

Chapter 1

NUMBER SYSTEMS

The set of real numbers can be written as $R = Q \cup Q'$ where Q is the set of rational numbers and Q' is the set of irrational numbers.

RATIONAL NUMBER

Rational number is a number which can be put in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z} \wedge q \neq 0$.

Thus $\sqrt{16}, \frac{3}{4}, 2.7$ are rational numbers.

Decimal Representation of Rational Numbers

(1) Terminating Decimals

A decimal which has only a finite number of digits in its decimal part, is called a terminating decimal. Terminating decimal represents a rational number. Thus 3.7, 0.0005, 207.9 are rational numbers.

(2) Recurring Decimals

Recurring decimal is a decimal in which one or more digits repeat indefinitely. Every recurring decimal represents a rational number. Thus 0.333....., 1.57575757..... are rational numbers.

Irrational Number

Irrational number is a number which can not be put into the form $\frac{p}{q}$ where $p, q \in \mathbb{Z} \wedge q \neq 0$. Thus $\sqrt{2}, \sqrt{3}, \sqrt{\frac{5}{6}}$ are irrational numbers.

Note: A non terminating, non recurring decimal represents an irrational number.

Binary Operation

A binary operation in a set A is a rule usually denoted by $*$ that assigns to any pair of elements of A , taken in a definite order, another element of A .

Properties of Real Numbers

1. Addition Laws:

(i) Closure Law of Addition

$$\forall a, b \in \mathbb{R} \quad a + b \in \mathbb{R}$$

(ii) Associative Law of Addition

$$\forall a, b, c \in \mathbb{R}, \quad a + (b + c) = (a + b) + c$$

(iii) Additive Identity

$$\forall a \in \mathbb{R}, \quad \exists 0 \in \mathbb{R} \text{ such that } a + 0 = 0 + a = a$$

(v) Commutative Law for Addition

$$\forall a, b \in \mathbb{R}, \quad a + b = b + a$$

2. Multiplication Laws:

(vi) Closure Law of Multiplication

$$\forall a, b \in \mathbb{R}, \quad a \cdot b \in \mathbb{R}$$

(vii) Associative Law for Multiplication

$$\forall a, b, c \in \mathbb{R}, \quad a(bc) = (ab)c$$

(viii) Multiplicative Identity

$$\forall a \in \mathbb{R}, \quad \exists 1 \in \mathbb{R} \text{ such that } a \cdot 1 = 1 \cdot a = a$$

(ix) Multiplicative Inverse

$$\forall a (\neq 0) \in \mathbb{R}, \quad \exists a^{-1} \in \mathbb{R} \text{ such that } a \cdot a^{-1} = a^{-1} \cdot a = 1$$

(x) Commutative Law of Multiplication

$$\forall a, b \in \mathbb{R}, \quad ab = ba$$

3. Multiplication – Addition Law

(xi) Distributive Property

$$\forall a, b, c \in \mathbb{R}$$

$$a(b + c) = ab + ac \quad (\text{left distributive property})$$

$$(a + b)c = ac + bc \quad (\text{Right distributive property})$$

Any set possessing all the above 11 properties is called a field.

Any set possessing above 11 properties is called a field.

4. Properties of Equality

- (i) Reflexive property: $\forall a \in \mathbb{R}, a = a$
- (ii) Symmetric property: $\forall a, b \in \mathbb{R}, a = b \Rightarrow b = a$
- (iii) Transitive property: $\forall a, b, c \in \mathbb{R}, a = b \wedge b = c \Rightarrow a = c$
- (iv) Additive property: $\forall a, b, c \in \mathbb{R}, a = b \Rightarrow a + c = b + c \wedge c + a = b + c$
- (v) Multiplicative property: $\forall a, b, c \in \mathbb{R}, a = b \Rightarrow ac = bc \wedge ca = cb$
- (vi) Cancellation property w.r.t. addition: $\forall a, b, c \in \mathbb{R}, a + c = b + c \Rightarrow a = b$
- (vii) Cancellation property w.r.t. Multiplication: $\forall a, b, c \in \mathbb{R}, ac = bc \Rightarrow a = b, c \neq 0$

5. Properties of Inequalities

Trichotomy Property

$$\forall a, b \in \mathbb{R} \text{ either } a = b \text{ or } a > b \text{ or } a < b$$

Transitive Property

$$\forall a, b, c \in \mathbb{R}$$

- (i) $a > b \wedge b > c \Rightarrow a > c$
- (ii) $a < b \wedge b < c \Rightarrow a < c$

Additive Property

$$\forall a, b, c \in \mathbb{R}$$

- (a) (i) $a > b \Rightarrow a + c > b + c$ (ii) $a < b \Rightarrow a + c < b + c$
- (b) (i) $a > b \wedge c > d \Rightarrow a + c > b + d$ (ii) $a < b \wedge c < d \Rightarrow a + c < b + d$

Multiplicative Properties

(a) $\forall a, b, c \in \mathbb{R} \text{ and } c > 0$

- (i) $a > b \Rightarrow ac > bc$ (ii) $a < b \Rightarrow ac < bc$

(b) $\forall a, b, c \in \mathbb{R} \text{ and } c < 0$

- (i) $a > b \Rightarrow ac < bc$ (ii) $a < b \Rightarrow ac > bc$

(c) $\forall a, b, c, d \in \mathbb{R} \text{ and } a, b, c, d \text{ are all positive,}$

- (i) $a > b \wedge c > d \Rightarrow ac > bd$ (ii) $a < b \wedge c < d \Rightarrow ac < bd$

EXERCISE 1.1

Q.1 Which of the following sets have closure property w.r.t. addition and multiplication

(i) $\{0\}$

The set is closed w.r.t. addition because $0 + 0 = 0 \in \{0\}$

The set is closed w.r.t. multiplication because $0.0 = 0 \in \{0\}$

(ii) $\{1\}$

The set is not closed w.r.t. addition because $1 + 1 = 2 \notin \{1\}$

The set is closed w.r.t. multiplication because $1.1 = 1 \in \{1\}$

(iii) $\{0, -1\}$

+	0	-1
0	0	-1
-1	-1	-2

The set is not closed w.r.t. addition because $-2 \notin \{0, -1\}$

•	0	-1
0	0	0
-1	0	1

The set is not closed w.r.t. multiplication because $1 \notin \{0, -1\}$

(iv) $\{1, -1\}$

+	1	-1
1	2	0
-1	0	-2

The set is not closed w.r.t. addition because $-2, 0, 2 \notin \{-1, 1\}$

•	1	-1
1	1	-1
-1	-1	1

The set is closed w.r.t. multiplication.

Q.2 Name the properties used in the following equations (letters, where used, represents real numbers)

Solution:

(i) $4 + 9 = 9 + 4$

Commutative property w.r.t. '+'

$=$

Associative property w.r.t. '+'

(ii)	$(a + 1) + \frac{3}{4} = a + \left(1 + \frac{3}{4}\right)$	Associative property w.r.t. ‘+’
(iii)	$(\sqrt{3} + \sqrt{5}) + \sqrt{7} = \sqrt{3} + (\sqrt{5} + \sqrt{7})$	Associative property w.r.t. ‘+’
(iv)	$100 + 0 = 100$	Additive Identity
(v)	$100 \times 1 = 100$	Multiplicative Identity
(vi)	$4.1 + (-4.1) = 0$	Additive Inverse
(vii)	$a - a = 0$	Additive Inverse.
(viii)	$\sqrt{2} \times \sqrt{5} = \sqrt{5} \times \sqrt{2}$	Commutative property w.r.t. ‘.’
(ix)	$a(b - c) = ab - ac$	Left distributive property.
(x)	$(x - y)z = xz - yz$	Right distributive property.
(xi)	$4 \times (5 \times 8) = (4 \times 5) \times 8$	Associative property w.r.t. ‘.’
(xii)	$a(b + c - d) = ab + ac - ad$	Left distributive property

Q.3 Name the properties used in the following inequalities.

Solution:

(i)	$-3 < -2 \Rightarrow 0 < 1$	Additive property.
(ii)	$-5 < -4 \Rightarrow 20 > 16$	Multiplication property.
(iii)	$1 > -1 \Rightarrow -3 > -5$	Additive property.
(iv)	$a < 0 \Rightarrow -a > 0$	Multiplicative property.
(v)	$a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$	Multiplicative property.
(vi)	$a > b \Rightarrow -a < -b$	Multiplicative property.

Q.4 Prove the following Rules of Addition

(i) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

Solution:

L.H.S

$$= \frac{a}{c} + \frac{b}{c}$$

$$= a \cdot \frac{1}{c} + b \cdot \frac{1}{c} \quad \simeq \quad \frac{a}{b} = a \cdot \frac{1}{b}$$

$$= (a + b) \cdot \frac{1}{c} \quad \text{Distributive property}$$

$$= \frac{a+b}{c} \quad \simeq \quad a \cdot \frac{1}{c} = \frac{a}{c}$$

$$(ii) \quad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Solution:

$$\text{L.H.S.} = \frac{a}{b} + \frac{c}{d}$$

$$= \frac{a}{b} \cdot 1 + \frac{c}{d} \cdot 1$$

Multiplicative Identity

$$= \frac{a}{b} \cdot \left(d \cdot \frac{1}{d} \right) + \frac{c}{d} \cdot \left(b \cdot \frac{1}{b} \right)$$

Multiplicative Inverse

$$= \frac{a}{b} \cdot \frac{d}{d} + \frac{c}{d} \cdot \frac{b}{b}$$

$$\because d \cdot \frac{1}{d} = \frac{d}{d}, \quad b \cdot \frac{1}{b} = \frac{b}{b}$$

$$= \frac{ad}{bd} + \frac{cb}{db}$$

$$= \frac{ad}{bd} + \frac{bc}{bd}$$

Commutative Property w.r.t. ‘.’

$$= ad \cdot \frac{1}{bd} + bc \cdot \frac{1}{bd}$$

$$\because \frac{a}{b} = a \cdot \frac{1}{b}$$

$$= (ad + bc) \cdot \frac{1}{bd}$$

Distributive Property

$$= \frac{ad + bc}{bd}$$

$$\because a \cdot \frac{1}{b} = \frac{a}{b}$$

$$= \text{R.H.S.}$$

Q.5 Prove that $-\frac{7}{12} - \frac{5}{18} = \frac{-21 - 10}{36}$

Solution:

$$\text{L.H.S.} = -\frac{7}{12} - \frac{5}{18}$$

$$= -\frac{7}{12} \cdot 1 - \frac{5}{18} \cdot 1$$

Multiplicative Identity

$$= -\frac{7}{12} \cdot \left(3 \cdot \frac{1}{3} \right) - \frac{5}{18} \cdot \left(2 \cdot \frac{1}{2} \right)$$

Multiplicative Inverse

$$= -\frac{7}{12} \cdot \frac{3}{3} - \frac{5}{18} \cdot \frac{2}{2}$$

$$\because a \cdot \frac{1}{b} = \frac{a}{b}$$

$$= -\frac{21}{36} - \frac{10}{36}$$

$$= -21 \cdot \frac{1}{36} - 10 \cdot \frac{1}{36}$$

$$= (-21 - 10) \cdot \frac{1}{36}$$

$$= \frac{-21 - 10}{36}$$

$$= \text{R.H.S.}$$

$$\therefore \frac{a}{b} = a \cdot \frac{1}{b}$$

Distributive Property

Q.6 Simplify by justifying each step:

(i) $\frac{4 + 16x}{4}$

Solution:

$$\frac{4 + 16x}{4}$$

$$= \frac{1}{4} \cdot (4 + 16x)$$

$$= \frac{1}{4} \cdot 4 + \frac{1}{4} \cdot 16x$$

$$= \frac{1}{4} \cdot 4 \cdot 4x$$

$$= 1 + 1 \cdot 4x$$

$$= 1 + 4x$$

$$\therefore \frac{a}{b} = \frac{1}{b} \cdot a$$

Distributive Property

Multiplicative Inverse

Multiplicative Inverse

Multiplicative Identity

(ii) $\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}$

Solution:

$$\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}}$$

$$= \left(\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}} \right) \cdot 1$$

Multiplicative Identity

$$= \left(\frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{4} - \frac{1}{5}} \right) \cdot 20 \cdot \frac{1}{20}$$

Multiplicative Inverse

$$= \frac{\left(\frac{1}{4} + \frac{1}{5} \right) \cdot 20}{\left(\frac{1}{4} - \frac{1}{5} \right) \cdot 20}$$

$$\therefore 20 \cdot \frac{1}{20} = \frac{20}{20}$$

$$= \frac{\frac{1}{4} \cdot 20 + \frac{1}{5} \cdot 20}{\frac{1}{4} \cdot 20 + \frac{1}{5} \cdot 20}$$

Distributive Property

$$= \frac{\left(\frac{1}{4} \cdot 4 \right) 5 + \left(\frac{1}{5} \cdot 5 \right) 4}{\frac{1}{4} \cdot 4 \cdot 5 + \frac{1}{5} \cdot 5 \cdot 4} = \frac{1 \cdot 5 + 1 \cdot 4}{1 \cdot 5 - 1 \cdot 4}$$

Multiplicative Inverse

$$= \frac{5 + 4}{5 - 4}$$

Multiplicative Identity

$$= \frac{9}{1} = 9$$

(iii)

$$\frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}}$$

$$= \left(\frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}} \right) \cdot 1$$

Multiplicative Identity

$$= \left(\frac{\frac{a}{b} + \frac{c}{d}}{\frac{a}{b} - \frac{c}{d}} \right) \cdot bd \cdot \frac{1}{bd}$$

Multiplicative Inverse

$$= \frac{\left(\frac{a}{b} + \frac{c}{d} \right) \cdot bd}{\left(\frac{a}{b} - \frac{c}{d} \right) \cdot bd}$$

$$\therefore bd \cdot \frac{1}{bd} = \frac{bd}{bd}$$

$$= \frac{\frac{a}{b} \cdot bd + \frac{c}{d} \cdot bd}{\frac{a}{b} \cdot bd - \frac{c}{d} \cdot bd}$$

Distributive Property

$$= \frac{a \cdot \frac{1}{b} \cdot b \cdot d + c \cdot \frac{1}{d} \cdot b \cdot d}{a \frac{1}{b} \cdot b \cdot d - c \frac{1}{d} \cdot b \cdot d}$$

$$= \frac{a \left(\frac{1}{b} \cdot b \right) d + c \cdot \left(\frac{1}{d} \cdot d \right) b}{a \left(\frac{1}{b} \cdot b \right) d - c \left(\frac{1}{d} \cdot d \right) b}$$

$$= \frac{a \cdot 1 \cdot d + c \cdot 1 \cdot b}{a \cdot 1 \cdot d - c \cdot 1 \cdot b}$$

$$= \frac{ad + cb}{ad - cb}$$

(iv)
$$\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}}$$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}} \right) \cdot 1$$

$$= \left(\frac{\frac{1}{a} - \frac{1}{b}}{1 - \frac{1}{a} \cdot \frac{1}{b}} \right) \cdot ab \cdot \frac{1}{ab}$$

$$= \frac{\left(\frac{1}{a} - \frac{1}{b} \right) \cdot ab}{\left(1 - \frac{1}{a} \cdot \frac{1}{b} \right) \cdot ab}$$

$$= \frac{\frac{1}{a} \cdot ab - \frac{1}{b} \cdot ab}{ab - \frac{1}{a} \cdot \frac{1}{b} \cdot ab}$$

$$= \frac{\left(\frac{1}{a} \cdot a \right) b - \left(\frac{1}{b} \cdot b \right) a}{ab - \left(\frac{1}{a} \cdot a \right) \left(\frac{1}{b} \cdot b \right)}$$

$$= \frac{1 \cdot b - 1 \cdot a}{ab - 1 \cdot 1}$$

$$= \frac{b - a}{ab - 1} \quad \text{Multiplicative Identity}$$

$$= \quad \text{Multiplicative Identity}$$

$$\therefore \frac{1}{b} = a \cdot \frac{1}{b} \quad , \quad \frac{c}{d} = c \cdot \frac{1}{d}$$

Commutative Property

Multiplicative Inverse

Multiplicative Identity

Multiplicative Identity

Multiplicative Inverse

$$\therefore ab \cdot \frac{1}{ab} = \frac{ab}{ab} \cdot$$

Distributive Property

Commutative Property

Multiplicative Inverse

EXERCISE 1.2

Q.1 Verify the addition properties of complex numbers.

Solution:

Addition properties of complex numbers are:

(i) Closure property

$$\forall (a, b), (c, d) \in \mathbb{C}$$

$$(a, b) + (c, d) = a + ib + c + id = a + c + i(b + d) = (a + c, b + d) \in \mathbb{C}$$

(ii) Associative Property

$$\forall (a, b), (c, d), (e, f) \in \mathbb{C}$$

$$[(a, b) + (c, d)] + (e, f) = (a, b) + [(c, d) + (e, f)]$$

L.H.S.

$$\begin{aligned} [(a, b) + (c, d)] + (e, f) &= [a + ib + c + id] + (e + if) \\ &= a + ib + c + id + e + if \\ &= a + c + e + i(b + d + f) \\ &= (a + c + e, b + d + f) \end{aligned}$$

R.H.S.

$$\begin{aligned} (a, b) + [(c, d) + (e, f)] &= (a + ib) + [c + id + e + if] \\ &= a + ib + c + id + e + if \\ &= a + c + e + i(b + d + f) \\ &= (a + c + e, b + d + f) \end{aligned}$$

$$\Rightarrow \text{L.H.S.} = \text{R.H.S.}$$

(iii) Additive Identity

$$\forall (a, b) \in \mathbb{C} \quad \exists (0, 0) \in \mathbb{C}$$

$$\text{such that } (a, b) + (0, 0) = (0, 0) + (a, b) = (a, b)$$

(iv) Additive Inverse

$$\forall (a, b) \in \mathbb{C} \quad \exists (-a, -b) \in \mathbb{C}$$

$$\text{such that } (a, b) + (-a, -b) = (0, 0) = (-a, -b) + (a, b)$$

(v) Commutative Property

$$\forall (a, b), (c, d) \in \mathbb{C}$$

$$\begin{aligned} (a, b) + (c, d) &= a + ib + c + id \\ &= a + c + i(b + d) \\ &= c + a + i(d + b) = (c + id) + (a + ib) = (c, d) + (a, b) \end{aligned}$$

Q.2 Verify the multiplication properties of complex numbers.**Solution:**

Multiplication Properties of complex numbers are:

Closure property

$$\forall (a, b), (c, d) \in \mathbb{C}$$

$$\begin{aligned} (a, b) \cdot (c, d) &= (a + ib)(c + id) = ac + aid + ibc + i^2bd \\ &= ac - bd + i(ad + bc) = (ac - bd, ad + bc) \in \mathbb{C} \end{aligned}$$

Associative Property

$$\forall (a, b), (c, d), (e, f) \in \mathbb{C}$$

$$[(a, b) \cdot (c, d)] \cdot (e, f) = (a, b) [(c, d) \cdot (e, f)]$$

L.H.S.

$$\begin{aligned} [(a, b) \cdot (c, d)] \cdot (e, f) &= [(a + ib)(c + id)] \cdot (e + if) \\ &= [ac + iad + ibc + i^2bd] \cdot (e + if) \\ &= [ac - bd + iad + ibc] \cdot (e + if) \\ &= ace + iacf - bde - ibdf + iade + i^2adf + ibce + i^2bcf \\ &= ace - adf - bcf - bde + i(acf - bdf + ade + bce) \end{aligned}$$

Now R.H.S.

$$\begin{aligned} (a, b) [(c, d) \cdot (e, f)] &= (a + ib) [(c + id)(e + if)] \\ &= (a + ib) [ce + icf + ide + i^2df] \\ &= (a + ib) [ce - df + icf + ide] \\ &= ace - adf + iacf + iade + ibce - ibdf + i^2bcf + i^2bde \\ &= ace - adf - bcf - bde + i(acf - bdf + ade + bce) \end{aligned}$$

$$\Rightarrow \text{L.H.S.} = \text{R.H.S.}$$

Multiplicative Identity

$\forall (a, b) \in \mathbb{C} \quad \exists (1, 0) \in \mathbb{C}$ such that

$$(a, b) \cdot (1, 0) = (a + ib) \cdot (1 + 0i) = a + 0i + ib + 0 = a + ib = (a, b)$$

and

$$(1, 0) (a, b) = (1 + 0i) \cdot (a + ib) = a + ib + 0i + 0 = a + ib = (a, b)$$

$\Rightarrow (1, 0)$ is multiplicative identity in $\mathbb{C}$.

Multiplicative Inverse

$\forall (a + ib) \in \mathbb{C} \quad \exists (a + ib)^{-1} \in \mathbb{C}$

where

$$\begin{aligned}(a + ib)^{-1} &= \frac{1}{a + ib} = \frac{1}{a + ib} \cdot \frac{a - ib}{a - ib} \\&= \frac{a - ib}{(a)^2 - (ib)^2} = \frac{a - ib}{a^2 - i^2 b^2} = \frac{a - ib}{a^2 + b^2} = \frac{a}{a^2 + b^2} - i \frac{b}{a^2 + b^2} \\&= \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)\end{aligned}$$

Commutative Property

$\forall (a, b), (c, d) \in \mathbb{C}$

$$(a, b) \cdot (c, d) = (c, d) \cdot (a, b)$$

L.H.S.

$$\begin{aligned}(a, b) \cdot (c, d) &= (a + ib)(c + id) = ac + iad + ibc + i^2 bd \\&= (ac - bd) + i(ad + bc)\end{aligned}$$

R.H.S.

$$\begin{aligned}(c, d) \cdot (a, b) &= (c + id)(a + ib) = ca + icb + ida + i^2 db \\&= (ac - bd) + i(bc + ad)\end{aligned}$$

$\Rightarrow$ L.H.S. = R.H.S

Q.3 Verify the distributive law of complex numbers.

$$(a, b) [(c, d) + (e, f)] = (a, b) (c, d) + (a, b) (e, f)$$

Solution:

To show

$$(a, b) [(c, d) + (e, f)] = (a, b) (c, d) + (a, b) (e, f)$$

L.H.S.

$$(a, b) [(c, d) + (e, f)]$$

$$\begin{aligned}&= (a + ib)[c + id + e + if] = ac + iad + ae + iaf + ibc + i^2 bd + ibe + i^2 bf \quad \because i^2 = -1 \\&= ac + ae - bd - bf + i(ad + af + bc + be)\end{aligned}$$

R.H.S.

$$(a, b) (c, d) + (a, b) (e, f)$$

$$= (a + ib) (c + id) + (a + ib) (e + if)$$

$$= ac + iad + ibc + i^2 bd + ae + iaf + ibe + i^2 bf \quad \because i^2 = -1$$

$$= ac + ae - bd - bf + i (ad + bc + af + be)$$

L. H. S. = R.H.S.

Hence Proved.

Q.4 Simplify the following:

(i) i^9

Solution:

$$i^9 = i^8 \cdot i = (i^2)^4 \cdot i = (-1)^4 \cdot i = (1) (i) = i \quad \because i^2 = -1$$

(ii) i^{14}

Solution:

$$i^{14} = (i^2)^7 = (-1)^7 = -1 \quad \because i^2 = -1$$

(iii) $(-i)^{19}$

(Lahore Board 2004)

Solution:

$$(-i)^{19} = (-1) i^{19} = -(1) i^{18} \cdot i = -(i^2)^9 \cdot i = -(-1)^9 i = (-1) \cdot i = i \quad \because i^2 = -1$$

(iv) $(-1)^{-21/2}$

(Lahore Board 2007)

Solution:

$$(-1)^{-21/2} = [(-1)^{1/2}]^{-21} = [(i^2)^{1/2}]^{-21} = i^{-21}$$

$$= \frac{1}{i^{21}} = \frac{1}{i^{20} \cdot i} = \frac{1}{(i^2)^{10} \cdot i}$$

$$= \frac{1}{(-1)^{10} i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i \quad \because i^2 = -1$$

Q.5 Write in terms of i

Solution:

(i) $\sqrt{-1} b$

Solution:

$$\sqrt{-1} b = ib \quad \because \sqrt{-1} = i$$

(ii) $\sqrt{-5}$

Solution:

$$= \sqrt{-1} \cdot \sqrt{5} = i\sqrt{5} \because \sqrt{-1} = i$$

$$(iii) \sqrt{\frac{-16}{25}}$$

Solution:

$$\sqrt{\frac{-16}{25}} = \sqrt{-1} \sqrt{\frac{16}{25}} = i \left(\frac{4}{5} \right) = \frac{4}{5} i \quad \because \sqrt{-1} = i$$

$$(iv) \sqrt{\frac{1}{-4}}$$

Solution:

$$\sqrt{\frac{1}{-4}} = \frac{1}{\sqrt{-1}} \sqrt{\frac{1}{4}} = \frac{1}{i} \cdot \frac{1}{2} = \frac{i}{i \cdot i^2} \frac{1}{2} = \frac{i}{i^2 \cdot 2} = \frac{+i}{-2} = \frac{i}{2} \quad \because i^2 = -1$$

Q.6 Simplify the following:

$$(7, 9) + (3, -5)$$

Solution:

$$(7, 9) + (3, -5) = 7 + 9i + 3 - 5i = 10 + 4i = (10, 4)$$

$$\text{Q.7 } (8, -5) - (-7, 4)$$

Solution:

$$(8, -5) - (-7, 4) = 8 - 5i - (-7 + 4i) = 8 - 5i + 7 - 4i = 15 - 9i = (15, -9)$$

$$\text{Q.8 } (2, 6) (3, 7)$$

Solution:

$$\begin{aligned} (2, 6) (3, 7) &= (2 + 6i) \cdot (3 + 7i) = 6 + 14i + 18i + 42 i^2 \\ &= 6 + 32i - 42 = -36 + 32i = (-36, 32) \quad \because i^2 = -1 \end{aligned}$$

$$\text{Q.9 } (5, -4) (-3, -2)$$

Solution:

$$\begin{aligned} (5, -4) (-3, -2) &= (5 - 4i) (-3 - 2i) \\ &= -15 - 10i + 12i + 8i^2 = -15 + 2i - 8 = -23 + 2i = (-23, 2) \end{aligned}$$

$$\text{Q.10 } (0, 3) (0, 5)$$

Solution:

$$\begin{aligned} (0, 3) (0, 5) &= (0 + 3i) (0 + 5i) \\ &= 0 + 0 + 0 + 15i^2 = 0 + 15(-1) = -15 = -15 + 0i = (-15, 0) \end{aligned}$$

$$\text{Q.11 } (2, 6) \div (3, 7)$$

Solution:

$$\begin{aligned} (2, 6) \div (3, 7) &= (2 + 6i) \div (3 + 7i) = \frac{(2 + 6i)}{(3 + 7i)} \times \frac{(3 - 7i)}{(3 - 7i)} \quad \text{Rationalizing} \\ &= \frac{(2 + 6i)(3 - 7i)}{(3 + 7i)(3 - 7i)} = \frac{6 - 14i + 18i - 42 i^2}{(3)^2 - (7i)^2} \quad \because i^2 = -1 \\ &= \frac{6 + 4i + 42}{9 - 49i^2} = \frac{48 + 4i}{9 + 49} = \frac{48 + 4i}{58} \\ &= = = = = \end{aligned}$$

$$= \left(\frac{48}{58}, \frac{4i}{58} \right) = \left(\frac{24}{29}, \frac{2i}{29} \right)$$

Q.12 $(5, -4) \div (-3, -8)$

Solution:

$$\begin{aligned} (5, -4) \div (-3, -8) &= (5 - 4i) \div (-3 - 8i) \\ &= \frac{5 - 4i}{-3 - 8i} \times \frac{-3 + 8i}{-3 + 8i} \quad \text{Rationalizing} \\ &= \frac{(5 - 4i)(-3 + 8i)}{(-3 - 8i)(-3 + 8i)} = \frac{-15 + 40i + 12i - 32i^2}{(-3)^2 - (8i)^2} \\ &= \frac{-15 + 52i + 32}{9 - 64i^2} = \frac{17 + 52i}{9 + 64} = \frac{17 + 52i}{73} \\ &= \frac{17}{73} + \frac{52}{73}i = \left(\frac{17}{73}, \frac{52}{73} \right) \end{aligned}$$

Q.13 Prove that the sum as well as product of any two conjugate complex numbers is a real number.

Solution:

Let $z = a + bi$ is a complex number then its conjugate is $\bar{z} = \overline{a + ib} = a - ib$

Sum = $z + \bar{z} = a + ib + a - ib = 2a$ (real number)

Product = $z \cdot \bar{z} = (a + ib)(a - ib) = (a)^2 - (ib)^2 = a^2 - i^2 b^2 \quad \because i^2 = -1$
 $= a^2 + b^2$ (real number)

Hence the sum as well as the product of any two conjugate complex numbers is a real number.

Q.14 Find the multiplicative inverse of each of the following numbers.

(i) $(-4, 7)$

(Lahore Board 2007)

Solution:

(i) $(-4, 7)$

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$

$$\begin{aligned} \text{So multiplicative inverse of } (-4, 7) &= \left(\frac{-4}{(-4)^2 + (7)^2}, \frac{-7}{(-4)^2 + (7)^2} \right) \\ &= \left(\frac{-4}{16 + 49}, \frac{-7}{16 + 49} \right) = \left(\frac{-4}{65}, \frac{-7}{65} \right) \end{aligned}$$

(ii) $(\sqrt{2}, -\sqrt{5})$

Solution:

As multiplicative inverse of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$

$$\text{So multiplicative inverse of } (\sqrt{2}, -\sqrt{5}) = \left(\frac{\sqrt{2}}{(\sqrt{2})^2 + (-\sqrt{5})^2}, \frac{+\sqrt{5}}{(\sqrt{2})^2 + (-\sqrt{5})^2} \right)$$

$$= \left(\frac{\sqrt{2}}{2+5}, \frac{\sqrt{5}}{2+5} \right) = \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7} \right)$$

(iii) (1, 0)

As multiplicative inverse of (a, b) = $\left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$

So multiplicative inverse of (1, 0) = $\left(\frac{1}{(1)^2 + (0)^2}, \frac{-0}{(1)^2 + (0)^2} \right)$

$$= \left(\frac{1}{1+0}, 0 \right) = (1, 0)$$

Q.15 Factorize the following:

(i) $a^2 + 4b^2$

Solution:

$$\begin{aligned} a^2 + 4b^2 \\ = a^2 - (-4b^2) &= (a^2) - (i^2 4b^2) \quad \because i^2 = -1 \\ &= (a)^2 - (2bi)^2 = (a + 2bi)(a - 2bi) \end{aligned}$$

(ii) $9a^2 + 16b^2$

(Lahore Board 2006)

Solution:

$$\begin{aligned} 9a^2 + 16b^2 &= 9a^2 - (-16b^2) \\ &= 9a^2 - (i^2 16b^2) \quad \because i^2 = -1 \\ &= (3a)^2 - (4bi)^2 = (3a + 4bi)(3a - 4bi) \end{aligned}$$

(iii) $3x^2 + 3y^2$

(Gujranwala Board 2007)

Solution:

$$\begin{aligned} 3x^2 + 3y^2 &= 3[x^2 - (-y)^2] = 3[x^2 - (i^2 y^2)] \\ &= 3[(x)^2 - (iy)^2] = 3(x + iy)(x - iy) \end{aligned}$$

Q.16 Separate into real and imaginary parts (write as a simple complex number)

(i) $\frac{2-7i}{4+5i}$

(Lahore Board 2011)

Solution:

$$\begin{aligned} \frac{2-7i}{4+5i} &= \frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i} \quad \text{Rationalizing} \\ &= \frac{(2-7i)(4-5i)}{(4+5i)(4-5i)} = \frac{8-10i-28i+35i^2}{(4)^2-(5i)^2} \quad \because i^2 = -1 \end{aligned}$$

$$= \frac{8 - 38i - 35}{16 - 25i^2} = \frac{-27 - 38i}{16 + 25}$$

$$= \frac{-27 - 38i}{41} = \frac{-27}{41} - \frac{38}{41}i$$

(ii) $\frac{(-2 + 3i)^2}{(1 + i)}$

(Lahore Board 2003)

Solution:

$$\begin{aligned} \frac{(-2 + 3i)^2}{(1 + i)} &= \frac{(-2)^2 + (3i)^2 + 2(-2)(3i)}{1 + i} \\ &= \frac{4 + 9i^2 - 12i}{1 + i} = \frac{4 - 9 - 12i}{1 + i} = \frac{-5 - 12i}{1 + i} \\ &= \frac{-5 - 12i}{1 + i} \times \frac{1 - i}{1 - i} \\ &= \frac{(-5 - 12i)(1 - i)}{(1 + i)(1 - i)} = \frac{-5 + 5i - 12i + 12i^2}{(1)^2 - (i)^2} \\ &= \frac{-5 + 5i - 12i - 12}{1 - i^2} = \frac{-5 - 7i - 12}{1 - (-1)} \\ &= \frac{-17 - 7i}{2} = -\frac{17}{2} - \frac{7}{2}i \end{aligned}$$

(iii) $\frac{i}{1 + i}$

(Gujranwala Board 2007)

Solution:

$$\begin{aligned} \frac{i}{1 + i} &= \frac{i}{1 + i} \times \frac{1 - i}{1 - i} = \frac{i(1 - i)}{(1 + i)(1 - i)} = \frac{i - i^2}{(1)^2 - i^2} = \frac{i - (-1)}{1 - (-1)} \\ &= \frac{i + 1}{1 + 1} = \frac{i + 1}{2} \\ &= \frac{i}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}i \end{aligned}$$

GEOMETRICAL REPRESENTATION OF COMPLEX NUMBERS

The Complex Plane:

EXERCISE 1.3

Q.1 Graph the following numbers on the complex plane:

(i) $2 + 3i = (2, 3)$

(ii) $2 - 3i = (2, -3)$

(iii) $-2 - 3i = (-2, -3)$

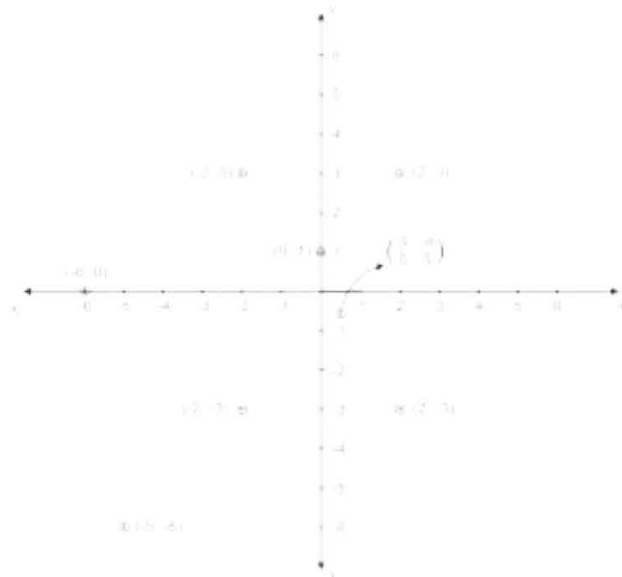
(iv) $-2 + 3i = (-2, 3)$

(v) $-6 = (-6, 0)$

(vi) $i = (0, 1)$

(vii) $\frac{3}{5} - \frac{4}{5}i = \left(\frac{3}{5}, -\frac{4}{5}\right) = (0.6, -0.8)$

(viii) $-5 - 6i = (-5, -6)$



Q.2 Find the multiplicative inverse of the following numbers:

(i) $-3i$

Solution:

$$-3i = (0 - 3)$$

$$\text{As multiplicative inverse of } (a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$$

$$\begin{aligned} \text{So multiplicative inverse of } (0, -3) &= \left(\frac{0}{(0)^2 + (-3)^2}, \frac{3}{(0)^2 + (-3)^2} \right) \\ &= \left(0, \frac{3}{9} \right) = \left(0, \frac{1}{3} \right) = 0 + \frac{1}{3}i = \frac{1}{3}i \end{aligned}$$

(ii) $1 - 2i$

(Lahore Board 2006)

Solution:

$$1 - 2i = (1, -2)$$

$$\text{As multiplicative inverse of } (a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$$

$$\begin{aligned}
 \text{So multiplicative inverse of } (1, -2) &= \left(\frac{1}{(1)^2 + (-2)^2}, \frac{-(-2)}{(1)^2 + (-2)^2} \right) \\
 &= \left(\frac{1}{1+4}, \frac{2}{1+4} \right) \\
 &= \left(\frac{1}{5}, \frac{2}{5} \right) = \frac{1}{5} + \frac{2}{5}i
 \end{aligned}$$

(iii) $-3 - 5i$

(Gujranwala Board 2004)

Solution:

$$-3 - 5i = (-3, -5)$$

$$\text{As multiplicative inverse of } (a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$$

$$\begin{aligned}
 \text{So multiplicative inverse of } (-3, -5) &= \left(\frac{-3}{(-3)^2 + (-5)^2}, \frac{-(-5)}{(-3)^2 + (-5)^2} \right) \\
 &= \left(\frac{-3}{9 + 25}, \frac{5}{9 + 25} \right) \\
 &= \left(\frac{-3}{34}, \frac{5}{34} \right) = \frac{-3}{34} + \frac{5}{34}i
 \end{aligned}$$

(iv) $(1, 2)$

Solution:

$$\text{As multiplicative inverse of } (a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$$

$$\begin{aligned}
 \text{So multiplicative inverse of } (1, 2) &= \left(\frac{1}{(1)^2 + (2)^2}, \frac{-2}{(1)^2 + (2)^2} \right) \\
 &= \left(\frac{1}{1+4}, \frac{-2}{1+4} \right) \\
 &= \left(\frac{1}{5}, \frac{-2}{5} \right)
 \end{aligned}$$

Q.3 Simplify

(i) i^{101}

(Gujranwala Board 2007)

Solution:

$$i^{101} = i^{100} \cdot i = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = i \qquad \because i^2 = -1$$

(ii) $(-ai)^4, a \in \mathbf{R}$

Solution:

$$(-ai)^4 = (-1)^4 \cdot a^4 i^4 = a^4 (i^2)^2 = a^4 (-1)^2 = a^4 \qquad \because i^2 = -1$$

(iii) i^{-3}

Solution:

$$i^{-3} = \frac{1}{i^3} = \frac{1}{i^2 \cdot i} = \frac{1}{-i \cdot i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = i$$

(iv) i^{-10}

Solution:

$$i^{-10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1$$

Q.4 Prove that $\bar{z} = z$ iff z is real.

(Lahore Board 2009)

Solution:

Let $z = a + ib$ then $\bar{z} = a - ib$

Suppose $z = \bar{z}$

$$\Rightarrow a + ib = a - ib$$

$$\Rightarrow a + ib - a + ib = 0$$

$$\Rightarrow 2ib = 0 \Rightarrow b = 0 \quad \because 2i \neq 0$$

$$\Rightarrow z = a + i0$$

$$\Rightarrow z = a$$

$$\Rightarrow z \text{ is real.}$$

Now conversely;

Suppose that z is real

Such that $z = a$

then $\bar{z} = a$

$$\Rightarrow z = \bar{z}$$

Hence proved.

Q.5 Simplify by expressing in form $a + bi$

(i) $5 + 2\sqrt{-4}$

Solution:

$$5 + 2\sqrt{-4} = 5 + 2\sqrt{-1} \cdot \sqrt{4} = 5 + 2i(2) = 5 + 4i \quad \because \sqrt{-1} = i$$

(ii) $(2 + \sqrt{-3})(3 + \sqrt{-3})$

Solution:

$$\begin{aligned} (2 + \sqrt{-3})(3 + \sqrt{-3}) &= 6 + 2\sqrt{-3} + 3\sqrt{-3} + (\sqrt{-3})^2 \\ &= 6 + 2\sqrt{-1} \cdot \sqrt{3} + 3\sqrt{-1} \cdot \sqrt{3} + (-3) \\ &= 6 + 2i\sqrt{3} + 3i\sqrt{3} - 3 \\ &= 3 + 5\sqrt{3}i \end{aligned}$$

(iii) $\frac{2}{\sqrt{5} + \sqrt{-8}}$

$$\begin{aligned}
\frac{2}{\sqrt{5} + \sqrt{-8}} &= \frac{2}{\sqrt{5} + \sqrt{-8}} \cdot \frac{\sqrt{5} - \sqrt{-8}}{\sqrt{5} - \sqrt{-8}} && \text{Rationalizing} \\
&= \frac{2(\sqrt{5} - \sqrt{-8})}{(\sqrt{5})^2 - (\sqrt{-8})^2} = \frac{2\sqrt{5} - 2\sqrt{-8}}{5 - (-8)} \\
&= \frac{2\sqrt{5} - 2\sqrt{-1} \cdot \sqrt{8}}{5 + 8} = \frac{2\sqrt{5} - 2i\sqrt{8}}{13} \\
&= \frac{2\sqrt{5}}{13} - \frac{4i\sqrt{2}}{13}
\end{aligned}$$

(iv) $\frac{3}{\sqrt{6} - \sqrt{-12}}$

Solution:

$$\begin{aligned}
\frac{3}{\sqrt{6} - \sqrt{-12}} &= \frac{3}{\sqrt{6} - \sqrt{-12}} \times \frac{\sqrt{6} + \sqrt{-12}}{\sqrt{6} + \sqrt{-12}} && \text{Rationalizing} \\
&= \frac{3(\sqrt{6} + \sqrt{-12})}{(\sqrt{6})^2 - (\sqrt{-12})^2} \\
&= \frac{3\sqrt{6} + 3\sqrt{-12}}{6 - (-12)} = \frac{3\sqrt{6} + 3\sqrt{-1} \cdot \sqrt{12}}{6 + 12} \\
&= \frac{3\sqrt{6} + 3i\sqrt{12}}{18} = \frac{3\sqrt{6}}{18} + \frac{3i \cdot 2\sqrt{3}}{18} \\
&= \frac{\sqrt{6}}{6} + \frac{\sqrt{3}}{3}i = \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}i
\end{aligned}$$

Q.6 Show that $\forall z \in \mathbb{C}$

(i) $z^2 + \bar{z}^2$ is real number

Solution:

Let $z = a + bi$

Then $\bar{z} = a - bi$

Now

$$\begin{aligned}
z^2 + \bar{z}^2 &= (a + bi)^2 + (a - bi)^2 \\
&= (a)^2 + (ib)^2 + 2abi + a^2 + (ib)^2 - 2abi \\
&= a^2 + i^2b^2 + a^2 + i^2b^2 && \because i^2 = -1 \\
&= a^2 - b^2 + a^2 - b^2 = 2a^2 - 2b^2 = 2(a^2 - b^2)
\end{aligned}$$

which is real number

Hence proved.

(ii) $(z - \bar{z})^2$ is real number

(Lahore Board 2009)

Solution:

Let $z = a + bi$

then $\bar{z} = a - ib$

$$\begin{aligned}\Rightarrow (z - \bar{z})^2 &= [(a + ib) - (a - ib)]^2 \quad \because i^2 = -1 \\ &= [a + ib - a + ib]^2 = (2ib)^2 = 4i^2b^2 = -4b^2\end{aligned}$$

which is a real number.

Hence proved.

Q.7 Simplify the followings

(i) $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3$ (Gujranwala Board 2003, 2005)

Solution:

$$\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 \quad \because (a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

$$\begin{aligned}\left(\frac{-1 + \sqrt{3}i}{2}\right)^3 &= \frac{(-1 + \sqrt{3}i)^3}{(2)^3} \\ &= \frac{1}{8} [(-1)^3 + (\sqrt{3}i)^3 + 3(-1)^2(\sqrt{3}i) + 3(-1)(\sqrt{3}i)^2] \\ &= \frac{1}{8} [-1 + (\sqrt{3})^3 i^3 + 3(1)\sqrt{3}i - 3(3i^2)] \\ &= \frac{1}{8} [-1 + \sqrt{3}^3 i^2 + 3\sqrt{3}i - 3(3)(-1)] \\ &= \frac{1}{8} [-1 + \sqrt{27}(-1)i + 3\sqrt{3}i + 9] \\ &= \frac{1}{8} [-1 - 3\sqrt{3}i + 3\sqrt{3}i + 9] \\ &= \frac{1}{8} (8) = 1\end{aligned}$$

(ii) $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^3$

Solution:

$$\begin{aligned}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^3 &= \left(\frac{-1 - \sqrt{3}i}{2}\right)^3 = \left(\frac{(-1)(1 + \sqrt{3}i)}{2}\right)^3 = \frac{(-1)^3 (1 + \sqrt{3}i)^3}{2^3} \\ &= -\frac{1}{8} [(1)^3 + (\sqrt{3}i)^3 + 3(1)^2(\sqrt{3}i) + 3(1)(\sqrt{3}i)^2]\end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{8} [1 + (\sqrt{3})^3 i^3 + 3\sqrt{3}i + 3(3i^2)] \\
&= -\frac{1}{8} [1 + \sqrt{3^3} i^2 + 3\sqrt{3}i + 9(-1)] \\
&= -\frac{1}{8} [1 + \sqrt{27}(-1)i + 3\sqrt{3}i - 9] \\
&= -\frac{1}{8} [1 - 3\sqrt{3}i + 3\sqrt{3}i - 9] \\
&= -\frac{1}{8} [-8] = 1
\end{aligned}$$

(iii) $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

Solution:

$$\begin{aligned}
\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) &= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-1} \\
&= \left(\frac{-1 - \sqrt{3}i}{2}\right)^{-1} \\
&= \frac{2}{-1 - \sqrt{3}i} \cdot \frac{-1 + \sqrt{3}i}{-1 + \sqrt{3}i} \quad \text{Rationalizing} \\
&= \frac{2(-1 + \sqrt{3}i)}{(-1)^2 - (\sqrt{3}i)^2} = \frac{-2 + 2\sqrt{3}i}{1 - 3i^2} = \frac{-2 + 2\sqrt{3}i}{1 + 3} \quad \because i^2 = -1 \\
&= \frac{-2 + 2\sqrt{3}i}{4} = \frac{-2}{4} + \frac{2\sqrt{3}i}{4} = -\frac{1}{2} + \frac{\sqrt{3}i}{2}
\end{aligned}$$

(iv) $(a + bi)^2$

Solution:

$$(a + bi)^2 = (a)^2 + 2(a)(bi) + (bi)^2 = a^2 + b^2 i^2 + 2abi$$

(v) $(a + bi)^2$

Solution:

$$\begin{aligned}
(a + bi)^2 &= \frac{1}{(a + bi)^2} = \left(\frac{1}{a + bi} \cdot \frac{a - bi}{a - bi}\right)^2 \quad \text{Rationalizing} \\
&= \left(\frac{a - bi}{a^2 - b^2 i^2}\right)^2 = \left(\frac{a - bi}{a^2 + b^2}\right)^2 \quad \because i^2 = -1 \\
&= \frac{(a - bi)^2}{(a^2 + b^2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(a - bi)^2}{(a^2 + b^2)^2} \\
&= \frac{a^2 + b^2 i^2 - 2abi}{(a^2 + b^2)^2} \\
&= \frac{a^2 - b^2 - 2abi}{(a^2 + b^2)^2} \qquad \because i^2 = -1 \\
&= \frac{a^2 - b^2}{(a^2 + b^2)^2} - \frac{2abi}{(a^2 + b^2)^2}
\end{aligned}$$

(vi) $(a + bi)^3$

Solution:

$$\begin{aligned}
(a + bi)^3 &= a^3 + b^3 i^3 + 3a^2 (bi) + 3a (bi)^2 \\
&= a^3 + b^3 \cdot i^2 \cdot i + 3a^2 bi + 3ab^2 i^2 \\
&= a^3 - b^3 i + 3a^2 bi - 3ab^2 \qquad \because i^2 = -1 \\
&= a^3 - 3ab^2 + i (3a^2 b - b^3)
\end{aligned}$$

(vii) $(a - bi)^3$

Solution:

$$\begin{aligned}
(a - bi)^3 &= (a)^3 - (bi)^3 - 3(a)^2 (bi) + 3(a) (bi)^2 \\
&= a^3 - b^3 \cdot i^3 - 3a^2 bi + 3ab^2 i^2 \qquad \because (a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2 \\
&= a^3 - b^3 i^2 \cdot i - 3a^2 bi + 3ab^2 (-1) \\
&= a^3 + b^3 i - 3a^2 bi - 3ab^2 \\
&= a^3 - 3ab^2 + i (b^3 - 3a^2 b)
\end{aligned}$$

(viii) $(3 - \sqrt{-4})^{-3}$

Solution:

$$\begin{aligned}
(3 - \sqrt{-4})^{-3} &= (3 - \sqrt{4} \cdot \sqrt{-1})^{-3} \\
&= (3 - i 2)^{-3} = (3 - 2i)^{-3} \\
&= \frac{1}{(3 - 2i)^3} = \left(\frac{1}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i} \right)^3 \qquad \text{Rationalizing} \\
&= \left(\frac{3 + 2i}{(3)^2 - (2i)^2} \right)^3 \\
&= \frac{(3 + 2i)^3}{(9 - 4i^2)^3} = \frac{(3 + 2i)^3}{(9 + 4)^3} \\
&= \frac{1}{(13)^3} (3 + 2i)^3 \\
&= \left(\frac{1}{13^3} \right) (3 + 2i)^3
\end{aligned}$$

$$= \frac{1}{2197} [(3)^3 + (2i)^3 + 3 (3)^2 (2i) + 3 (3) (2i)^2]$$

$$= \frac{1}{2197} [27 + 8i^3 + 54i + 9 \cdot 4 i^2]$$

$$= \frac{1}{2197} [27 + 8i^2 \cdot i + 54i + 36 (-1)] = \frac{1}{2197} [27 - 8i + 54i - 36]$$

$$= \frac{1}{2197} [-9 + 46i]$$

$$= \frac{-9}{2197} + \frac{46}{2197} i$$

NUMBER SYSTEMS

MULTIPLE CHOICE QUESTIONS

(1) Modulus of $a + bi$ is:

- | | |
|-----------------|------------------------|
| (a) $a^2 + b^2$ | (b) $\sqrt{a^2 + b^2}$ |
| (c) $a^2 - b^2$ | (d) $a - ib$ |

[Lahore Board 2005]

(2) $\left\{x \mid x = \frac{p}{q}, p, q \in \mathbb{Z} \wedge q \neq 0\right\}$ is set of all:

- | | |
|------------------------|----------------------|
| (a) Natural numbers | (b) Integers |
| (c) Irrational numbers | (d) Rational numbers |

[Lahore Board 2005]

(3) $|2 - 3i| = \underline{\hspace{2cm}}$

- | | |
|-----------------|-----------------|
| (a) $\sqrt{2}$ | (b) $\sqrt{3}$ |
| (c) $\sqrt{13}$ | (d) $\sqrt{17}$ |

[Lahore Board 2005]

(4) $9a^2 + 64b^2 = \underline{\hspace{2cm}}$

- | | |
|----------------------------|--------------------------|
| (a) $3a + 8b$ | (b) $(3a + 8b)(3a - 8b)$ |
| (c) $(3a + 8bi)(3a - 8bi)$ | (d) None of these |

[Lahore Board 2005][Lahore Board 2015]

(5) $|a + ib|$ is equal to:

- | | |
|-------------------------|------------------------|
| (a) $\sqrt{a^2 + b^2}$ | (b) $\sqrt{a^2 - b^2}$ |
| (c) $\sqrt{-a^2 - b^2}$ | (d) None of these |

[Gujranwala Board 2005]

- (6) The property of inequality used in $a > b$ and $b > c \Rightarrow a > c$ is:
- | | |
|-------------------------|-----------------------------|
| (a) Additive property | (b) Multiplicative property |
| (c) Transitive property | (d) Additive inverse |

[Gujranwala Board 2005]

- (7) Which set has closure property with respect to addition:

- | | |
|-----------------|--------------|
| (a) $\{-1, 1\}$ | (b) $\{-1\}$ |
| (c) $\{1\}$ | (d) $\{0\}$ |

[Gujranwala Board 2006]

- (8) $a, b \in \mathbb{R}$ $a = b \Rightarrow b = a$ is called:

- | | |
|-----------------------------|------------------------|
| (a) Reflexive property | (b) Symmetric property |
| (c) Multiplicative property | (d) None of these |

[Gujranwala Board 2006]

- (9) $(i)^{-10}$ is equal to:

- | | |
|----------|-------------------|
| (a) i | (b) $-i$ |
| (c) -1 | (d) None of these |

[Gujranwala Board 2006]

- (10) If 'n' is a prime number then $\sqrt{n}$ is:

- | | |
|--------------------------|-----------------------|
| (a) An irrational number | (b) A rational number |
| (c) Prime number | (d) None of these |

[Lahore Board 2006-07]

- (11) $\forall a, b, c \in \mathbb{R}, c > 0$

$a > b \Rightarrow ac > bc$ is:

- | | |
|------------------------|-----------------------------|
| (a) Reflexive property | (b) Symmetric property |
| (c) Additive property | (d) Multiplicative property |

[Lahore Board 2006]

- (12) The number of the form $\frac{1}{7}$ is called a/an:

- | | |
|---------------------|-----------------------|
| (a) Rational number | (b) Irrational number |
| (c) Integer | (d) None of these |

[Lahore Board 2006]

- (13) $9a^2 + 16b^2 = \underline{\hspace{2cm}}$
- (a) $(3a + 4bi)(3a - 4bi)$ (b) $(3a + 4b)(3a - 4b)$
 (c) $(3a + 4bi)(3a + 4bi)$ (d) None of these

[Lahore Board 2006]

- (14) Geometrically the modulus of a complex number represents its distance from the point:
- (a) $(1, 0)$ (b) $(0, 1)$
 (c) $(0, 0)$ (d) $(1, 1)$

[Gujranwala Board 2007]

- (15) The simplified form of $(i)^{101}$ is:
- (a) -1 (b) 1
 (c) i (d) $-i$

[Gujranwala Board 2007]

- (16) $(-1)^{-21/2}$ is equal to:
- (a) $-i$ (b) i
 (c) 1 (d) -1

[Lahore Board 2007]

- (17) The number 1 is a:
- (a) Prime number (b) Irrational
 (c) Even number (d) Odd number

[Lahore Board 2008]

- (18) $\sqrt{3}$ is:
- (a) Odd number (b) Natural number
 (c) Rational (d) Irrational number

[Lahore Board 2008]

- (19) Multiplicative property of order of real numbers is that $\forall a, b, c \in \mathbb{R}$:
- (a) $a > b \wedge c > 0 \Rightarrow ac > bc$ (b) $a > b \wedge c > 0 \Rightarrow ac < bc$
 (c) $a > b \wedge c > 0 \Rightarrow ac = bc$ (d) $a > b \wedge c < 0 \Rightarrow ac > bc$

[Gujranwala Board 2008]

- (20) $\overline{-a - ib}$ is equal to:
- (a) $a + ib$ (b) $-a + ib$
 (c) $a - ib$ (d) $-a - ib$

[Gujranwala Board 2008]

(21) Multiplicative inverse of (a, b) is:

- (a) $\left(\frac{1}{a}, \frac{1}{b}\right)$ (b) $\left(\frac{a}{a^2 + b^2}, \frac{b}{a^2 + b^2}\right)$
(c) $\left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2}\right)$ (d) $\left(\frac{1}{a^2 + b^2}, \frac{b}{a^2 + b^2}\right)$

[Gujranwala Board 2009]

(22) If $Z = x + iy$, $x, y \in \mathbb{R}$ then $|Z|$ equals:

- (a) $x^2 + y^2$ (b) $\sqrt{x^2 + y^2}$
(c) $\sqrt{x^2 - y^2}$ (d) $\sqrt{x + y}$

[Lahore Board 2009]

(23) $\sqrt{\frac{5}{16}}$ is:

- (a) Rational number (b) Irrational number
(c) Prime number (d) Whole number

[Lahore Board 2009]

(24) Trichotomy is property of:

- (a) In-equality (b) Equality
(c) Division (d) Subtraction

[Lahore Board 2010]

(25) The multiplicative inverse of $(\sqrt{2}, -\sqrt{5})$ is:

- (a) $\left(\frac{\sqrt{2}}{\sqrt{7}}, \frac{\sqrt{5}}{\sqrt{7}}\right)$ (b) $\left(\frac{\sqrt{2}}{7}, \frac{-\sqrt{5}}{7}\right)$
(c) $\left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$ (d) $\left(\frac{-\sqrt{2}}{7}, \frac{-\sqrt{5}}{7}\right)$

[Lahore Board 2010]

(26) The property used in $4 + 9 = 9 + 4$:

- (a) Commutative property w.r.t multiplication
(b) Commutative property w.r.t addition.
(c) Associative property w.r.t addition.
(d) Associative property w.r.t multiplication.

[Lahore Board 2011]

- (27) If $Z = 3 + 4i$, then $|z| = \underline{\hspace{2cm}}$
- (a) 25 (b) 5
(c) -25 (d) -5

[Lahore Board 2012]

- (28) $\sqrt{3}$ is a _____ number.
- (a) Rational (b) Irrational
(c) Prime (d) Even

[Lahore Board 2012]

- (29) $\frac{22}{7}$ is:
- (a) Natural number (b) Whole number
(c) Rational number (d) Irrational number

[Lahore Board 2012]

- (30) Multiplicative inverse of $-3i$ is:
- (a) $3i$ (b) $\frac{1}{3}i$
(c) $-\frac{1}{3}i$ (d) $-3i$

- (31) $a > 0 \Rightarrow$
- (a) $-a > 0$ (b) $2a < 0$
(c) $\frac{1}{a} > 0$ (d) $\frac{-1}{a} < 0$

[Lahore Board 2013]

- (32) Multiplication inverse of $(1, 0)$ is:
- (a) $(-1, 0)$ (b) $(0, -1)$
(c) $(0, 1)$ (d) $(1, 0)$

[Lahore Board 2013]

[Lahore Board 2012]

- (33) $(7, 9) + (3, -5) =$
- (a) $(7, 9)$ (b) $(3, -5)$
(c) $(10, 4)$ (d) $(4, 10)$

- (34) Inverse of b^{-1} is:
- (a) b (b) b^{-1}
(c) $\frac{1}{b}$ (d) None of these

- (35) Tracheotomy property does not exists in:
- | | |
|-------|-------|
| (a) R | (b) Q |
| (c) C | (d) W |
- (36) $\sqrt{-i} = \underline{\hspace{2cm}}$
- | | |
|----------------------------------|------------------|
| (a) $\frac{1}{\sqrt{2}} (1 - i)$ | (b) $i\sqrt{-i}$ |
| (c) $i\sqrt{i}$ | (d) $1 + i$ |
- (37) $\sqrt{x + y}$ is modulus of:
- | | |
|------------------|-----------------------------|
| (a) $x + iy$ | (b) $(\sqrt{x}, -\sqrt{y})$ |
| (c) (x^2, y^2) | (d) (x, y) |
- (38) The number $1 + i\sqrt{3}$ in polar form is:
- | | |
|--|--|
| (a) $2 \operatorname{cis} \frac{\pi}{6}$ | (b) $2 \operatorname{cis} \frac{\pi}{3}$ |
| (c) $\operatorname{cis} \frac{\pi}{3}$ | (d) None of these |
- (39) $\left(\frac{b^2}{b^4 + b^2}, \frac{-b}{b^4 + b^2} \right)$ is multiplicative inverse of:
- | | |
|-------------------------------------|----------------|
| (a) (b^3, b) | (b) (b^2, b) |
| (c) $\left(b, \frac{1}{b} \right)$ | (d) (b, b^2) |
- (40) If $x > 0$ then:
- | | |
|-----------------------|--------------|
| (a) $\frac{1}{x} > 0$ | (b) $-x > 0$ |
| (c) $2x < 0$ | (d) $-x < 0$ |
- (41) If $Z = a + ib$ and $a = r \cos \theta$, $b = r \sin \theta$ then:
- | | |
|----------------|---------------|
| (a) $r = Z$ | (b) $r = -Z$ |
| (c) $r = - Z $ | (d) $r = Z $ |
- (42) Multiplicative inverse of $(0, 1)(-1, 0)$ is:
- | | |
|--------------|--------------|
| (a) $(0, 1)$ | (b) $(1, 0)$ |
| (c) $(0, 0)$ | (d) $(1, 1)$ |
- (43) $a(x, y) = \underline{\hspace{2cm}}$ where $a \in \mathbb{R}$:
- | | |
|----------------|---------------|
| (a) (ax, y) | (b) (x, ay) |
| (c) (ax, ay) | (d) (x, y) |

- (44) Imaginary part of a real number 'a' is:
- (a) 0 (b) 1
(c) i (d) None of these
- (45) In the set $C - \{0\}$, associative law does not holds w.r.t.
- (a) $\times$ (b) $+$
(c) $-$ (d) None of these
- (46) $|Z_1 + Z_2|$ —————
- (a) $< |Z_1 + Z_2|$ (b) $> |Z_1| > |Z_2|$
(c) $\leq |Z_1| + |Z_2|$ (d) $\geq |Z_1| + |Z_2|$
- (47) Modulus of two conjugate complex numbers are —————
- (a) Same (b) Different
(c) Not exists (d) None of these
- (48) $(\cos\theta - i \sin\theta)^{-n} =$ —————
- (a) $\cos n\theta - i \sin n\theta$ (b) $\cos(-n)\theta + i \sin(-n)\theta$
(c) $\cos\theta + i \sin\theta$ (d) $\cos n\theta + i \sin n\theta$
- (49) The members of cartesian product $A \times B$ are called:
- (a) Ordered pairs (b) Sets
(c) Subsets (d) Elements
- (50) If $Z = 3 - 2i$ then $\overline{Z} =$
- (a) $3 + 2i$ (b) $3 - 2i$
(c) $-3 - 2i$ (d) $-3 + 2i$
- (51) π is a/an:
- (a) Rational number (b) Irrational number
(c) Even number (d) Whole number
- (52) Better approximate value of π is:
- (a) $\frac{22}{7}$ (b) $\frac{355}{113}$
(c) 3.14159 (d) All of these
- (53) $Q \cup Q' =$ —————
- (a) C (b) R
(c) $C - \{0\}$ (d) $R - \{0\}$

- (54) If $Z = 4 - 3i$ then imaginary part of Z is:
 (a) $3i$ (b) $-3i$
 (c) 3 (d) -3
- (55) Which set does not satisfy the order axioms:
 (a) $\mathbb{Q}$ (b) $\mathbb{C}$
 (c) $\mathbb{N}$ (d) $\mathbb{Q}'$
- (56) $\forall Z_1, Z_2 \in \mathbb{C} \quad |Z_1| - |Z_2| \text{ ————— }$
 (a) $= |Z_1 - Z_2|$ (b) $\leq |Z_1 \cdot Z_2|$
 (c) $\leq |Z_1 + Z_2|$ (d) $\geq |Z_1| + |Z_2|$
- (57) $(-1)^{-55 \left| \frac{7}{3} - \frac{15}{6} \right|} =$
 (a) i (b) $-i$
 (c) 1 (d) -1
- (58) Which number is not an imaginary number:
 (a) $\sqrt{-9}$ (b) $(-1)^{-1/2}$
 (c) $7 + 6i$ (d) $(1 + i)^2$
- (59) $(-1, 0)^{2n+1/2} =$
 (a) i (b) $-i$
 (c) 1 (d) -1
- (60) Solution of the equation $\frac{x^2}{2} + a = b$ belongs to set of complex numbers if:
 (a) $a = 0$ (b) $a > b$
 (c) $a < b$ (d) $a = b$
- (61) $\sqrt{-5}$ belongs to set of:
 (a) Real numbers (b) Prime numbers
 (c) Even numbers (d) Complex numbers
- (62) Multiplicative inverse of $-i$ is: **[Lahore Board 2014]**
 (a) i (b) $-i$
 (c) 1 (d) -1
- (63) Types of ratios fractions are: **[Lahore Board 2015]**
 (a) 3 (b) 2
 (c) 4 (d) 1
- (64) $(-i)^{19} = \text{—————}$
 (a) $-i$ (b) i
 (c) 1 (d) -1
- (65) An improper fraction can be reduced to proper fraction by an addition:
 (a) Addition (b) Subtraction
 (c) Multiplication (d) Division

Answers

(1)	b	(2)	d	(3)	c	(4)	c	(5)	a
(6)	c	(7)	d	(8)	b	(9)	c	(10)	a
(11)	d	(12)	b	(13)	a	(14)	c	(15)	c
(16)	a	(17)	d	(18)	d	(19)	a	(20)	b
(21)	c	(22)	b	(23)	b	(24)	a	(25)	c
(26)	b	(27)	b	(28)	b	(29)	c	(30)	b
(31)	d	(32)	d	(33)	c	(34)	a	(35)	c
(36)	c	(37)	b	(38)	b	(39)	b	(40)	d
(41)	d	(42)	a	(43)	c	(44)	a	(45)	c
(46)	c	(47)	a	(48)	d	(49)	a	(50)	b
(51)	b	(52)	c	(53)	b	(54)	d	(55)	b
(56)	c	(57)	b	(58)	c	(59)	a	(60)	b
(61)	d	(62)	a	(63)	b	(64)	b	(65)	d

Q.1 Modulus of $a + ib = |a + ib|$
 $= \sqrt{(a)^2 + (b)^2}$
 $= \sqrt{a^2 + b^2}$

Q.2 As we know that:

The number which can be written in the form of $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$ is called a rational number so the given set is a set of rational numbers.

Q.3 $|2 - 3i| = \sqrt{(2)^2 + (-3)^2} = \sqrt{4 + 9} = \sqrt{13}$

Q.4 $9a^2 + 64b^2 = 9a^2 - (-64b^2) = (3a)^2 - (8bi)^2$
 $= (3a + 8bi)(3a - 8bi)$

Q.5 $|a + ib|$
 By definition of modulus, $|a + ib| = \sqrt{a^2 + b^2}$.

Q.7 As $0 + 0 = 0 \in \{0\}$.

$\Rightarrow$ Only set $\{0\}$ has closure property with respect to addition.

Q.9 $(i)^{-10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1$.

Q.10 If 'n' is a prime number then $\sqrt{n}$ is an irrational number. Its proof can be seen at pg. 5 (Textbook).

Q.12 As $\frac{1}{7}$ is in $\frac{p}{q}$ form where $1, 7 \in \mathbb{Z}$ and $7 \neq 0 \Rightarrow \frac{1}{7}$ is a rational number.

Q.13 $9a^2 + 16b^2$
 $= 9a^2 - (-16b^2)$
 $= (3a)^2 - (4ib)^2$
 $= (3a + 4ib)(3a - 4ib)$

Q.14 Geometrically, the modulus of a complex number represents its distance from origin (0, 0) so correct option is (0, 0).

Q.15 $(i)^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = i$.

Q.16 $(-1)^{-\frac{21}{2}} = [(-1)^{1/2}]^{-21} = (i)^{-21} = \frac{1}{i^{21}} = \frac{1}{(i^2)^{10} \cdot i}$
 $= \frac{1}{(-1)^{10} \cdot i} = \frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = -i$

Q.17 1 is an odd number.

Q.18 As $\sqrt{n}$ (where 'n' is a prime number) is an irrational number.

$\Rightarrow \sqrt{3}$ is also irrational number.

Q.20 $\overline{-a - ib}$

$= -a + ib$ (by definition of conjugate)

Q.21 As multiplicative inverse of (a, b) is $\frac{1}{(a, b)} = \frac{1}{a + ib}$

$$= \frac{1}{a + ib} \times \frac{a - ib}{a - ib}$$

$$= \frac{a - ib}{(a)^2 - (ib)^2}$$

$$= \frac{a - ib}{a^2 + b^2}$$

$$= \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$$

Q.22 $|Z| = \sqrt{(x)^2 + (y)^2}$
 $= \sqrt{x^2 + y^2}$ (by def of modulus)

Q.23 $\sqrt{\frac{5}{16}}$
 $= \frac{\sqrt{5}}{\sqrt{16}} = \frac{\sqrt{5}}{4}$

As $\sqrt{5} \notin \mathbb{Z} \Rightarrow \frac{\sqrt{5}}{4}$ is not a rational number.

$\Rightarrow \frac{\sqrt{5}}{4}$ is an irrational number.

Q.25 As we know that:

Multiplicative inverse of (a, b) $= \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$

Multiplicative inverse of $(\sqrt{2}, -\sqrt{5}) = \left(\frac{\sqrt{2}}{2 + 5}, \frac{\sqrt{5}}{2 + 5} \right)$
 $= \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7} \right)$

Q.32 Inverse of $b^{-1} = (b^{-1})^{-1} = b$.

Q.33 Trichotomy property does not exist in $\mathbb{C}$.

Q.34 $\sqrt{-i}$
 $= \sqrt{(-1)i} = \sqrt{-1} \sqrt{i} = i \sqrt{i}$

Q.35 As, $(\sqrt{x}, -\sqrt{y})$
 $= \sqrt{x} - \sqrt{y} i$
and $|\sqrt{x} - \sqrt{y} i| = \sqrt{(\sqrt{x})^2 + (-\sqrt{y})^2}$
 $= \sqrt{x+y}$
 $\Rightarrow \sqrt{x+y}$ is modulus of $(\sqrt{x}, -\sqrt{y})$

Q.36 Let, $Z = 1 + i\sqrt{3}$
Let, $1 = r \cos \theta$ and $\sqrt{3} = r \sin \theta$
 $r^2 \cos^2 \theta + r^2 \sin^2 \theta = 1 + (\sqrt{3})^2$
 $r^2 = 1 + 3 = 4$
 $r = 2$

And $\theta = \tan^{-1} \frac{\sqrt{3}}{1} = \tan^{-1} \sqrt{3} = 60^\circ = \frac{\pi}{3}$

So,

$$1 + i\sqrt{3} = r (\cos \theta + i \sin \theta) = 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$
$$= 2 \operatorname{cis} \frac{\pi}{3}$$

Q.37 As, Multiplicative inverse of $(a, b) = \left(\frac{a}{a^2 + b^2}, \frac{-b}{a^2 + b^2} \right)$

$$\text{Multiplicative inverse of } (b^2, b) = \left(\frac{b^2}{(b^2)^2 + (b)^2}, \frac{-b}{(b^2)^2 + (b)^2} \right)$$
$$= \left(\frac{b^2}{b^4 + b^2}, \frac{-b}{b^4 + b^2} \right)$$

Q.38 If $x > 0$
 $(-1)x < (-1)0$
 $-x < 0$

Q.39 As,

$$Z = a + ib$$

$$Z = r \cos\theta + i r \sin\theta$$

$$|Z| = \sqrt{(r \cos\theta)^2 + (r \sin\theta)^2}$$

$$= \sqrt{r^2 \cos^2\theta + r^2 \sin^2\theta}$$

$$= \sqrt{r^2 (\cos^2\theta + \sin^2\theta)}$$

$$= \sqrt{r^2(1)}$$

$$= \sqrt{r^2}$$

$$|Z| = r$$

Q.40 As,

$$(0, 1)(-1, 0) = (0 + i)(-1 + 0i)$$

$$= (i)(-1)$$

$$= -i = (0, -1)$$

$$\text{Multiplicative inverse of } (0, -1) = \left(\frac{0}{(0)^2 + (-1)^2}, \frac{-(-1)}{(0)^2 + (-1)^2} \right)$$

$$= \left(0, \frac{1}{0 + 1} \right)$$

$$= (0, 1)$$

Q.41 $a(x, y) = a(x + iy) = ax + iay = (ax, ay)$

Q.42 As,

$$a = a + 0i$$

$\Rightarrow$ Imaginary part of a is 0.

Q.43 Associative law does not hold w.r.t ‘-’.

Q.44 $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

Q.45 Let,

$$Z = a + ib \text{ and its conjugate } \bar{Z} = a - ib$$

$$|Z| = \sqrt{a^2 + b^2} \quad \text{and} \quad |\bar{Z}| = \sqrt{a^2 + b^2} \text{ (same)}$$

Q.46 As,

$$(\cos\theta - i \sin\theta)^{-n}$$

$$= \cos(-n)\theta - i \sin(-n)\theta \quad \cos(-\theta) = \cos\theta$$

$$= \cos n\theta + i \sin n\theta \quad \sin(-\theta) = -\sin\theta$$

Q.47 The members of cartesian product are called ordered pairs.

Q.48 If

$$Z = 3 - 2i$$

$$\bar{Z} = 3 + 2i$$

$$\overline{\bar{Z}} = 3 - 2i$$

Q.49 π is an irrational number.

Q.50 Better approximate value of Π is 3.14159.

Q.51 $Q \cup Q' = R$

Q.52 If $Z = 4 - 3i$

then imaginary part of Z is -3 .

Q.53 Set of complex numbers 'C' does not satisfy the order axioms.

Q.54 $|Z_1| - |Z_2| \leq |Z_1 + Z_2|$

Q.55 $(-1)^{-55 \left| \frac{7}{3} - \frac{15}{6} \right|} = (-1)^{-55 \left| \frac{14-15}{6} \right|} = (-1)^{-55 \left| -\frac{1}{6} \right|}$

$$= (-1)^{-55 \cdot \frac{1}{6}}$$

$$= (-1)^{-\frac{55}{6}}$$

$$= [(-1)^2]^{-\frac{55}{3}}$$

$$= (i)^{-\frac{55}{3}}$$

$$= \frac{1}{(i)^{\frac{55}{3}}}$$

$$= \frac{1}{[i^{55}]^{1/3}}$$

$$\begin{aligned}
&= \frac{1}{[(i^2)^{27} \cdot i]^{1/3}} \\
&= \frac{1}{((-1)^{27} \cdot i)^{1/3}} \\
&= \frac{1}{(-i)^{1/3}} \\
&= \frac{1}{(-1)^{1/3} \cdot (i)^{1/3}} \\
&= \frac{1}{(-1)^{1/3} \cdot (\sqrt{-1})^{1/3}} = \frac{1}{(-1)^{1/3} (-1)^{1/6}} \\
&= \frac{1}{(-1)^{\frac{1}{3} + \frac{1}{6}}} = \frac{1}{(-1)^{3/6}} = \frac{1}{(-1)^{1/2}} = \frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i
\end{aligned}$$

Q.57 $(-1, 0)^{2n+\frac{1}{2}} = (-1)^{2n+\frac{1}{2}} = (-1)^{2n} \cdot (-1)^{1/2} = (-1)^{2n} \cdot i = i \quad \because 2n \text{ is even}$

Q.58 Let, $a = 2$, $b = 1$ then $\frac{x^2}{2} + 2 = 1$.

$$\Rightarrow \frac{x^2}{2} = -1 \Rightarrow x^2 = -2 \Rightarrow x = \sqrt{-2} \in \mathbb{C}$$

$$\Rightarrow \text{If } a > b \text{ then solution belongs to } \mathbb{C}.$$

Q.61 $\sqrt{5}i \in \mathbb{C}$

Q.62 $\frac{1}{-i} \times \frac{i}{i}$

$$= \frac{i}{-i^2} = \frac{i}{-(-1)} = i$$

Q.64 $(-i)^{19} = -i^{19} = -i^{16} \cdot i^3 = -(1)(-i) = i$