not so close.

Ans. MOMENT OF INERTIA

Consider a mass m attached to a massless rod at O as shown in Fig. (i). Let us assume that the bearing at the pivot O is frictionless and that the mass of the rod is negligible. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence this will accelerate the mass according to

$$F = ma$$

In doing so the force will cause the mass to rotate about O. Since tangential acceleration a_T is related to angular acceleration α by the equation.

$$a_T = r\alpha$$

So, $$F = m r \alpha$$

Multiply both sides by r

$$\therefore \quad r F = \tau = \text{torque} = m r^2 \alpha$$

which is rotational analogue of the Newton's second law of motion, F = ma.

Here F is replaced by τ, a by α and m by mr^2. The quantity $m r^2$ is known as the moment of inertia and is represented by I. It may be defined as the product of mass and square of distance from the point of rotation. The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2.

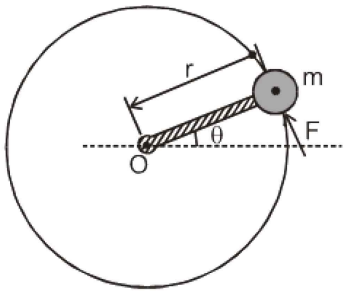
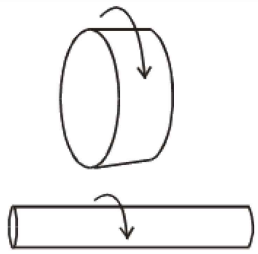


Fig. The force F causes a torque about the axis O and gives the mass m an angular acceleration about the pivot point.

Do You Know?

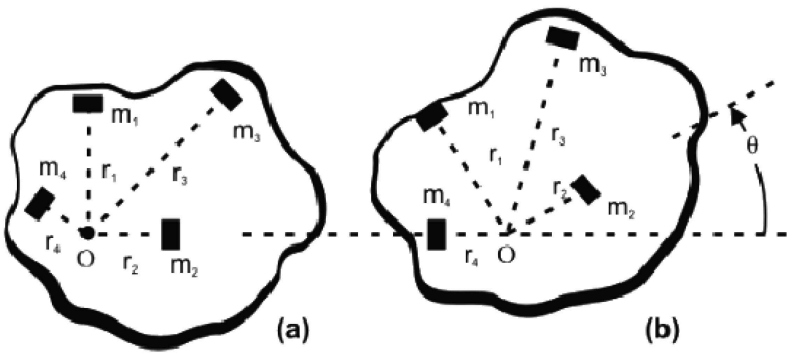


Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

Moment of Inertia of a Rigid Body

Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means mass distribution is not uniform. As shown in figure, rigid body is made up of n small pieces of masses.

For Your Information



$m_1, m_2, \ldots\ldots, m_n$ at distance $r_1, r_2, \ldots\ldots, r_n$ from the axis of rotation O. Let the body be rotating with the angular acceleration α, so the magnitude of the torque acting on m_1 is

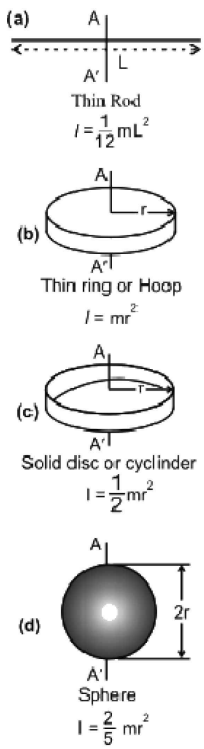
$$\tau_2 = m_2 r_2^2 \alpha_2$$

and so on.

Hence the body is rigid, so all the masses are rotating with same angular acceleration α,

Total torque τ_{total} is then given by

$$\tau_{total} = (m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2)\,\alpha$$



$$= \left(\sum_{i=1}^{n} m_i r_i^2\right)\alpha$$

$$\tau = I\alpha$$

where $I = \sum_{i=1}^{n} m_i r_i^2$

Q.11 Explain angular momentum. Derive a relation between angular momentum and moment of inertia.

Ans. ANGULAR MOMENTUM

Angular momentum is the product of linear momentum and moment arm for momentum. **(OR)** A particle is said to possess an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum L of a particle of mass m moving with velocity V and momentum p relative to origin O is defined as

$$\vec{L} = \vec{r} \times \vec{P}$$

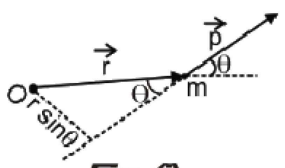


Fig. (i)

where $\vec{r}$ is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity. Its magnitude

$$L = rp \sin\theta = m r V \sin\theta \quad (\because P = mV)$$

If the particle is moving in a circle of radius r with uniform angular velocity ω, then angle between r and tangent velocity is 90^o. Hence

$$L = m\,r\,V \sin 90^o = m\,r\,V$$

But $V = r\,\omega$

Hence $L = m\,r^2\,\omega$

$$L = I\omega \qquad (\because I = mr^2)$$

Now consider a symmetric rigid rotating about a fixed axis through the centre of mass as shown in Fig. (ii). Each particle of the rigid body rotates about the same axis in a circle with an angular velocity ω. The magnitude of the angular momentum of the particle of mass m_i is $m_i\,V_i\,r_i$ about the origin O. The direction of L_i is the same as that of ω. Since $V_i = r_i\,\omega$, the angular momentum of the ith particle is $m_i\,r_i^2\,\omega$. Summing this over all particles gives the total angular momentum of the rigid body.

$$L = \left(\sum_{i=1}^{n} m_i\,r_i^2\right) = I\,\omega$$

Where I is the moment of inertia of the rigid body about the axis of rotation.

Q. Show that $Kgm^2/s = Js$.

Ans. Taking:

L.H.S = $Kg\,m^2/s$

Multiply and divide by s

$= Kgm^2/s \times \frac{s}{s}$

$= Kgm^2/s^2 \times s$

$= Kgm/s^2\ ms$

$= Nms \quad (\because Nm = J)$

$= Js$

= R.H.S

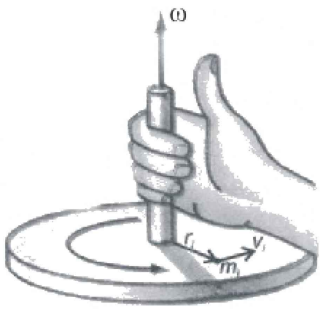


Fig. (ii)

Physicists usually make a distinction between spin angular momentum (L_s) and orbital angular momentum (L_c). The spin angular momentum is the angular momentum of spinning body, while orbital angular momentum is associated with the motion of a body along a circular path.

The difference is illustrated in Fig. (iii). In the usual circumstances concerning orbital angular momentum, the orbital radius is large as compared to the size of the body, hence, the body may be considered to be a point object.

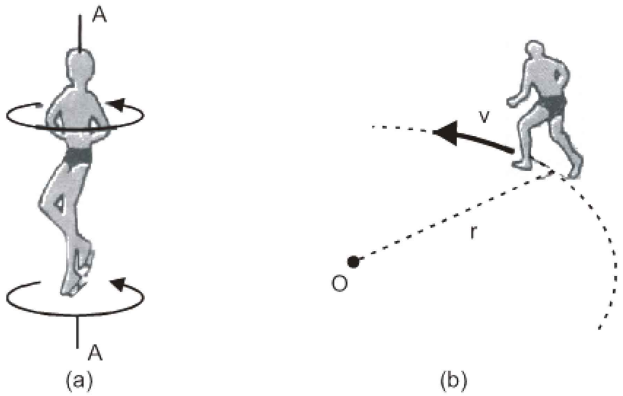
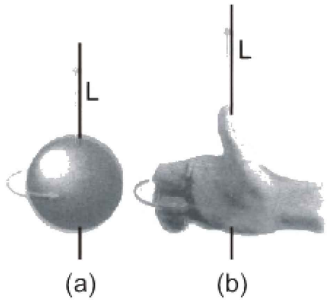


Fig. (iii)

For Your Information



The sphere in (a) is rotating in the sense given by the gold arrow. Its angular velocity and angular momentum are taken to be upward along the rotational axis, as shown by the right-hand rule in (b).

Ans. LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum states that if no external torque acts on a system, the total angular momentum of the system remains constant.

$$L_{total} = L_1 + L_2 + \ldots\ldots = \text{constant}$$

or $$I\,\omega = \text{Constant}$$

This is illustrated by the diver in Fig. (i). The diver pushes off the board with a small angular velocity about a horizontal axis through his centre gravity G. Upon lifting off from the board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about this axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into the closed position. As the angular momentum is conserved, so

$$I_1\,\omega_1 = I_2\,\omega_2$$

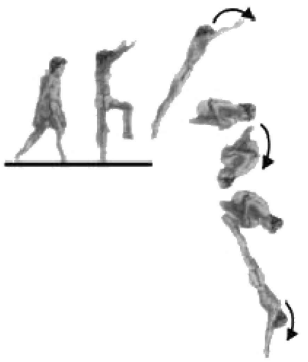
Hence, the diver must spin faster when moment of inertia becomes smaller to conserve angular momentum. This enables the diver to take extra somersaults.

The angular momentum is a vector quantity with direction along axis of rotation.

The direction of angular momentum along axis of rotation also remain fixed. This is illustrated by the fact given below.

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

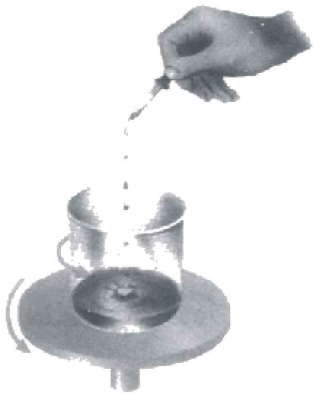
This fact is of great importance for the Earth as it moves around the Sun. no other sizeable torque is experienced by the Earth, because the major force acting on it is the pull of the Sun. The Earth's axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.



A man diving from a diving board.

Fig. (i)

Point to Ponder



Why does the coasting rotating system slow down as water drips into the beaker?

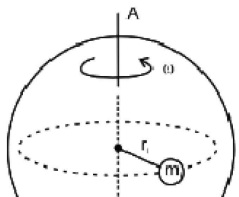
Do You Know?

Telawofconservationofangla nonertumisinpotatinmay spots, paticulaly in diving gymnaticsandiceskating

Q.13 Define rotational kinetic energy. Also calculate the expression for rotational K.E.

Ans. ROTATIONAL KINETIC ENERGY

If a body is spinning about an axis with constant angular velocity ω, each point of the body is moving in a circular path and, therefore, has some K.E. To determine the total K.E. of a spinning body, we imagine it to be composed of tiny pieces of mass m_1, m_2, …… If a piece of mass m_1 is at a distance r_i from the axis of rotation,



Thus the K.E. of this piece is

$$K.E_i = \frac{1}{2} m_i V_i^2 = \frac{1}{2} m_i (r_i \omega)^2$$

$$K.E_i = \frac{1}{2} m_i r_i^2 \omega^2$$

The rotational K.E. of the whole body is the sum of the kinetic energies of all the parts. So we have

$$K.E_{rot} = \frac{1}{2} (m_1 r_1^2 \omega^2 + m_2 r_2^2 \omega^2 + \ldots\ldots)$$

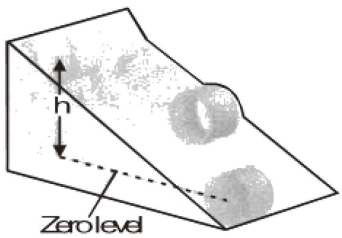
$$K.E_{rot} = \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots) \omega^2$$

$$\therefore \quad K.E_{rot} = \frac{1}{2} I \omega^2$$

Q.14 Calculate the rotational K.E of a disc and a hoop with its velocities.

Ans. **ROTATIONAL KINETIC ENERGY OF A DISC AND A HOOP**

As, $K.E_{rot} = \frac{1}{2} I \omega^2$ ..... (1)



For disc

$$I = \frac{1}{2} m r^2$$

Putting in eq (1)

$$\therefore \quad K.E_{rot} = \frac{1}{2} \left(\frac{1}{2} m r^2 \right) \omega^2$$

$$K.E_{rot} = \frac{1}{4} m r^2 \omega^2$$

As, $V = r \omega$

Squaring

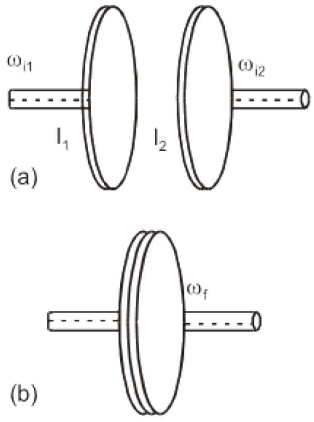
$$V^2 = r^2 \omega^2$$

$$\therefore \quad K.E_{rot} = \frac{1}{4} m V^2$$

For Hoop

$$I = m r^2$$

Interesting Information



$$\therefore \quad K.E_{rot} = \frac{1}{2} m r^2 \omega^2$$

$$K.E_{rot} = \frac{1}{2} m V^2$$

When both starts moving down an inclined plane of height h, their motion consists of both rotational and transnational motions. If no energy is lost against friction, the total K.E. of disc or hoop on reaching the bottom of incline must be equal to its P.E. at top.

$$P.E = K.E_{rot} + K.E_{trans}$$

Velocity of a Disc

$$mgh = \frac{1}{4} m V^2 + \frac{1}{2} m V^2$$

$$mgh = m V^2 \left(\frac{1}{4} + \frac{1}{2} \right)$$

$$mgh = m V^2 \left(\frac{1+2}{4} \right)$$

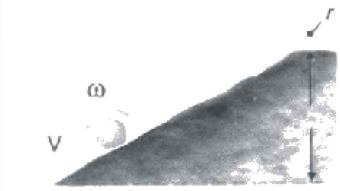
$$mgh = \frac{3}{4} m V^2$$

$$gh = \frac{3}{4} V^2$$

$$V^2 = \frac{4}{3} g h$$

$$V = \sqrt{\frac{4gh}{3}}$$

For Your Information



As the sphere rolls to the bottom of the incline, its gravitational potential energy is changed to kinetic energy of rotation and translation.

Velocity of a Hoop

$$P.E. = K.E_{rot} + K.E_{trans}$$

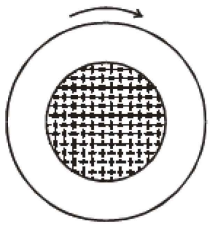
$$mgh = \frac{1}{2} m V^2 + \frac{1}{2} m V^2$$

$$mgh = m V^2 \left(\frac{1}{2} + \frac{1}{2} \right)$$

$$mgh = m V^2 \left(\frac{1+1}{2} \right)$$

$$mgh = m V^2$$

Do You Know?



As the wheel rolls, it has both rotational and translational kinetic energy.

Q.15 What are artificial satellite? Find the expression for minimum velocity and period to put a satellite into the orbit.

Ans. ARTIFICIAL SATELLITES

"Satellites are objects that orbit around the Earth." They are put into orbit by rockets and are held in orbit by the gravitational pull of the Earth. The low flying Earth satellites have acceleration 9.8 ms^{-2} towards the centre of the Earth. If they do not, they would fly off in a straight line tangent to the Earth.

When the satellite is moving in a circle, it has an acceleration

$$a_c = \frac{V^2}{r}$$

In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity and we have;

$$g = \frac{V^2}{R} \qquad \ldots\ldots\ldots (1)$$

where V is the orbital velocity and R is the radius of earth, which is 6400 km.

From equation (1)

$$V^2 = g\,h$$

$$V = \sqrt{g\,R}$$

$$= \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= \sqrt{62.72 \times 10^6}$$

$$= 7.9 \times 10^3 \text{ m/s}$$

$$V = 7.9 \text{ km/s}$$

This is the minimum velocity necessary to put a satellite into orbit and called critical velocity.

Time Period of a Satellite

The period T is given by

$$T = \frac{2\pi R}{V} \quad \left(\because \; S = V\,T, \; S = 2\pi R \right)$$

$$T = \frac{2\,(3.14)\,6400}{7.9}$$

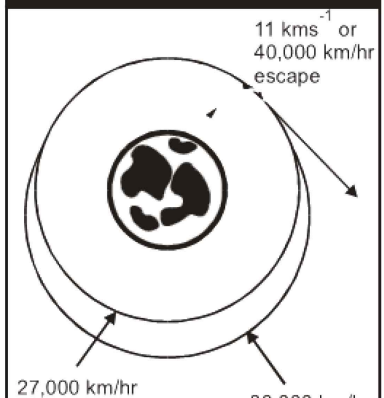
$$= 5060 \text{ sec}$$

$$= \frac{5060}{60} \text{ min}$$

$$T = 84 \text{ min}$$

$$\frac{GMm}{}$$

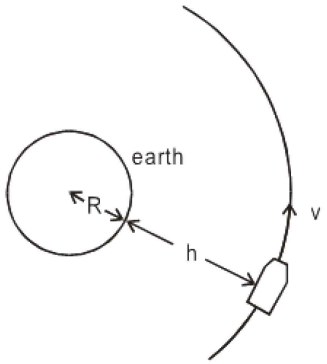
For Your Information



$$a \propto \frac{1}{r^2}$$

Higher the satellite the slower will the required speed and longer it will take to complete one revolution around the earth.

Close orbiting satellites orbit the Earth at a height of about 400 km. Twenty four such satellites form the Global positioning system. An airline pilot, sailor or any other person can now use a pocket size instrument or mobile phone to find his position on the Earth's surface to within 10 m accuracy.



Q.16 What do you understand by real and apparent weight?

Ans. **REAL AND APPARENT WEIGHT**

On the Earth the weight of an object is the gravitational pull of the Earth on the object. The weight of an object is measured by a spring balance. The force exerted by the object on the scale is equal to the pull due to gravity on the object i.e., weight of the object.

Consider an object of mass 'm' suspended by a string and spring balance in a lift as shown in fig. The reading of spring balance indicates tension in string and it shows apparent weight of object. Its value depends upon acceleration of lift.

Tidbits

Tension at you with on your mobile phone, your location can be tracked immediately by global positioning system

Case I

When lift is at rest or moving with uniform velocity.

In this case

$$a = 0$$

As $F = ma$

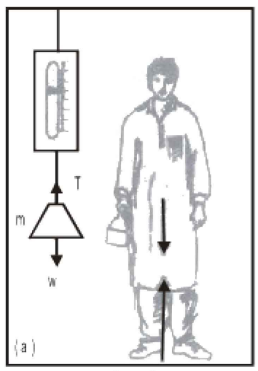
$\therefore$ $F_{net} = m(0) = 0$

As $F_{net} = T - w$

$0 = T - w$

$T = w$

The scale shows real weight of the object.



at rest
a = 0
T = w

Case II

When lift is moving upward with acceleration 'a'.

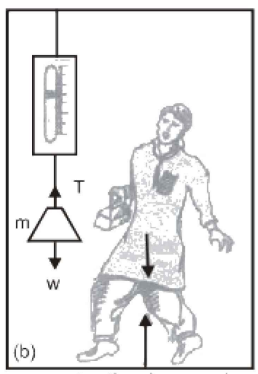
In this case

$$F_{net} = T - w$$

$$T - w = ma \qquad (\because F_{net} = ma)$$

$$T = w + ma$$

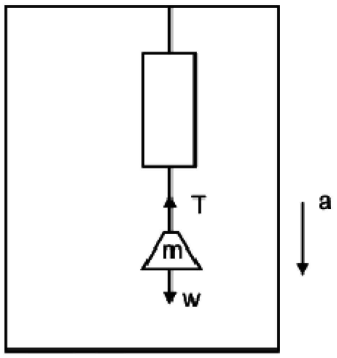
$$T = mg + ma$$



acceleration downward
w - T = ma

amount ma.

Case III



When lift is moving downward with acceleration 'a'.

In this case

$$F_{net} = w - T$$

$$ma = w - T$$

$$T = w - ma \quad \ldots\ldots\ldots (1)$$

The tension is less than w by an amount ma. To a person in the accelerated lift the object appears to weight less than w.

Case IV

Do You Know?

Your apparent weight differs from your true weight when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ridge when that velocity is constant.

When lift is falling freely under gravity

i.e., $a = g$

From equation (1)

$$T = mg - ma$$

$$T = m(g - a)$$

$\because \quad a = g$

$\therefore \quad T = m(g - g)$

$$T = m(0)$$

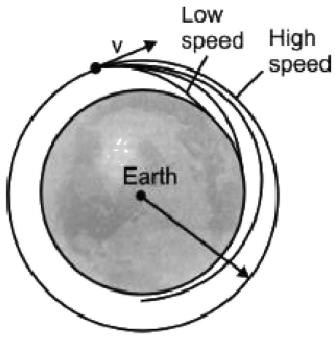
$$T = 0$$

The apparent weight of the object will be shown by the scale to be zero. It is understood from these considerations that apparent weight of the object is not equal to its true weight in an accelerating system. It is equal and opposite to the force required to stop it from falling in that frame of reference.

Q.17 Explain the phenomena of weightless in a satellite.

Ans. **WEIGHTLESSNESS IN A SATELLITE AND GRAVITY FREE SYSTEM**

When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless. It does not matter where the object is whether it is falling under force of attraction of the Earth, the Sun, or some distant star.

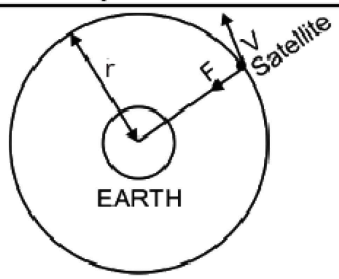


An Earth's satellite is freely falling object. The statement may be surprising at first, but it is easily seen to be correct. Consider the behaviour of a projectile shot parallel to horizontal surface of the Earth in the absence of air friction. If the projectile is thrown at successively larger speed then during its free fall to the Earth, the curvature of path decreases with increasing horizontal speeds. If object is thrown fast enough parallel to the Earth, curvature of its path will match the curvature of the Earth as shown in figure. In this case the space ship will simply circle the Earth.

The space ship is accelerating towards the centre of the Earth at all times since it circles round the Earth. Its radial acceleration is simply g, the free fall acceleration. In fact the space ship is falling towards the centre of the Earth at all the times, but the curvature of the Earth prevents space ship from hitting. Since the space ship is in free fall all the objects within it appear to be weightless. Thus no force is required to hold an object falling in the frame of reference of the space craft or satellites. Such a system is called gravity free system.

Q.18 What is orbital velocity? Define an expression for orbital velocity.

Ans. **ORBITAL VELOCITY**



The earth and some other planets revolve round the Sun nearly circular paths. The artificial satellites launched by men also adopt nearly circular course around the Earth. This type of motion is called orbital motion.

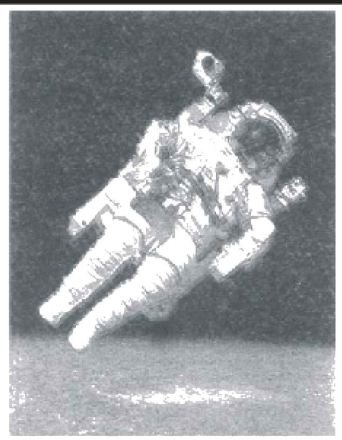
Figure shows a satellite going round the Earth in a circular path. The mass of the satellite is m_s and V is its orbital speed. The mass of the Earth is M and r represents radius of the orbit. This force $\frac{m_s V^2}{r}$ is required to hold the satellite in orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equation the gravitational force to the required centripetal force, gives

$$\frac{G\, m_s\, M}{r^2} = \frac{m_s V^2}{r}$$

$$V = \sqrt{\frac{GM}{r}} \qquad \ldots\ldots\ldots (1)$$

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus any satellite orbiting at distance r from Earth's centre must have the orbital speed given by equation (1). Any speed less than this will bring the satellite tumbling back to the Earth.

Tid-bits

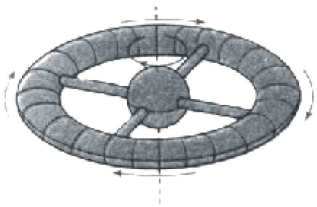


In 1984, at a height of 100 km above Hawaii island with a speed of 29000 kmh^{-1} Bruce McCandless stepped into space from a space shuttle and became the first human satellite of the Earth.

Q.19 What do you understand by artificial gravity? Derive an expression for

In a gravity free space satellite there will be no force that will force any body to any side of the spacecraft. If this satellite is to stay in orbit over an extended period of time this weightlessness may affect the performance of astronauts present in that spacecraft. To over come this difficulty, an artificial gravity is created in the spacecraft. This could enable the crew of the spaceships to function in an almost normal manner. For

this the spaceship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.



Consider a spacecraft of the shape as shown in Fig. The outer radius of the spaceship is R and it rotates around its own central axis with angular speed ω.

Then its angular acceleration is given by

$$a_c = R\omega^2$$

But $$\omega = \frac{2\pi}{t}$$

Where t is period of revolution of spaceship.

Hence, $$a_c = R\left(\frac{2\pi}{t}\right)^2 = \frac{4\pi^2 R}{t^2}$$

As frequency

$$f = \frac{1}{t}$$

$$a_c = R\,4\pi^2 f^2$$

or $$f^2 = \frac{a_c}{4\pi^2 R}$$

or $$f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

Here the force of gravity provides required centripetal acceleration, therefore,

$$a_c = g$$

and $$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

When the spaceship rotates with this frequency, artificial gravity like Earth is provided to the inhabitants of the spaceship.

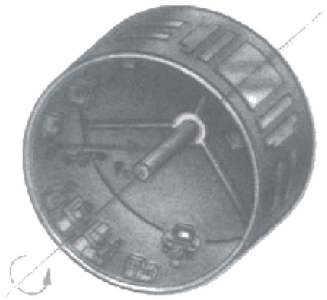
$$\therefore \quad a_c = \frac{V^2}{R}$$

$$V = R\omega$$

$$\therefore \quad a_c = \frac{V^2}{R}$$

$$a_c = R\omega^2$$

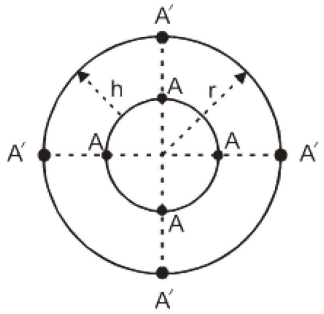
Do You Know?



The surface of the rotating space ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

Q.20 What are geo stationary satellites? Derive an expression for the orbital radius of

"The satellite whose orbital motion is synchronized with the rotation of Earth is called geo stationary satellite and the orbit in which geo stationary satellite revolves around the Earth is called geo stationary orbit."



In this way the synchronous satellite remains always over same point on equator as Earth spins on its axis.

Such a satellite is very useful for worldwide communication, weather observations, navigation and other military uses.

Radius of Geo Stationary Orbit

As, $V = \sqrt{\frac{GM}{r}}$ ..... (1)

But this speed must be equal to the average speed of the satellite in one day.

i.e. $V = \frac{S}{T}$

$V = \frac{2\pi r}{T}$ ..... (2)

Where T is the revolution of satellite, that is equal to one day. This means that the satellite must move in one complete orbit in a time of exactly one day. As the Earth rotates in one day and the satellite will revolve around. Earth is one day, the satellite at A will always stay over same point A on Earth as shown in figure.

Equating above equation (1) and (2),

$$\frac{2\pi r}{T} = \sqrt{\frac{GM}{r}}$$

Squaring both sides;

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

or $r^3 = \frac{GM T^2}{4\pi^2}$

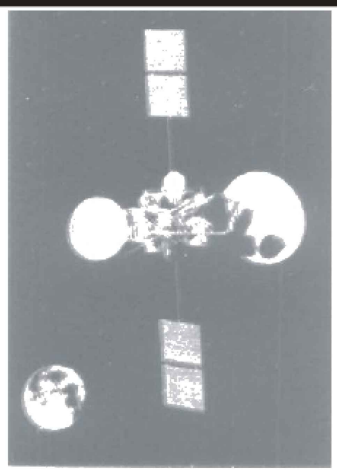
or $r = \left(\frac{GM T^2}{4\pi^2}\right)^{1/3}$

Putting the value we get

$$r = 4.23 \times 10^4 \text{ km}$$

which is orbital radius measured from centre of the Earth, for a geostationary satellite. A satellite at this height will always stay directly above a particular point on the surface of the Earth. The height of such satellite above equator is 36000 km.

Do You Know?



A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationery. It is used now for international communications.

Ans. COMMUNICATION SATELLITES



The whole Earth can be covered by just three geostationary satellite

A satellite communication system can be set up by placing several geo stationary satellites in orbit over different point on the surface of the Earth. One such satellite covers 120^{o} of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites shown in figure.

Since these geo stationary satellites seem to hover over one place on the Earth, continuous communication with any place on the surface of the Earth can be made. Microwaves are used because they travel in a narrow beam, in a straight line and pass easily through the atmosphere of the Earth. The energy needed to amplify and retransmit the signals is provided by large solar cell panels fitted on the satellites. There are over 200 Earth stations which transmit signals to satellites and receive signals via satellites from other countries can also pick up the signals from the satellite using a dish antenna on your house.

Do You Know?

The largest satellite system is managed by 126 countries, international Telecommunication satellite organization (INTELSAT). An INTELSAT VI satellite is shown in Fig. It operates at microwave frequency of 4, 6, 11 and 14 GHz and has capacity of 30,000 two way telephone circuits plus three T.V. channels.

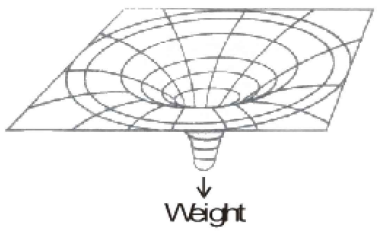
Q.22 Discuss Newton's and Einstein's views of gravitation.

Ans. NEWTON'S AND EINSTEIN'S VIEWS OF GRAVITATION

Do You Know?

According to Newton, the gravitation is the intrinsic property of matter that every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

According to Einstein's theory, space time is curved, especially locally near massive bodies. To visualize this, we might think of space as a thin rubber sheet; if a heavy weight is hung from it, curves as shown in Fig. The weight corresponds to a huge mass that causes space itself to curve. Thus, in Einstein's theory we do not speak of the force of gravity acting on bodies; instead we say that bodies and light rays move along geodesics (equivalent to straight lines in plane geometry) in curved space time. Thus a body at rest or moving slowly near the great mass would follow a geodesic towards that body.

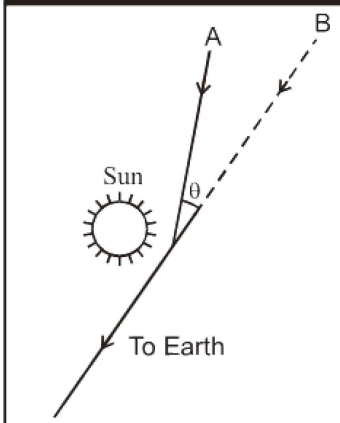


Rubber sheet analogy for curved space-time.

Einstein's theory gives us a physical picture of how gravity works. Newton discovered the inverse square law of gravity; but explicitly said that he offered no explanation of why gravity should follow an inverse square law. Einstein's theory also says that gravity follow inverse square law (Except in strong gravitational fields), but it tells us why this should be so that is why Einstein's theory is better

Einstein inferred that if gravity and acceleration are precisely equivalent, gravity must bend light, by a precisely amount that could be calculated. This was not entirely startling suggestions Newton's theory, based on the idea light beam would be deflected by gravity. But in Einstein's theory, the deflection of light is predicted to be exact twice as great as it is according to Newton's theory. Why the bending to straight caused by the gravity of the Sun was measured during a solar eclipse in 1919, and found, match Einstein's prediction rather than Newton's, then Einstein's theory was hailed as a scientific triumph.

Interesting Information



Bending of starlight by the Sun. Light from the star A is deflected as it passes close to the Sun on its way to Earth. We see the star in the apparent direction B, shifted by the angle ϕ = 1.745 seconds of angle which was found to be the same during the solar eclipse of 1919.

SOLVED EXAMPLES

EXAMPLE 5.1

An electric fan rotating at 3 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming declaration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Data

Initial angular velocity $= \omega_i = 3$ rev / s

Final angular velocity $= \omega_f = 0$

Time $= \Delta t = 18$ sec

To Find

Angular declaration $= \alpha = ?$

Number of revolutions $= \theta = ?$

SOLUTION

Using $\alpha = \dfrac{\Delta\omega}{\Delta t}$

$\alpha = \dfrac{\omega_f - \omega_i}{\Delta t}$

$\alpha = \dfrac{0 - 3}{18}$

$\alpha = -0.167$ rev/s^2

For angular motion

$\theta = \omega_i t + \dfrac{1}{2}\alpha t^2$

$\theta = 3 \times 18 + \dfrac{1}{2}(-0.167)(18)^2$

$\theta = 27$ rev

Result

Angular declaration $= \alpha = 0.167$ rev/s^2

Number of revolutions $= \theta = 27$ rev

EXAMPLE 5.2

A 1000 kg car is turning round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of circular path is 10 m, how large a force must be exerted by the pavement on the tyres to hold the car in the circular path?

Data

Mass of car = m = 1000 kg

Radius of circular path = r = 10 m

Speed of car = v = 10 ms^{-1}

To Find

Force required = F_C = ?

SOLUTION

Using $F_C = \frac{m V^2}{r}$

$F_C = \frac{1000\ (10)^2}{10}$

$F_C = 100 \times 100$

$F_C = 10^4$ N

Result

Force required = $F_C = 1.0 \times 10^4$ N

This force must be supplied by the frictional force of the pavement on the wheels.

EXAMPLE 5.3

A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. What will be the tension in the spring when the ball is at the point A of the path and its speed is v at this point?

SOLUTION

At point A two forces are acting on the ball.

(1) Tension in string.

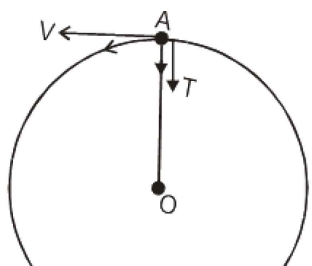
(2) Weight of ball w.

As these force act along the radius at point A, therefore,

$T + w = \frac{m V^2}{r}$ ..... (1)

$T = \frac{m V^2}{r} - w$

$T = \frac{m V^2}{r} - mg$



If $\frac{V^2}{r} = g$

Then $T = m(g - g) = 0$

$T = 0$ and the centripetal force just equal to weight.

EXAMPLE 5.4

The mass of Earth is 6.00×10^{24} kg. The distance r from Earth to the Sun is 1.50×10^{11} m. As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7 s).

Data

Mass of earth $= M_e = 6 \times 10^{24}$ kg

Distance between Earth and Sun $= r = 1.50 \times 10^{11}$ m

Time period $= T = 3.16 \times 10^7$ sec

To Find

Orbital angular momentum $= L_o = ?$

SOLUTION

Using $L_o = I\omega$ ..... (1)

Here $I = Mer^2$

As $T = \frac{2\pi}{T}$

$\therefore \quad \omega = \frac{2\pi}{T}$

$\therefore$ equation (1) becomes

$$L_o = M_e r^2 \frac{2\pi}{T}$$

Putting values

$$L_o = 6 \times 10^{24} (1.50 \times 10^{11})^2 \frac{2(3.14)}{3.16 \times 10^7}$$

$$= \frac{37.68 \times 2.25 \times 10^{24} \times 10^{22}}{3.16 \times 10^7}$$

$= 26.83 \times 10^{39}$

$L_o = 2.68 \times 10^{40}$ Js

Result

Orbital angular momentum $= L_o = 2.68 \times 10^{40}$ Js

EXAMPLE 5.5

A disc without slipping rolls down a hill of height 10.0 m. If the disc stars from rest at the top of the hill, what is its speed at the bottom?

Data

Height of hill $= h = 10$ m

To Find

Speed of disc at bottom $= V = ?$

SOLUTION

Using $V = \sqrt{\frac{4gh}{3}}$

$V = \sqrt{\frac{4 \times 9.8 \times 10}{3}}$

$V = \sqrt{\frac{392}{3}}$

$V = \sqrt{130.66}$

$V = 11.4$ m/s

Result

Speed of disc at bottom $= V = 11.4$ m/s

EXAMPLE 5.6

An Earth satellite is in circular orbit at a distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius $R = 6400$ km.

Data

Distance of satellite from surface of Earth $= h = 384000$ km

Mass of earth $= M_e = 6 \times 10^{24}$ kg

Radius $= R = 6400$ km

To Find

As distance of satellite from centre of Earth is

$$r = h + R$$

$$\therefore \quad \text{Using} \quad V = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{40.02 \times 10^{13}}{390400000}}$$

$$= \sqrt{0.000001025 \times 10^{12}}$$

$$= 0.00101 \times 10^{6}$$

$$= 1.01 \times 10^{3} \text{ m/s}$$

$$= 1.01 \text{ km/s}$$

Now using:

$$T = \frac{2 \times 3.14 \times 290400000}{1.01 \times 10^{3}}$$

$$= \frac{1 \text{ day}}{60 \times 60 \times 24}$$

$$T = 27.5 \text{ day}$$

Result

Period of one revolution = T = 27.5 days

EXAMPLE 5.7

Radio and T.V. signals bounce from a synchronous satellite. This satellite circles the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays over the same spot on the Earth because the Earth is rotating at the same rate.

(a) What is the orbital radius for a synchronous satellite?

(b) What is its speed?

Data

$$T = 24 \text{ h}$$

$$T = 24 \times 60 \times 60 \text{ sec}$$

$$T = 86400 \text{ sec}$$

To Find

(a) Orbital radius = r = ?

(b) Speed of satellite = V = ?

SOLUTION

Putting values

$$r = \left[\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} (86400)^2}{4 (3.14)^2} \right]^{1/3}$$

$$= \left[\frac{40.02 \times 10^{13} \times 7.46 \times 10^{9}}{4 \times 9.86} \right]^{1/3}$$

$$= \left[\frac{298.55 \times 10^{22}}{39.44} \right]^{1/3}$$

$$= [7.57 \times 10^{22}]^{1/3}$$

$$= (75.7 \times 10^{21})^{1/3}$$

$$= (75.7)^{1/3} \ 10^{21 \times 1/3}$$

$$= [(4.23)^3]^{1/3} \ 10^{7}$$

$$r = 4.23 \times 10^{7} \text{ m}$$

(b) Now using

$$V = \frac{2\pi r}{T}$$

Putting values

$$= \frac{2 (3.14) (4.23 \times 10^{7})}{86400}$$

$$= \frac{26.56 \times 10^{7}}{86400}$$

$$= 3.07 \times 10^{-4} \times 10^{7}$$

$$= 3.07 \times 10^{3} \text{ m/s}$$

$$V = 3.1 \text{ km/s}$$

Result

(a) Orbital radius = r = 4.23×10^{7} m

(b) Speed of satellite = V = 3.1 km/s



CIRCULAR MOTION

Each question has four possible answers, encircled the correct answer:

1. When a body is moving along a circular path, it covers a certain angle in given time. Such a type of motion is called:

(a) Linear motion **(b)** Angular motion

(c) Vibratory motion **(d)** Rotatory motion

2. When a body moves in such a way that its distance from the mean position remains constant is called:

(a) Linear motion **(b)** Circular motion

(c) Vibratory motion **(d)** Rotatory motion

3. The angle through which a body moves is called:

(a) Angular displacement **(b)** Angular velocity

(c) Angular acceleration **(d)** None of these

4. The SI unit of angular displacement is:

(a) Metre **(b)** Kilometre

(c) Radian **(d)** None of these

5. The angle subtended at the center of a circle by an arc equal to its radius is called:

(a) One radian **(b)** One degree

(c) One rotation **(d)** None of these

6. One radian is equal to:

(a) 47.3° **(b)** 57.3°

(c) 67.3° **(d)** 59.3°

7. The linear acceleration of bodies moving in circular path is:

(a) Constant **(b)** Negative

(c) Positive **(d)** Zero

8. The angular displacement is assign positive sign when the rotation is:

(a) Clockwise **(b)** Anti-clock wise

(c) Perpendicular **(d)** Parallel

9. The direction associated with angular displacement is given by:

(a) Left hand rule **(b)** Head to tail rule

(c) Right hand rule **(d)** None of these

10. Angular displacement is a:

(a) Scalar **(b)** Vector

(c) Neither scalar nor vector **(d)** None of these

11. Radian is defined as the angle subtended at the center of a circle by an:

(a) Arc whose length equal to radius of circle

(b) Arc whose length less than radius of circle

(c) Arc whose length greater than radius of circle

(d) None of these

12. Which one is the correct relation for angular displacement θ, radius r and length of arc S is:

(a) $S = r\theta$ **(b)** $\frac{S}{r} = \theta$

(c) $Sr = \theta$ **(d)** $S\theta = r$

13. The rate of change of angular displacement is called:

(a) Angular speed **(b)** Angular acceleration

(c) Angular velocity **(d)** None of these

14. Angular velocity is a:

(a) Scalar **(b)** Vector

(c) Neither scalar nor vector **(d)** None of these

15. The direction of angular velocity of a body moving in a circle is:

(a) Towards the axis of rotation **(b)** Away from the axis of rotation

(c) Along the axis of rotation **(d)** None of these

16. Average angular velocity is defined by the relation:

(a) $< \omega > = \theta t$ **(b)** $< \omega > = \frac{\theta}{t}$

(c) $< \omega > = \frac{t}{\theta}$ **(d)** $< \omega > = \frac{\theta^2}{t}$

17. When an object moves in a circle, the angle between angular velocity ω and linear velocity v is:

(a) 90° **(b)** 0°

(c) 45° **(d)** 60°

18. The angular velocity at any instant is called:

(a) Instantaneous angular velocity **(b)** Instantaneous angular speed

(c) Instantaneous angular displacement **(d)** None of these

19. If a particle moves in a circle of radius r with uniform angular velocity ω then the angle between radius r and linear velocity v is:

(a) 45° **(b)** 90°

(c) 60° **(d)** 180°

20. The rate of change of angular velocity is called:

(a) Angular velocity **(b)** Angular acceleration

(c) Angular displacement **(d)** None of these

21. SI unit of angular acceleration is:

(a) rad/s^2 **(b)** rad/s

(c) $rad.s^2$ **(d)** rad.s

22. The relation between linear acceleration and angular acceleration is:

(a) $\vec{\alpha} = \vec{a} \times \vec{r}$ **(b)** $\vec{a} = \vec{r} \times \vec{\alpha}$

(c) $\vec{r} = \vec{a} \times \vec{\alpha}$ **(d)** $\vec{a} = \vec{\alpha} \times \vec{r}$

23. The dimensions of angular acceleration are:

(a) $[LT^{-2}]$ **(b)** $[T^{-2}]$

(c) $[LT^{-1}]$ **(d)** $[T^{-3}]$

24. The relation between linear velocity and angular velocity is:

(a) $\vec{v} = \vec{r} \times \vec{\omega}$ **(b)** $\vec{v} = \vec{\omega} \times \vec{r}$

(c) $\vec{\omega} = \vec{v} \times \vec{r}$ **(d)** $\vec{r} = \vec{v} \times \vec{\omega}$

25. The dimensions of angular velocity are:

(a) $[LT^{-1}]$ **(b)** [LT]

(c) $[LT^{-2}]$ **(d)** $[T^{-1}]$

26. If a rotating body is moving anti-clockwise, the direction of angular velocity is:

(a) Towards the centre **(b)** Along the linear velocity

(c) Away from the centre **(d)** Perpendicular to both radius and linear velocity

27. Angular acceleration is produced due to:

(a) Centripetal force **(b)** Torque

(c) Force **(d)** None of these

28. The period of circular motion is given by:

(a) $T = \frac{2\pi}{\omega}$ **(b)** $T = \frac{\omega}{2\pi}$

(c) $T = 2\pi\omega$ **(d)** None of these

29. The force needed to bend the normally straight path of particle into a circular path is called:

(a) Inertia **(b)** Torque

(c) Centripetal force **(d)** None of these

30. When a body is moving in a circle of radius r with constant linear velocity v, its centripetal force is:

(a) $\frac{mv^2}{r}$ **(b)** $\frac{m^2v}{r}$

(c) $\frac{mv}{r^2}$ **(d)** None of these

31. When a body moves in a circle of radius r with angular speed ω, its centripetal force is:

(a) $\frac{m\omega}{r}$ **(b)** $mr\omega^2$

(c) $\frac{mr}{\omega^2}$ **(d)** $\frac{m\omega^2}{r}$

32. A body is moving in a circle of radius r with constant speed v, its centripetal acceleration is:

(a) $\frac{v^2}{r}$ **(b)** v^2r

(c) $\frac{r}{v^2}$ **(d)** $\frac{v}{r}$

33. A body is moving in a circle of radius r with constant angular velocity ω, its centripetal acceleration is:

(a) $\frac{\omega}{r}$ **(b)** ωr

(c) $\omega^2 r$ **(d)** ωr^2

34. The force required to keep a body into a circle is called:

(a) Centrifugal force **(b)** Centripetal force

(c) Gravitational force **(d)** None of these

35. A body is moving in a circle at constant speed. Which of the following statement is correct:

(a) There is no force acting away from the centre of the circle

(b) There is no force acting towards the centre of the circle

(c) There is no acceleration

(d) None of these

36. The vectorial form of centripetal force is:

(a) $-m\omega^2\vec{r}$ **(b)** $m\omega^2\vec{r}$

(c) $-m\omega\vec{r}$ **(d)** $m^2\omega\vec{r}$

37. An object is traveling in a circle with constant speed. Its acceleration is constant in:

(a) Direction only **(b)** Magnitude only

(c) Both magnitude and direction **(d)** None of these

38. A body of mass 4 kg moves along a circle of radius 2m with a constant speed of 4 m/s. The centripetal force is:

(a) 32 N **(b)** 16 N

(c) 20 N **(d)** 64 N

39. In case of planets, the necessary acceleration is provided by:

(a) Coulombs force **(b)** Centripetal force

(c) Gravitation force **(d)** None of these

40. The necessary centripetal force to the moving car round a corner track is provided by:

(a) Centrifugal force **(b)** Gravitational force

(c) Coulombs force **(d)** Frictional force

41. The mud flies off the tyre of a moving bicycle in the direction of:

(a) Towards the centre **(b)** Tangent to the tyre

(c) Along the radius **(d)** Along the motion

42. The angular momentum is defined as:

(a) $\vec{L} = m\vec{v}$ **(b)** $\vec{L} = \vec{r} \times \vec{F}$

(c) $\vec{L} = \vec{r} \times \vec{P}$ **(d)** $\vec{L} = \vec{P} \times \vec{r}$

43. The SI unit of angular momentum is:

(a) $kg\ m^2/s$ **(b)** $kg\ m^2/s^2$

(c) $kg^2\ m/s$ **(d)** $kg\ m/s$

44. The dimensions of angular momentum $\vec{L}$ are:

(a) $[M^2LT^{-1}]$ **(b)** $[M^2L^2T]$

(c) $[ML^2T^2]$ **(d)** $[ML^2T^{-2}]$

45. Angular momentum of a body under central force is:

(a) Minimum **(b)** Zero

(c) Maximum **(d)** Constant

46. A body rotating in a circle of radius 1 m with an angular speed 10 rad/s has the tangential velocity:

(a) 2 m/s **(b)** 5 m/s

(c) 10 m/s **(d)** 15 m/s

47. The rate of change of angular momentum of a body is:

(a) The applied torque **(b)** The applied force

(c) Both (a) and (b) **(d)** None of these

48. Angular momentum is conserved under:

(a) Variable force **(b)** Constant force

(c) Central force **(d)** None of these

49. The direction of angular momentum $\vec{L}$ is:

(a) Along the direction of $\vec{P}$

(b) Along the direction $\vec{r}$

(c) Parallel to the plane containing $\vec{r}$ and $\vec{P}$

(d) Perpendicular to the plane containing $\vec{r}$ and $\vec{P}$

50. The angular momentum of a body about a fixed point is conserved if its velocity:

(a) Increases **(b)** Decreases

(c) Constant **(d)** None of these

51. Angular momentum of a rigid body is equal to:

(a) $I\omega$ **(b)** $I\alpha$

(c) $\frac{1}{2} I\omega$ **(d)** $\frac{1}{2} I^2\omega$

52. In rotational motion, the quantity which plays the same role as the inertial mass in term of linear motion is known as:

(a) Angular momentum **(b)** Linear momentum

(c) Momentum of inertia **(d)** Torque

53. The value of angular momentum is maximum when θ is:

(a) $60°$ **(b)** $45°$

(c) $0°$ **(d)** $90°$

54. The angular momentum $\vec{L}$ in terms of angular velocity ω is equal to:

(a) $mr^2\omega$ **(b)** $m^2r\omega$

(c) $mr\omega^2$ **(d)** $m^2r^2\omega^2$

55. The product of the mass m of the rotating body and the square of radius of gyration is called:

(a) Moment of inertia **(b)** Torque

(c) Linear momentum **(d)** None of these

56. Mathematically moment of inertia is equal to:

(a) $I = mr^2$ **(b)** $I = mr$

(c) $I = m^2r$ **(d)** $I = m^2r^2$

57. The moment of inertia depends upon:

(a) Angular momentum

(b) Mass of the body and its angular speed

(c) Mass of the body and its radius

(d) Mass as well as its distribution with respect to axis of rotation

58. Moment of inertia of a thin rod about mid length is:

(a) $I = \frac{1}{12} mL^2$ **(b)** $I = \frac{2}{5} mr^2$

(c) $I = \frac{1}{3} mr^2$ **(d)** $I = \frac{1}{2} mr^2$

59. Moment of inertia of a hoop about its axis is:

(a) $I = \frac{1}{3} mr^2$ **(b)** $I = \frac{1}{2} mr^2$

(c) $I = \frac{2}{3} mr^2$ **(d)** $I = \frac{2}{5} mr^2$

60. The moment of inertia of a body comes in action in:

(a) Circular motion **(b)** Straight line motion

(c) Curved path **(d)** None of these

61. The SI unit of angular momentum is:

(a) kgm^3 **(b)** kgm

(c) kgm^2 **(d)** kgm^{-2}

62. The physical quantity which plays the same part in angular motion as mass does in linear motion is called:

(a) Moment of inertia **(b)** Momentum

(c) Torque **(d)** None of these

63. The physical quantity which produces angular acceleration is known as:

(a) Force **(b)** Torque

(c) Inertia **(d)** None of these

64. Moment of inertia of a solid disc about its axis is:

(a) $\frac{1}{2}mr^2$ **(b)** $\frac{2}{5}mr^3$

(c) $\frac{2}{3}mr^2$ **(d)** $\frac{1}{3}mr^2$

65. Moment of inertia of a sphere is:

(a) $\frac{1}{2}mr^2$ **(b)** $\frac{2}{5}mr^3$

(c) $\frac{2}{5}mr^2$ **(d)** $\frac{1}{3}mr^2$

66. The dimensions of moment of inertia are:

(a) $[ML^2]$ **(b)** $[ML^{-2}]$

(c) $[ML^{-1}]$ **(d)** $[M^2L]$

67. The force and torque are analogues to:

(a) Each other **(b)** Moment of inertia

(c) Velocity **(d)** None of these

68. The total weight of the body acts at:

(a) its center of gravity **(b)** its centre

(c) its one end **(d)** its other end

69. The center of gravity is also called:

(a) Centre of body **(b)** Centre of mass

(c) Both (a) and (b) **(d)** None of these

70. The acceleration due to gravity:

(a) Same value at every place **(b)** Same value everywhere on the surface of earth

(c) Varies with altitude **(d)** None of these

71. The value of 'g' at the center of the Earth is:

(a) Infinite **(b)** Double

(c) Zero **(d)** None of these

72. Law of conservation of angular momentum states that if no ———— acts on a system, the total angular momentum of the system remains constant:

(a) External torque **(b)** External couple

(c) External force **(d)** None of these

73. The axis of rotation of a body will not change its orientation unless an external ———— causes it to do so:

(a) Force **(b)** Torque

(c) Work **(d)** Energy

74. A cricketer spins the ball before bowling to:

(a) Give it downward deflection **(b)** Give it upward deflection

(c) Keep it straight **(d)** None of these

75. An elevator is accelerated downward with acceleration a, the apparent weight of a body of mass m in it will be:

(a) $m(a - g)$ **(b)** $m(g + a)$

(c) $m(g - a)$ **(d)** mg

76. An elevator is accelerated upward with acceleration a, the apparent weight of a body of mass m in it will be:

(a) $m(a - g)$ **(b)** $m(a + g)$

(c) $m(g - a)$ **(d)** mg

77. If the rope of an elevator moving downward with acceleration a breaks, the apparent weight of a body of mass m in it will be:

(a) $m(a - g)$ **(b)** $m(a + g)$

(c) $m(g - a)$ **(d)** Zero

78. The K.E of a disc of mass m rolling down on an included plane is:

(a) $\frac{1}{2}mv^2$ **(b)** $\frac{1}{4}mv^2$

(c) $\frac{3}{4}mv^2$ **(d)** None of these

79. The rotation K.E of any hoop of radius r is given by:

(a) $\frac{1}{2}mr^2\omega^2$ **(b)** $\frac{1}{2}r\omega^2$

(c) $\frac{1}{2}r^2\omega^2$ **(d)** None of these

80. Suppose a body of cylindrical shape is called down on an inclined plane of height h. It contains:

(a) Translation K.E **(b)** Rotational K.E

(c) Both (a) and (b) **(d)** None of these

81. Speed of hoop at the bottom can be given by:

(a) $v = \sqrt{gh}$ **(b)** $v = \sqrt{2gh}$

(c) $v = \sqrt{\frac{3}{4}gh}$ **(d)** None of these

82. Orbital velocity of a satellite orbiting closer to the planet is given by:

(a) $v = \sqrt{\frac{GM}{R}}$ **(b)** $v = \sqrt{\frac{GMm}{R}}$

(c) $v = \sqrt{gR}$ **(d)** None of these

83. If the earth stops rotating weight of a body on the equator:

(a) Increases **(b)** Remains constant

(c) Decreases **(d)** None of these

84. A body becomes weightless:

(a) Spaceship orbiting the earth **(b)** Outside the field free region

(c) On the earth's centre **(d)** All of above

85. The axis of rotation of a rotating body in the absence of external torque:

(a) Continuously changes **(b)** Remain fixed in direction

(c) Both (a) and (b) **(d)** None of these

86. The number of satellite included in the Global positioning system is:

(a) 20 **(b)** 48

(c) 24 **(d)** None of these

87. Orbital speed of a satellite at large enough height h above the earth's surface is:

(a) $\sqrt{gR}$ **(b)** $\sqrt{\frac{GM}{R+h}}$

(c) $\sqrt{\frac{GM}{R}}$ **(d)** None of these

88. In a spaceship orbiting the earth, the apparent weight of the body in it is:

(a) Less than its real weight **(b)** Greater than its real weight

(c) Weightlessness **(d)** None of these

89. One communication satellite covers:

(a) 240 longitudinal lines **(b)** 360 longitudinal lines

(c) 120 longitudinal lines **(d)** None of these

90. International telecommunication in satellite organization (INTELSAT):

(a) Operates at microwave frequency 4.611 and 149 Hz

(b) Has capability of 30,000 two-way telephone circuits

(c) Provides facility of three T.V channels

(d) All of above

91. Height of the closest orbit of the satellite above the earth is:

(a) 300 km **(b)** 250 km

(c) 500 km **(d)** 400 km

92. Radius of the geo-stationary orbit from the earths center is:

(a) 4.24×10^4 km **(b)** 3.23×10^4 km

(c) 4.23×10^3 km **(d)** None of these

93. Height of geo-stationary satellite above the equator is:

(a) 40,000 km **(b)** 24,000 km

(c) 30,000 km **(d)** 36,000 km

94. Entire populated earth, surface is covered by:

(a) Four **(b)** Three

(c) Two **(d)** None of these

95. The geo-stationary satellite are used for:

(a) World communications **(b)** Weather observations

(c) Navigation **(d)** All of above

96. Einstein theory was considered as a:

(a) Scientific trump **(b)** Simple trump

(c) Natural trump **(d)** None of these

97. Which theory of gravitation is better:

(a) Newton **(b)** Einstein

(c) Compton **(d)** Plank

98. The time period T of the artificial satellite is given by:

(a) $T = \frac{2\pi v}{R}$ **(b)** $T = \frac{2v}{\pi R}$

(c) $T = \frac{2\pi R}{v}$ **(d)** None of these

99. $1° =$ _____ radian.

(a) 0.01745 **(b)** 57

(c) 0.1745 **(d)** 0.2

100. If $r = 1$ m and $\theta = 1°$ then what is value of S:

(a) 0.01745 m **(b)** 1 m

(c) 2 m **(d)** None

101. Dimensions of angular velocity:

(a) $[LT^{-1}]$ **(b)** $[T^{-1}]$

(c) [LT] **(d)** None

102. An electric fan rotating at 3 rev/s is switched off. It comes to rest in 18 s. Assuming deceleration to be uniform. What is its value?

(a) $0.167\ rev/s^2$ **(b)** $-0.167\ rev/s^2$

(c) $-0.176\ rev/s^2$ **(d)** $0.176\ rev/s^2$

103. Unit of moment of inertia is:

(a) kg/m^2 **(b)** $kg\ m^2$

(c) gm^2 **(d)** $kg\ cm^2$

104. The product of linear momentum and moment arm for momentum is called:

(a) Momentum **(b)** Angular momentum

(c) Torque **(d)** Moment of inertia

105. The axis of rotation of an object will not change its orientation unless an external ———— causes it to do so.

(a) Force **(b)** Torque

(c) Both (a), (b) **(d)** None

106. Moment of inertia of a thin rod of mass 'm' and length 'L' is:

(a) $\frac{2}{5}mL^2$ **(b)** $\frac{1}{12}mL^2$

(c) $\frac{1}{12}mL$ **(d)** $\frac{1}{12}m^2L$

107. If an artificial satellite move with 30000 km/h then it follow:

(a) Circular orbit **(b)** Elliptical orbit

(c) Escape **(d)** None of these

108. Global positioning system can be formed by ———— artificial satellites.

(a) 22 **(b)** 42

(c) 3 **(d)** 24

109. When the lift is moving upwards with an acceleration $\vec{a}$ then weight of the object appears to be ———— than real weight.

(a) Less **(b)** More

(c) No change **(d)** None

110. If distance between satellite and centre of Earth is 2R then orbital velocity of satellite:

(a) $\sqrt{2gR}$ **(b)** $\sqrt{\frac{gR}{2}}$

(c) $\sqrt{\frac{GM}{r}}$ **(d)** None of these

111. To create artificial gravity spaceship rotates with frequency:

(a) $2\pi\sqrt{\frac{g}{R}}$ **(b)** $\frac{1}{2\pi}\sqrt{\frac{R}{g}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ **(d)** All

112. The Earth rotates on its axis once a day. Suppose by some process the Earth contracts so that its radius is only half as large as at present. The Earth would complete its rotation in:

(a) 24 h **(b)** 6 h

(c) 12 h **(d)** 18 h

113. The minute hand of a large clock is 3.0 m long. What is its mean angular speed?

(a) 1.4×10^{-4} rad s^{-1} **(b)** 1.7×10^{-3} rad s^{-1}

(c) 5.2×10^{-3} rad s^{-1} **(d)** 1.0×10^{-1} rad s^{-1}

114. When an aircraft is moving in a horizontal plane at a constant speed of 650 ms^{-1}, its turning circle has a radius of 80 km. What is the ratio of the centripetal force to the weight of the aircraft?

(a) 8.3×10^{-4} **(b)** 0.54

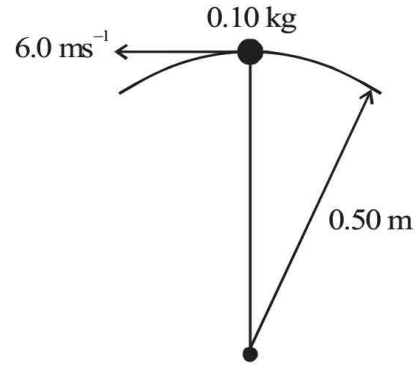
(c) 1.9 **(d)** 52

115. An object travels at constant speed around a circle of radius 1.0 m in 1.0 s. What is the magnitude of its acceleration?

(a) Zero **(b)** 1.0 ms^{-2}

(c) 2π ms^{-2} **(d)** $4\pi^2$ ms^{-2}

116. A ball of mass 0.10 kg is attached to a string and swung in a vertical circle of radius 0.50 m, as shown. Its speed at the top of the circle is 6.0 ms^{-1}. (Take g as 10 ms^{-2})



What is the tension in the string at this moment?

(a) 1.0 N **(b)** 6.2 N

(c) 7.2 N **(d)** 8.2 N

117. An object on the end of a spring oscillates with simple harmonic motion of angular frequency 2.0 rad s^{-1}. What is the period of the oscillation?

(a) 0.080 s **(b)** 0.32 s

(c) 0.50 s **(d)** 3.1 s

118. An object is travelling in a circle of radius r with angular velocity ω and speed v. Which expression gives the centripetal acceleration?

(a) $r\omega$ **(b)** $v\omega$

(c) v/r **(d)** v/r^2

119. When brakes of a car are applied, angular velocity of a flywheel reduces from 900 cycle/min. to 720 cycle/min. in 6sec. Angular retardation is:

(a) π rad/s^2 **(b)** 9π rad/s^2

(c) 8π rad/s^2 **(d)** None of these

120. A motor operates at 20 rev/s. What is power delivered by it? If it supply a torque of 75 Nm:

(a) 75 W **(b)** 85 W

(c) 90 W **(d)** 9.4×10^3 W

121. A wheel starts from rest and gets a rotational speed of 240 rev/s in time 2 min. Its average acceleration will be:

(a) 1 rev/s^2 **(b)** 2 rev/s^2

(c) 5 rev/s^2 **(d)** 8 rev/s^2

122. Ratio of magnitude of angular velocity of hour hand of a watch to that of earth's rotation about its own axis is:

(a) 3 : 1 **(b)** 1 : 3

(c) 2 : 1 **(d)** 1 : 4

123. If a solid sphere rolls on a horizontal surface, the ratio of its rotational K.E. to its total K.E. is:

(a) 1 : 3 **(b)** 2 : 7

(c) 7 : 3 **(d)** None of these

124. A solid cylinder rolls down an inclined plane of height 1 m in 10 s. Its acceleration is:

(a) 0.3 m/s^2 **(b)** 0.7 m/s^2

(c) 5 m/s^2 **(d)** 1 m/s^2

125. A particle of mass m tied to a string of length l is rotating in a circular path with constant speed v. Torque on it is:

(a) Zero **(b)** mv$l$

(c) $\frac{mv}{l}$ **(d)** $\frac{mv^2}{l}$

126. A motor runs at 20 rev/s and supplies a torque of 75 Nm. Power delivered by it is:

(a) 6.12 hp **(b)** 12.6 hp

(c) 7.3 hp **(d)** 8.3 hp

127. A body is executing uniform circular motion. Its angular acceleration is:

(a) 2 **(b)** 4

(c) Zero **(d)** None of these

ANSWERS

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(c)
5.	(a)	**6.**	(b)	**7.**	(d)	**8.**	(b)
9.	(c)	**10.**	(b)	**11.**	(a)	**12.**	(a)
13.	(c)	**14.**	(b)	**15.**	(c)	**16.**	(b)
17.	(a)	**18.**	(a)	**19.**	(b)	**20.**	(b)
21.	(a)	**22.**	(d)	**23.**	(b)	**24.**	(a)
25.	(d)	**26.**	(d)	**27.**	(b)	**28.**	(a)
29.	(c)	**30.**	(a)	**31.**	(b)	**32.**	(a)
33.	(c)	**34.**	(b)	**35.**	(b)	**36.**	(b)
37.	(b)	**38.**	(a)	**39.**	(c)	**40.**	(d)
41.	(b)	**42.**	(c)	**43.**	(a)	**44.**	(d)
45.	(d)	**46.**	(c)	**47.**	(a)	**48.**	(c)
49.	(d)	**50.**	(b)	**51.**	(a)	**52.**	(c)
53.	(d)	**54.**	(a)	**55.**	(a)	**56.**	(a)
57.	(d)	**58.**	(a)	**59.**	(b)	**60.**	(a)
61.	(c)	**62.**	(a)	**63.**	(b)	**64.**	(a)
65.	(c)	**66.**	(a)	**67.**	(a)	**68.**	(a)
69.	(b)	**70.**	(c)	**71.**	(c)	**72.**	(c)
73.	(b)	**74.**	(b)	**75.**	(c)	**76.**	(b)
77.	(d)	**78.**	(b)	**79.**	(a)	**80.**	(c)
81.	(a)	**82.**	(c)	**83.**	(c)	**84.**	(d)
85.	(b)	**86.**	(c)	**87.**	(b)	**88.**	(c)
89.	(c)	**90.**	(d)	**91.**	(d)	**92.**	(a)
93.	(d)	**94.**	(b)	**95.**	(d)	**96.**	(a)
97.	(b)	**98.**	(c)	**99.**	(a)	**100.**	(a)
101.	(b)	**102.**	(b)	**103.**	(b)	**104.**	(b)
105.	(b)	**106.**	(b)	**107.**	(a)	**108.**	(d)
109.	(b)	**110.**	(b)	**111.**	(c)	**112.**	(b)
113.	(b)	**114.**	(b)	**115.**	(d)	**116.**	(b)
117.	(d)	**118.**	(b)	**119.**	(a)	**120.**	(d)
121.	(b)	**122.**	(c)	**123.**	(c)	**124.**	(a)
125.	(a)	**126.**	(b)	**127.**	(c)		

SHORT QUESTIONS

5.1 Explain the difference between tangential velocity and the angular velocity, if one of these is given for a wheel of known radius, how will you find the other?

Ans. **Tangential Velocity:** The tangential velocity v_T is the linear velocity of a particles moving along a curve or a circle directed along the tangent at any point on the curve.

Angular Velocity: The angular velocity ω is the rate of change of angular displacement moving along a curved path.

Relation: The tangential velocity v_T, angular velocity ω and the radius r of the wheel are related by the relation

$$v_T = r\omega$$

If one of these is given for a wheel of known radius r then the other one can be find by using the above relation.

5.2 Explain what is meant by centripetal force and why it must be furnished to an object if the object to follow a circular path?

Ans. The force which is required to keep the body moving in a circular path and directed towards the centre of the circular path is called centripetal force. It is denoted by F_c and mathematically, it can be written as

$$F_c = \frac{mv^2}{r}$$

For uniform circular motion, it must be under the influence of a force which changes the direction of motion continuously. So the centripetal force is always needed if a body is to maintain its circular motion.

5.3 What is meant by moment of inertia? Explain its significance.

Ans. **Moment of Inertia:** It is defined as the product of mass of particle and square of its perpendicular distance from axis of rotation. It is denoted by I. Mathematically it can be written as

$$I = mr^2$$

Where m is the mass of the particle and r is the perpendicular distance of particle from the axis of rotation.

Significance: The moment of inertia plays the same role in angular motion as the mass in linear motion. As mass of a body is called measure of inertia but moment of inertia is the property of a body which resists to change its uniform circular motion.

5.4 What is meant by angular momentum? Explain the law of conservation of angular momentum.

Ans. **Angular Momentum:** The angular momentum of an object is defined as the cross product of position vector $\vec{r}$ with respect to the axis of rotation and the linear momentum $\vec{P}$. It is denoted by $\vec{L}$. Mathematically

$$\vec{L} = \vec{r} \times \vec{P}$$

$$= rP \sin\theta \hat{n}$$

It is a vector quantity so the direction of angular momentum $\vec{L}$ is perpendicular to the plane contained by $\vec{r}$ and $\vec{P}$.

Law of Conservation of Angular Momentum: This law states that if no external torque acts on a system, the total angular momentum of a system remains constant.

Mathematically

$$\vec{L} = \vec{L_1} + \vec{L_2} + \vec{L_3} \ \ldots\ldots = \text{Constant}$$

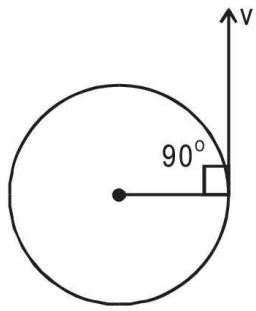
5.5 Show that orbital angular momentum $L_o = mvr$.

Ans. From the definition of angular momentum

$$\vec{L_o} = \vec{r} \times \vec{P}$$

$$L_o = rP \sin\theta = mvr \sin\theta$$

where θ is the angle between position vector $\vec{r}$ and velocity $\vec{v}$. In case of circular orbital motion, the angle between radius and tangential velocity is 90°. Hence



$$L_o = rP \sin\theta = mvr \sin 90° \quad \text{where} \quad \sin 90° = 1$$

$$L_o = mrv\ (1)$$

Hence $L_o = mvr$

5.6 Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

Ans. When a satellite is moving in a circular orbit, it has an acceleration $a_c = \frac{v^2}{r}$ which is supplied by gravity. The low flying earth satellite have an acceleration $a_c = g = 9.8\ m/s^2$. Thus

$$g = \frac{v^2}{r}$$

$$v^2 = gr$$

$$v = \sqrt{gr}$$

where r is the radius of earth = R = 6400 km.

So $v = \sqrt{9.8 \times 6400}$

$$= 7.9\ km/s$$

This is the minimum velocity necessary to put a satellite into orbit around the earth.

5.7 State the direction of the following vectors in simple situations; angular momentum and angular velocity.

Ans. **Direction of Angular Momentum:** As we know that the angular momentum is always directed perpendicular to the plane containing $\vec{v}$ and $\vec{r}$ according to right hand rule.

$$\vec{L} = \vec{r} \times \vec{P} = (\vec{r} \times \vec{v})m$$

Direction of Angular Velocity: The direction of angular velocity is taken along the axis of rotation given by right hand rule. If rotation is along the curl of the fingers of right hand, then the outward thumb gives the direction of angular velocity. According to this rule, for counter clockwise rotation, the direction of angular velocity is outward along the axis of rotation.

5.8 Explain why an object, orbiting the Earth, is said to be freely falling. Use you explanation to point out why objects appear weightless under certain circumstances.

Ans. An object such as an artificial satellite orbits the Earth due to force of gravity. Its centripetal acceleration is equal to the acceleration due to gravity directed towards the centre of the Earth. Hence, a satellite is always falling towards the centre of Earth with acceleration equal to "g" and is a freely falling object. It does not hit the ground because of its tangential velocity and curvature of the Earth.

In the fame of reference of a freely falling object the bodies appears weightless because both, the body inside and the frame of reference, are falling with the same acceleration. The relative acceleration of inside body with respect to its frame of reference is zero. Hence, it seems that no force is acting on the body and when it is suspended to a spring balance, the spring balance shows its weight as zero.

5.9 When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain.

Ans. The mud flies off the tyre of a moving bicycle along the tangent as it leaves the path of circular motion. Actually the mud is sticked with the surface of tyre due to adhesive force (the attraction force existing between the molecules of different surfaces). This adhesive force provides the necessary centripetal force to the mud to move in a circular path. If the speed of rotation is increased then at a certain instant this adhesive force becomes unable to meet the demand of centripetal force and hence, it will no longer move in circular path. As, in the circular path mud has linear velocity along the tangent at every point so when it leaves its circular path it moves along tangent at that point.

5.10 A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom?

Ans. The velocity of the disc on reaching the bottom of the inclined plane is given as

$$v_{Disc} = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4}{3}} \times \sqrt{gh}$$

and for a hoop, the velocity of hoop on reaching the bottom of the inclined plane is given as

$$v_{Hoop} = \sqrt{gh}$$

$$v_{Disc} = \sqrt{\frac{4}{3}} \times \sqrt{gh}$$

$$v_{Disc} = \sqrt{\frac{4}{3}} v_{Hoop}$$

As $\sqrt{\frac{4}{3}} > 1$

$\therefore$ $v_{Disc} > V_{Hoop}$

From these relations, it is clear that the velocity of disc is greater than the velocity of hoop so the disc moves faster as compared to hoop.

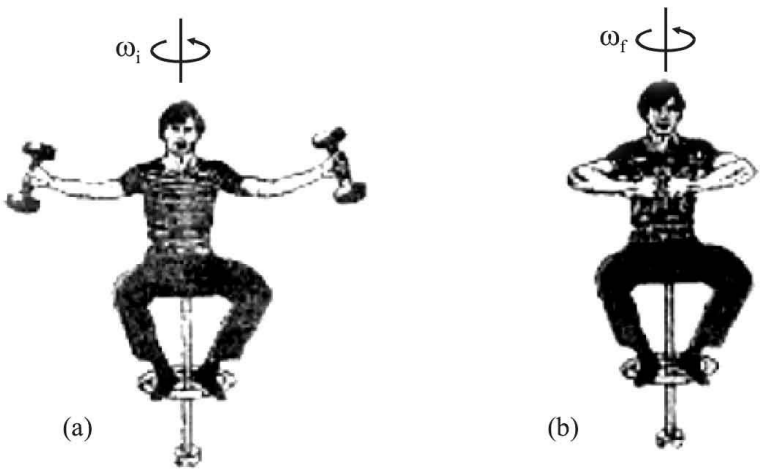
5.11 Why does a diver change his body position before diving in the pool?

Ans. Before lifting off the diving board, the diver's legs and arms are fully extended which means that the diver has a large moment of inertia I_1 about an axis. The moment of inertia is considerably reduced to a new value I_2, when the legs and arms are drawn into closed tuck position. As angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

where ω_1 and ω_2 are the angular speeds before and after diving. Hence the diver spins faster when its moment of inertia becomes smaller.

5.12 A student holds two dumb-bells without stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumbbell towards his chest (Fig. 5.25). What will be the effect on rate of rotation?



Ans. By using the law of conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

With out-stretched arms, the moment of inertia is I_1 and his angular speed is ω_1. Their product $I_1\omega_1$ is equal to angular moment which remains constant. When he pulls the dumbbells towards his chest, his moment of inertia decrease and he spins faster as ω_2 increases to keep the product $I_2\omega_2$ constant.

5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of T.V transmission?

Ans. A satellite communication system can be set up by placing several geo stationary satellites in orbit over different point on the surface of the Earth. One such satellite covers 120^o of longitude, so that whole of the populated Earth's surface can be covered by three correctly positioned satellites.

PROBLEMS WITH SOLUTIONS

PROBLEM 5.1

A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8m.

Data

Diameter of beam = Length of arc = S = 2.50 m

Distance of moon from the earth = r = 3.8×10^8 m

To Find

Divergence angle = θ = ?

SOLUTION

By formula

$$S = r\theta$$

$$\boxed{\theta = \frac{S}{r}}$$

$$\theta = \frac{2.50}{3.8 \times 10^8}$$

$$= 0.657 \times 10^{-8}$$

$$= 6.6 \times 10^{-9} \text{ rad}$$

Result

Divergence angle = θ = 6.6×10^{-9} rad

PROBLEM 5.2

A gramophone record turntable accelerate from rest to an angular velocity of 45.0 rev min^{-1} in 1.60s. What is its average angular acceleration?

Data

Initial angular velocity = ω_i = 0

Final angular velocity = ω_f = 45.0 rev/min.

$$= 45 \times \frac{2\pi}{60} \text{ rad/s}$$

$$= 1.5\pi \text{ rad/s}$$

Time interval = Δt = 1.60 sec.

To Find

Angular acceleration = α = ?

SOLUTION

By formula

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

$$= \frac{1.5\pi - 0}{1.6}$$

$$= \frac{1.5 \times 3.14}{1.6}$$

$$\alpha = 2.9 \text{ rad/s}^2$$

Result

Angular acceleration = α = 2.9 rad/s^2

PROBLEM 5.3

A body of moment of inertia I = 0.80 kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad s^{-1}. Calculate its angular momentum L and the torque to sustain this motion.

Data

Moment of inertia = I = 0.80 kgm^2

Constant angular velocity = ω = 100 rad/s

To Find

Angular momentum = $\vec{L}$ = ?

Torque = $\vec{\tau}$ = ?

SOLUTION

For angular momentum

$$L = I\omega$$

$$L = 0.80 \times 100$$

$$= 80 \text{ kgm}^2/\text{s}^2 \qquad \text{As} \quad \text{kgm}^2/\text{s}^2 = \text{J.s}$$

So $L = 80$ J.s

For torque to sustain its motion

$$\boxed{\tau = I \times \alpha}$$

Since the body is moving with constant angular velocity so $\alpha = 0$

$$\tau = I \times 0$$

$$\tau = 0$$

Result

Angular momentum $= \vec{L} = 80$ J.s

Torque $= \vec{\tau} = 0$

PROBLEM 5.4

Consider the rotating cylinder shown in Fig. 5.26. Suppose that m = 5.0 kg, F = 0.60 N and r = 0.20 m. Calculate (a) the torque acting on the cylinder, (b) the angular acceleration of the cylinder. (Moment of inertia of cylinder = $\frac{1}{2}$ mr^2)

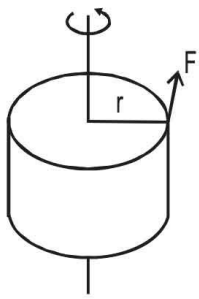


Fig. 5.26

Data

Mass of cylinder = m = 5.0 kg

Force acting on cylinder = F = 0.60 N

Radius of cylinder = r = 0.20 m

To Find

(a) Torque acting on the cylinder = τ = ?

(b) Angular acceleration = α = ?

SOLUTION

(a) For torque acting on the cylinder

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$\tau = rF \sin\theta$ But $\theta = 90°$

$$\boxed{\tau = rF}$$

$$= 0.20 \times 0.60$$

$$= 0.12 \text{ Nm}$$

(b) For angular acceleration

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I} \quad \text{But} \quad I = \frac{1}{2}mr^2$$

$$\alpha = \frac{\tau}{\frac{1}{2}mr^2}$$

$$\boxed{\alpha = \frac{2\tau}{mr^2}}$$

Putting the values

$$\alpha = \frac{2 \times 0.12}{5.0 \times (0.20)^2}$$

$$= 1.2 \text{ rad/s}^2$$

Result

(a) Torque acting on the cylinder $= \tau = 0.12$ N.m

(b) Angular acceleration $= \alpha = 1.2 \text{ rad/s}^2$

PROBLEM 5.5

Calculate the angular momentum of a star of mass 2.0×10^{30} kg and radius 7.0×10^5km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

Data

Mass of star $= M = 2.0 \times 10^{30}$ kg

Radius of star $= R = 7.0 \times 10^5$ km

$= 7.0 \times 10^8$ m

Time period of star $= T = 20$ days

$= 20 \times 24 \times 3600$

$= 1728000$ sec.

To Find

Angular momentum $= \vec{L} = ?$

Kinetic energy $= (K.E)_{rot} = ?$

SOLUTION

By using formula

$$L = I\omega \qquad \text{But} \qquad I = \frac{2}{5}MR^2$$

$$\text{and} \qquad \omega = \frac{2\pi}{T}$$

So $$\boxed{L = \frac{2\pi}{T} \times \frac{2}{5}MR^2}$$

$$= \frac{2(3.14)}{1728000} \times \frac{2 \times 2.0 \times 10^{30} \times (7.0 \times 10^8)^2}{5}$$

$$= \frac{1230.80 \times 10^{46}}{8640000}$$

$$= 1.42 \times 10^{46-4}$$

$$= 1.42 \times 10^{42}\ \text{J.s}$$

For rotational kinetic energy

$$(K.E)_{rot} = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} \times \frac{2}{5}MR^2 \times \left(\frac{2\pi}{T}\right)^2$$

$$= \frac{1}{5}MR^2 \times \frac{4\pi^2}{T^2}$$

$$= \frac{2.0 \times 10^{30} \times (7.0 \times 10^8)^2 \times 4(3.14)^2}{5 \times (1728000)^2}$$

$$= \frac{3864.9 \times 10^{30+16}}{1.49 \times 10^{13}}$$

$$= 2.58 \times 10^{46-10}$$

$$(K.E)_{rot} = 2.58 \times 10^{36}\ \text{J}$$

Result

Angular momentum $= \vec{L} = 1.42 \times 10^{42}\ \text{J.s}$

Kinetic energy $= (K.E)_{rot} = 2.58 \times 10^{36}\ \text{J}$

PROBLEM 5.6

A 1000 kg car traveling with a speed of 144 km h^{-1} rounds a curve of radius 100 m. Find the necessary centripetal force.

Data

Mass of car = m = 1000 kg

Speed of car = v = 144 km/h

$$= \frac{144 \times 1000}{3600} \text{ m/s}$$

$$= 40 \text{ m/s}$$

Radius of curve = r = 100 m

To Find

Centripetal force = F_c = ?

SOLUTION

By using formula

$$\boxed{F_c = \frac{mv^2}{r}}$$

$$F_c = \frac{1000 \times (40)^2}{100}$$

$$= 16000 \text{ N}$$

$$= 1.6 \times 10^4 \text{ N}$$

Result

Centripetal force = F_c = 1.6×10^4 N

PROBLEM 5.7

What is the least speed at which an aeroplane can execute a vertical loop of 1.0km radius so that there will be not tendency for the pilot to fall down at the highest point?

Data

Radius of loop = r = 1.0 km = 1000 m

Acceleration due to gravity = g = 9.8 m/s^2

To Find

Least speed of aeroplane = v = ?

SOLUTION

By formula

$$\boxed{v = \sqrt{gr}}$$

$$= \sqrt{9.8 \times 1000}$$

$$= \sqrt{9800}$$

$$= 98.9 \text{ m/s}$$

$$= 99 \text{ m/s}$$

Result

Least speed of aeroplane $= v = 99$ m/s

PROBLEM 5.8

The Moon orbit the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (In this case, treat Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is 3.85×10^8 m. Radius of the Moon is 1.74×10^6 m.

Data

Distance between earth and moon $= r = 3.85 \times 10^8$ m

Radius of moon $= R_m = 1.74 \times 10^6$ m

To Find

Ratio $= \dfrac{L_s}{L_o} = ?$

SOLUTION

The spin angular momentum of the moon is

$$L_s = I\omega \quad \text{where} \quad I = \frac{2}{5}MR^2$$

$$\boxed{L_s = \frac{2}{5}MR_m^2\omega} \qquad \ldots\ldots \text{(i)}$$

The orbital angular momentum is given by

$$\boxed{L_o = Mr^2\omega} \qquad \ldots\ldots \text{(ii)}$$

where $MR^2 = I$ (moment of inertia)

Divide eq. (i) by (ii)

$$\frac{L_s}{L_o} = \frac{\frac{2}{5}MR_m^2\omega}{Mr^2\omega}$$

$$\boxed{\frac{L_s}{L_o} = \frac{2R_m^2}{5r^2}}$$

$$= \frac{2 \times (1.74 \times 10^6)^2}{5 \times (3.85 \times 10^8)^2}$$

$$= \frac{6.05 \times 10^{12}}{74.11 \times 10^{16}}$$

$$= 0.0816 \times 10^{12-16}$$

$$= 0.0816 \times 10^{-4}$$

$$\frac{L_s}{L_o} = 8.2 \times 10^{-6}$$

Result

Ratio $= \frac{L_s}{L_o} = 8.2 \times 10^{-6}$

PROBLEM 5.9

The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then? (For sphere I = 2/5 MR^2).

Data

Time period $= T_1 = 1$ day

$= 24$ hr.

Moment of inertia $= I = \frac{2}{5} MR^2$

To Find

Time period $= T_2 = ?$

SOLUTION

According to law of angular momentum

$$\boxed{I_1\omega_1 = I_2\omega_2} \qquad \ldots\ldots (i)$$

where $I_1 = \frac{2}{5} MR_1^2$ According to question

$I_2 = \frac{2}{5} MR_2^2$ $R_2 = \frac{R_1}{2}$

$$I_2 = \frac{2}{5} M \left(\frac{R_1}{2}\right)^2$$

$$I_2 = \frac{2}{5} M \frac{R_1^2}{4} = \frac{1}{10} MR_1^2$$

and $\omega_1 = \frac{2\pi}{T_1}$ and $\omega_2 = \frac{2\pi}{T_2}$

Putting in eq. (i)

$$\frac{2}{5} MR_1^2 \times \frac{2\pi}{T_1} = \frac{1}{10} MR_1^2 \times \frac{2\pi}{T_2}$$

$$\frac{2}{5T_1} = \frac{1}{10T_2}$$

$$20T_2 = 5T_1$$

$$\boxed{T_2 = \frac{5T_1}{20}}$$

$$T_2 = \frac{5T_1}{20}$$

$$= \frac{5 \times 24}{20}$$

$$T_2 = 6 \text{ hr.}$$

Result

Earth completes its rotation = T_2 = 6 hr.

PROBLEM 5.10

What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24} kg and its radius as 6400 km).

Data

Radius of earth = R = 6400 km

= 6400 × 1000

= 6400000 m

Height of circular orbit = h = 900 km

= 900 × 1000

= 900000 m

Mass of earth = M = 6.0×10^{24} kg

To Find

Orbit speed = v = ?

SOLUTION

By formula

$$v = \sqrt{\frac{GM}{r}}$$

where $r = h + R$

$= 6400 + 900$

$= 7300 \text{ km}$

$= 7300 \times 1000$

$= 7300000 \text{ m} = 7300 \times 10^3 \text{ m}$

Then $v = \sqrt{\frac{6.63 \times 10^{-11} \times 6.0 \times 10^{24}}{7300 \times 10^3}}$

$= \sqrt{5.44 \times 10^{-11-3-3+24}}$

$= \sqrt{5.44 \times 10^7}$

$= \sqrt{54.4 \times 10^6}$

$= 7.37 \times 10^3 \text{ m/s}$

$= 7.4 \times 10^3 \text{ m/s}$

Result

Orbital speed $= v = 7.4 \text{ km/s}$