Chapter 7

PERMUTATION, COMBINATION AND PROBABILITY

FACTORIAL NOTATION

Let n be a positive integer. Then the product $n(n-1)(n-2)\ldots\ldots 3 \cdot 2 \cdot 1$ is denoted by n! or $\lfloor n$ and read as n factorial.

i.e. $n! = n(n-1)(n-2)\ldots\ldots 3 \cdot 2 \cdot 1$

and $n! = n(n-1)!$ where $0! = 1$

EXERCISE 7.1

Q.1 Evaluate each of the following:

(i) $4!$ **(ii)** $6!$ **(iii)** $\frac{8!}{7!}$ **(iv)** $\frac{10!}{7!}$ **(v)** $\frac{11!}{4!\,7!}$ **(vi)** $\frac{6!}{3!\,3!}$ **(vii)** $\frac{8!}{4!\,2!}$

(viii) $\frac{11!}{2!\,4!\,5!}$ **(ix)** $\frac{9!}{2!\,(9-2)!}$ **(x)** $\frac{15!}{15!\,(15-15)!}$ **(xi)** $\frac{3!}{0!}$ **(xii)** $4!\,0!\,1!$

Solution:

(i) $4! = 4.3.2.1 = 24$

(ii) $6! = 6.5.4.3.2.1 = 720$

(iii) $\frac{8!}{7!} = \frac{8.7.6.5.4.3.2.1}{7.6.5.4.3.2.1} = 8$

(iv) $\frac{10!}{7!} = \frac{10.9.8.7.6.5.4.3.2.1}{7.6.5.4.3.2.1} = 720$

(v) $\frac{11!}{4!\,7!} = \frac{11.10.9.8.7.6.5.4.3.2.1}{4.3.2.1\;7.6.5.4.3.2.1} = 330$

(vi) $\frac{6!}{3!\,3!} = \frac{6.5.4.3.2.1}{3.2.1.3.2.1} = 20$

(vii) $\frac{8!}{4!\,2!} = \frac{8.7.6.5.4.3.2.1}{4.3.2.1.2.1} = 840$

(viii) $\frac{11!}{2!\,4!\,5!} = \frac{11.10.9.8.7.6.5!}{2.1.4.3.2.1.5!} = 6930$

(ix) $\frac{9!}{2!\,(9-2)!} = \frac{9!}{2!\,7!} = \frac{9.8.7!}{2.1.7!} = 36$

(x) $\frac{15!}{15!\,(15-15)!} = \frac{15!}{15!\,0!} = \frac{15!}{15!.1} = 1$

(xi) $\frac{3!}{0!} = \frac{3.2.1}{1} = 6$

(xii) $4!\,0!\,1! = 4.3.2.1.1.1 = 24$

Q.2 Write each of the following in factorial form:

(i) 6.5.4 **(ii) 12, 11, 10** **(iii) 20.19.18.17**

(iv) $\frac{\mathbf{10.9}}{\mathbf{2.1}}$ **(v)** $\frac{\mathbf{8.7.6}}{\mathbf{3.2.1}}$ **(vi)** $\frac{\mathbf{52.51.50.49}}{\mathbf{4.3.2.1}}$

(vii) n (n – 1) (n – 2) **(viii) (n + 2) (n + 1) (n)** **(ix)** $\frac{\mathbf{(n+1)\,(n)\,(n-1)}}{\mathbf{3.2.1}}$

(x) n (n – 1) (n – 2) ……. (n – r + 1)

Solution:

(i) $6.5.4 = \frac{6.5.4.3.2.1}{3!} = \frac{6!}{3!}$

(ii) $12.11.10 = \frac{12.11.10.9!}{9!} = \frac{12!}{9!}$

(iii) $20.19.18.17 = \frac{20.19.18.17.16!}{16!} = \frac{20!}{16!}$

(iv) $\frac{20.9}{2.1} = \frac{10.9.8!}{2.8!} = \frac{10!}{2!\,8!}$

(v) $\frac{8.7.6}{3.2.1} = \frac{8.7.6.5!}{3.2.1.5!} = \frac{8!}{3!\,5!}$

(vi) $\frac{52.51.50.49}{4.3.2.1} = \frac{52.51.50.49.48!}{4.3.2.1.\,48!} = \frac{52!}{4!\,48!}$

(vii) $n\,(n-1)\,(n-2) = \frac{n\,(n-1)\,(n-2)\,(n-3)!}{(n-3)!} = \frac{n!}{(n-3)!}$

(viii) $(n+2)\,(n+1)\,(n) = \frac{(n+2)\,(n+1)\,n\,(n-1)!}{(n-1)!} = \frac{(n+2)!}{(n-1)!}$

(ix) $\dfrac{(n+1)(n)(n-1)}{3\,.\,2\,.\,1} = \dfrac{(n+1)\,n\,(n-1)\,(n-2)!}{3.2.1\,.\,(n-2)!} = \dfrac{(n+1)!}{3!\,(n-2)!}$

(x) $n\,(n-1)\,(n-2)\,\ldots..\,(n-r+1) = \dfrac{n\,(n-1)\,(n-2)\,\ldots..\,(n-r+1)\,(n-r)!}{(n-r)!} = \dfrac{n!}{(n-r)!}$

PERMUTATION

An ordering (arrangement) of n objects is called a permutation of the objects.

EXPLANATION

Think of three places as shown ☐ ☐ ☐ . Since we can write any one of three vertices A, B, C at first place, so it is written in 3 different ways as shown [3] ☐ ☐ .

Now two vertices are left. So, corresponding to each way of writing at first place, there are two ways of writing at second place as shown [3] [2] ☐ .

Now just one vertex is left. So, we can write at third place only one vertex in one way as shown [3] [2] [1] .

$\Rightarrow$ The total number of possible ways is the product 3.2.1. = 6.

FUNDAMENTAL PRINCIPLE OF COUNTING

Suppose A and B are two events. The first event A can occur in P different ways. After A has occurred, B can occur in q different ways. The number of ways that the two events can occur is the product p.q.

THEOREM

A permutation of n different objects taken r ($\leq$ n) at a time is an arrangement of the r objects. Generally it is denoted by ${}^{n}P_{r}$ or P (n, r) where

$${}^{n}P_{r} = n\,(n-1)\,(n-2)\,\ldots\ldots\,(n-r+1) = \frac{n!}{(n-r)!}$$

EXERCISE 7.2

Q.1 Evaluate the following:

(i) ${}^{20}P_3$ (ii) ${}^{16}P_4$ (iii) ${}^{12}P_5$ (Lahore Board 2006) (iv) ${}^{10}P_7$ (v) ${}^{9}P_8$

Solution:

Using formula ${}^{n}P_{r} = \dfrac{n!}{(n-r)!}$

(i) ${}^{20}P_3 = \dfrac{20!}{(20-3)!} = \dfrac{20.19.18.17!}{17!} = 6840$

(ii) $^{16}P_4 = \frac{16!}{(16-4)!} = \frac{16.15.14.13.12!}{12!} = 43680$

(iii) $^{12}P_5 = \frac{12!}{(12-5)!} = \frac{12.11.10.9.8.7!}{7!} = 95040$

(iv) $^{10}P_7 = \frac{10!}{(10-7)!} = \frac{10.9.8.7.6.5.4.3!}{3!} = 604800$

(v) $^{9}P_8 = \frac{9!}{(9-8)!} = \frac{9.8.7.6.5.4.3.2.1}{1!} = 362880$

Q.2 Find the value of n when:

(i) $^{n}P_2 = 30$ (Gujranwala Board 2004, 2005, 2007)

(ii) $^{11}P_n = 11.10.9$ (Lahore Board 2008, 2009) (iii) $^{n}P_4 : {}^{n-1}P_3 = 9:1$

Solution:

(i) $^{n}P_2 = 30$

$\Rightarrow \frac{n!}{(n-2)!} = 30$

$\Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 30$

$\Rightarrow n(n-1) = 30$

$\Rightarrow n^2 - n - 30 = 0$

$\Rightarrow n^2 - 6n + 5n - 30 = 0$

$\Rightarrow n(n-6) + 5(n-6) = 0$

$\Rightarrow (n+5)(n-6) = 0$

$\Rightarrow n = -5$ (not possible)

$\Rightarrow \boxed{n = 6}$

(ii) Solution

$^{11}P_n = 11.10.9$

$\Rightarrow \frac{11!}{(11-n)!} = 11 . 10 . 9$

$\Rightarrow \frac{11!}{11.10.9} = (11-n)!$

$\Rightarrow \frac{11!}{11.10.9} = (11-n)!$

$\Rightarrow \quad \frac{11!\,8!}{11.10.9.8!} = (11-n)!$

$\Rightarrow \quad \frac{11!\,8!}{11!} = (11-n)!$

$\Rightarrow \quad 8! = (11-n)!$

$\Rightarrow \quad (11-3)! = (11-n)!$

$\Rightarrow \quad \boxed{n = 3}$

(iii) Solution

${}^{n}P_4 : {}^{n-1}P_3 = 9 : 1$

$\Rightarrow \quad \frac{{}^{n}P_4}{{}^{n-1}P_3} = \frac{9}{1}$

$\Rightarrow \quad \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)!}} = \frac{9}{1}$

$\Rightarrow \quad \frac{n!}{(n-1)!} = \frac{9}{1}$

$\Rightarrow \quad \frac{n(n-1)!}{(n-1)!} = 9$

$\Rightarrow \quad \boxed{n = 9}$

Q.3 Prove from the first principle that:

(i) ${}^{n}P_r = n \,.\, {}^{n-1}P_{r-1}$

(ii) ${}^{n}P_r = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$ **(Lahore Board 2009, 2011)**

Solution:

(i) ${}^{n}P_r = n \,.\, {}^{n-1}P_{r-1}$

L.H.S. $= {}^{n}P_r = \frac{n!}{(n-r)!}$

R.H.S. $= n\,{}^{n-1}P_{r-1}$

$= n \cdot \frac{(n-1)!}{(n-1-r+1)!}$

$= \frac{n(n-1)!}{(n-r)!} = \frac{n!}{(n-r)!} =$ L.H.S.

$\Rightarrow \quad {}^{n}P_r = n \,.\, {}^{n-1}P_{r-1}$

Proved

(ii) Solution

$${}^{n}P_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$$

$$\text{L.H.S.} = {}^{n}P_r = \frac{n!}{(n-r)!}$$

$$\text{R.H.S.} = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$$

$$= \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{(n-1-r+1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + \frac{r \cdot (n-1)!}{(n-r)\,(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left(1 + \frac{r}{n-r}\right)$$

$$= \frac{(n-1)!}{(n-r-1)!}\left(\frac{n-r+r}{n-r}\right)$$

$$= \frac{n\,(n-1)!}{(n-r)\,(n-r-1)!}$$

$$= \frac{n!}{(n-r)!} = \text{L.H.S.}$$

$$\Rightarrow \quad {}^{n}P_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$$

Proved.

Q.4 How many signals can be given by 5 flags of different colours, using 3 at a time?

Solution:

Total number of flags = n = 5

Used number of flags = r = 3

$$\text{Required numbers of signals} = {}^{n}P_r = {}^{5}P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

Q.5 How many signals can be given by 6 flags of different colours when any number of flags can be used at a time?

(Lahore Board 2004, Gujranwala Board 2007)

Solution:

Total number of flags = 6

Number of signals using 1 flag = ${}^{6}P_1 = 6$

Number of signals using 2 flags = ${}^{6}P_2 = 30$

Number of signals using 3 flags $= {}^6P_3 = 120$

Number of signals using 4 flags $= {}^6P_4 = 360$

Number of signals using 5 flags $= {}^6P_5 = 720$

Number of signals using 6 flags $= {}^6P_6 = 720$

Total number of signals $= 6 + 30 + 120 + 360 + 720 + 720 = 1956$

Q.6 How many words can be formed by the letters of the following words using all letters when no letter is to be repeated?

(i) PLANE (ii) OBJECT (Lahore Board 2010) (iii) FASTING

Solution:

(i) PLANE

Permutation of 5 letters using all at a time $= {}^5P_5 = \frac{5!}{(5-5)!} = \frac{5!}{6!} = 120$

(ii) OBJECT

Permutation of 6 letters using all at a time $= {}^6P_6 = \frac{6!}{0!} = 6! = 720$

(iii) FASTING

Permutation of 7 letters using all at a time $= {}^7P_7 = \frac{7!}{0!} = 7! = 5040$

Q.7 How many 3 – digit numbers can be formed by using each one of the digits 2, 3, 5, 7, 9 only once? (Gujranwala Board 2005)

Solution:

Total number of digits $n = 5$

Used number of digits $r = 3$

Required no. of permutations $= {}^nP_r = {}^5P_3 = 60$

Q.8 Find the numbers greater than 23000 that can be formed from the digits 1, 2, 3, 5, 6 without repeating any digit. (Lahore Board 2005)

Solution:

Numbers greater than 23000 are of the form

$23\square\square\square = {}^3P_3 = 3! = 6$

$25\square\square\square = {}^3P_3 = 3! = 6$

$26\square\square\square = {}^3P_3 = 3! = 6$

$3\square\square\square\square = {}^4P_4 = 4! = 24$

$5\square\square\square\square = {}^4P_4 = 4! = 24$

$6\square\square\square\square = {}^4P_4 = 4! = 24$

Total numbers $= 6 + 6 + 6 + 24 + 24 + 24 = 90$

Q.9 Find the number of 5 – digit numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated), but

(i) the digits 2 and 8 are next to each other.

(ii) the digits 2 and 8 are not next to each other.

Solution:

(i) Let (2, 8) as one digit

Now the digits are 1, 28, 4, 6 or 1, 82, 4, 6

Permutation containing (28) $= {}^4P_4 = 4! = 24$

Permutation containing (82) $= {}^4P_4 = 4! = 24$

Total permutation in which 2 and 8 are next to each other $= 24 + 24 = 48$

(ii) Total permutations of 5 digits $= {}^5P_5 = 5! = 120$

Numbers in which (2, 8) are together $= 48$

Numbers in which (2, 8) are not together $= 120 - 48 = 72$

Q.10 How many 6–digit numbers can be formed without repeating any digit from the digits 0, 1, 2, 3, 4, 5? In how many of them will 0 be at the tens place?

(Gujranwala Board 2007)

Solution:

'0' cannot be at first place from left so this place can be filled by 5 ways then second place can be filled by 5 ways, third place by 4 ways, fourth place by 3 ways, fifth place by 2 ways six place by 1 way.

So required numbers $= 5.5.4.3.2.1 = 600$

If we fix 0 at tens place then remaining places are 5 and digits are also 5.

So required numbers $= {}^5P_5 = 5! = 120$

Q.11 How many 5 – digit multiples of 5 can be formed from the digits 2, 3, 5, 7, 9 when no digit is repeated?

Solution:

As multiples of 5 will have the digit 5 at unit place.

So fixing 5 at unit place we have to find permutations of remaining four digits using all at a time.

$\Rightarrow$ Multiples of 5 are $= {}^4P_4 = 4! = 24$

Q.12 In how many ways can 8 books including 2 on English be arranged on a shelf in such a way that the English books are never together?

Solution:

Consider the two English books E_1 and E_2 as one book.

Now we find permutation of 7 books taken all at a time.

Permutation containing $E_1\,E_2 = {}^7P_7 = 7! = 5040$

Permutation containing $E_2\,E_1 = {}^7P_7 = 7! = 5040$

Permutation in which English book are together $= 5040 + 5040 = 10080$

Total permutation of 8 books $= {}^8P_8 = 8! = 40320$

Permutation in which English books are not together $= 40320 - 10080 = 30240$

Q.13 Find the number of arrangements of 3 books on English and 5 books on Urdu for placing them on a shelf such that the books on the same subjects are together?

Solution:

Let E, U denote English and Urdu books.

Permutation in the form EEEEEUUU $= 5!\,3! = 720$

Permutation in the form UUUEEEEE $= 3!\,5! = 720$

Total permutation $= 720 + 720 = 1440$

Q.14 In how many ways can 5 boys and 4 girls be seated on a bench so that the girls & the boys occupy alternate seats?

Solution:

Let B, G denote boys and girls then

Permutation in the form BGBGBGBGB $= {}^5P_5\,{}^4P_4 = 5!\,4! = 120 \times 24 = 2880$

PERMUTATION OF THINGS NOT ALL DIFFERENT

If there are n_1 alike things of one kind, n_2 alike things of second kind and n_3 alike things of third kind, then the number of permutation of n things taken all at a time is given by

$$\frac{n!}{(n_1)! \times (n_2)! \times (n_3)!} = \binom{n}{n_1, n_2, n_3}$$

CIRCULAR PERMUTATION

The permutation of things which can be represented by the points on a circle are called circular permutation.

EXERCISE 7.3

Q.1 How many arrangements of the letters of the following words, taken all together can be made?

(i) PAKPATTAN (Lahore Board 2007) **(ii) PAKISTAN**

(iii) MATHEMATICS (Lahore Board 2004) **(iv) ASSASSINATION**

Solution:

(i) PAKPATTAN

$n = 9, \quad n_1 = 3\,(A), \quad n_2 = 2\,(P), \quad n_3 = 2\,(T)$

so using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\,n_2!\,n_3!} = \frac{9!}{3!\,2!\,2!} = \frac{9.8.7.6.5.3!}{3!\,2.1\,.\,2.1} = 15120$$

(ii) PAKISTAN

Here $n = 8$

$n_1 = 2$ (A)

using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{8!}{2!} = \frac{8.7.6.5.4.3.2.1}{2.1} = 20160$$

(iii) MATHEMATICS

Here $n = 11$, $n_1 = 2$ (A), $n_2 = 2$ (M), $n_3 = 2$ (T)

So using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{11!}{2!\, 2!\, 2!} = 4989600$$

(iv) ASSASSINATION

Here $n = 13$, $n_1 = 4$ (S), $n_2 = 3$ (A), $n_3 = 2$ (N), $n_4 = 2$ (I)

So using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3, n_4} = \frac{n!}{n_1!\, n_2!\, n_3!\, n_4!} = \frac{13!}{4!\, 3!\, 2!\, 2!} = 10810800$$

Q.2 How many permutation of the letters of the word PANAMA can be made, if P is to be the first letter in each arrangement? (Gujranwala Board 2007)

Solution:

If we fix P at first place then remaining number of letters $= n = 5$

with $n_1 = 3$ (A), $n_2 = 1$ (N), $n_3 = 1$ (M)

using formula

$$\text{Permutation} = \binom{n}{n_1, n_2, n_3} = \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{5!}{3!\, 1!\, 1!} = \frac{5!}{3!} = \frac{5.4.3!}{3!} = 20$$

Q.3 How many arrangements of the letters of the word ATTACKED can be made, if each arrangement begins with C and ends with K?

Solution:

If we fix C at start and K at the end then we have to find permutation of remaining 6 letters.

$\Rightarrow$ $n = 6$ with $n_1 = 2$ (A), $n_2 = 2$ (T) using formula

$$\text{Permutation} = \binom{n}{n_1, n_2} = \frac{n!}{n_1!\, n_2!} = \frac{6!}{2!\, 2!} = 180$$

Q.4 How many numbers greater than 1000000 can be formed from the digits 0, 2, 2, 2, 3, 4, 4?

Solution:

The numbers greater than 1000000 are of the forms:

$$2\square\square\square\square\square\square = \binom{6}{2,2} = \frac{6!}{2!\,2!} = 180$$

$$3\square\square\square\square\square\square = \binom{6}{3,2} = \frac{6!}{3!\,2!} = 60$$

$$4\square\square\square\square\square\square = \binom{6}{3} = \frac{6!}{3!} = 120$$

So required numbers $= 180 + 60 + 120 = 360$

Q.5 How many 6–digit numbers can be formed from the digits 2, 2, 3, 3, 4, 4? How many of them lie between 400000 and 430,000?

Solution:

$n = 6,\ n_1 = 2\,(2),\ n_2 = 2\,(3),\ n_3 = 2\,(4)$

So using formula

$$\text{Required permutation} = \binom{n}{n_1, n_2, n_3} = \binom{6}{2,2,2} = \frac{6!}{2!\,2!\,2!} = 90$$

(ii) The number lying between 400,000 and 430,000 are of the form $42\square\square\square\square$

$$\text{so required arrangements} = \binom{4}{2,1,1} = \frac{4!}{2!\,1!\,1!} = \frac{4!}{2!} = 12$$

Q.6 11 members of a club form 4 committees of 3, 4, 2, 2 members so that so that no member is the member of more than one committee. Find the number of committees?

Solution:

Total members $= n = 11$

with $n_1 = 4,\ n_2 = 3,\ n_3 = 2,\ n_4 = 2$

$$\text{So required number of committees} = \binom{11}{4,3,2,2} = \frac{11!}{4!\,3!\,2!\,2!} = 69300$$

Q.7 The D.C.O's of 11 districts meet to discuss the law and order situation in these districts. In how many ways can they be seated a round table, when two particular D.C.O's insist on sitting together?

Solution:

Consider the two D.C.O's as one man. Now the number of men $= 10$

and number of ways that 10 men can be seated at a round table $= 9!$ also the two particular D.C.O's can sit in two different ways so the total number of ways $= 9! \times 2 = 725760$

Q.8 The Governor of Punjab calls a meeting of 12 officers. In how many ways can they be seated at a round table?

Solution:

If we fix seat of one officer then there are 11 seats for 11 officers so

required no. of arrangements $= 11! = 39916800$

Q.9 Fatima invites 14 people to dinner. There are 9 males and 5 females who are seated at two different tables so that guests of one sex sit at one round table and the guests of the other sex at the second table. Find the number of ways in which all guest are seated?

Solution:

If we fix the seat of one male and one female then

the required number of arrangements $= 8! \times 4! = 967680$

Q.10 Find the number of ways in which 5 men and 5 women can be seated at a round table in such a way that no two persons of the same sex sit together?

Solution:

If we fix a seat of one man or one woman then the number of ways in which 5 males and 5 females can be seated $= 5! \times 4! = 2880$.

Q.11 In how many ways can 4 keys by arranged on a circular key ring?

Solution:

The number of keys $= 4$

The number of ways in which 4 keys can be arranged in a circular key ring $= \frac{1}{2} \times 3! = 3$

Q.12 How many necklaces can be made from 6 beads of different colours?

(Gujranwala Board 2007)

Solution:

The number of beads $= 6$

The number of ways in which 6 beads can be arranged in a necklace $= \frac{1}{2} \times 5! = 60$

COMBINATION

A group or a set in which the order is immaterial.

For example the combination of three letters A, B, C taken all at a time is just one set while the permutation of these three A, B, C taken all at a time are 6 i.e.

ABC, ACB, BAC, CBA, CAB, BCA

Then number of combinations of 'n' different objects taken r at a time is denoted by ${}^nC_r$ or $C(n, r)$ or $\binom{n}{r}$ and is given by

$${}^nC_r = \frac{n!}{r!\,(n-r)!} \quad \text{when} \quad r \leq n$$

Also ${}^nC_r = {}^nC_{n-r}$

EXERCISE 7.4

Q.1 Evaluate the following:

(i) $^{12}C_3$ (Lahore Board 2008) (ii) $^{20}C_{17}$ (iii) $^{n}C_4$

Solution:

Using formula $^{n}C_r = \dfrac{n!}{r!\,(n-r)!}$

(i) $^{12}C_3 = \dfrac{12!}{3!\,(12-3)!} = \dfrac{12!}{3!\,9!} = 220$

(ii) $^{20}C_{17} = \dfrac{20!}{17!\,(20-17)!} = \dfrac{20!}{17!\,3!} = 1140$

(iii) $^{n}C_4 = \dfrac{n!}{4!\,(n-4)!} = \dfrac{n\,(n-1)\,(n-2)\,(n-3)\,(n-4)!}{4!\,(n-4)!} = \dfrac{n\,(n-1)\,(n-2)\,(n-3)}{4!}$

Q.2 Find the value of n, when

(i) $^{n}C_5 = {}^{n}C_4$ (Gujranwala Board 2007) (ii) $^{n}C_{10} = \dfrac{12 \times 11}{2!}$

(iii) $^{n}C_{12} = {}^{n}C_6$ (Lahore Board 2007, 2011)

Solution:

(i) $^{n}C_5 = {}^{n}C_4$

$\Rightarrow \dfrac{n!}{5!\,(n-5)!} = \dfrac{n!}{4!\,(n-4)!}$

$\Rightarrow \dfrac{1}{5.4!\,(n-5)!} = \dfrac{1}{4!\,(n-4)\,(n-5)!}$

$\Rightarrow \dfrac{1}{5} = \dfrac{1}{n-4}$

$\Rightarrow n - 4 = 5$

$\Rightarrow \boxed{n = 9}$

(ii) $^{n}C_{10} = \dfrac{12 \times 11}{2!}$

$\dfrac{n!}{10!\,(n-10)!} = \dfrac{12 \times 11 \times 10!}{10!\,2!}$

$\dfrac{n!}{10!\,(n-10)!} = \dfrac{12!}{10!\,(12-10)!}$

$\Rightarrow {}^{n}C_{10} = {}^{12}C_{10}$

$\Rightarrow \boxed{n = 12}$

(iii) $^{n}C_{12} = {}^{n}C_{6}$ ..... (1)

Using $^{n}C_{r} = {}^{n}C_{n-r}$

$\Rightarrow$ $^{n}C_{12} = {}^{n}C_{n-12}$

$\Rightarrow$ $^{n}C_{6} = {}^{n}C_{n-12}$ from equation (1)

$\Rightarrow$ $6 = n - 12$

$\Rightarrow$ $\boxed{n = 18}$

Q.3 Find the values of n and r, when

(i) $^{n}C_{r} = 35$ **and** $^{n}P_{r} = 210$ **(Lahore Board 2010)**

(ii) $^{n-1}C_{r-1}$: $^{n}C_{r}$: $^{n+1}C_{r+1} = 3 : 6 : 11$

(Gujranwala Board 2003, Lahore Board 2007)

Solution:

(i) Given that

$^{n}C_{r} = 35$

$\Rightarrow$ $\dfrac{n!}{r!\,(n-r)!} = 35$

$\Rightarrow$ $n! = 35r!\,(n-r)!$ ..... (1)

Also $^{n}P_{r} = 210$

$\Rightarrow$ $\dfrac{n!}{(n-r)!} = 210$

$\Rightarrow$ $n! = 210\,(n-r)!$ ..... (2)

Dividing (2) by (1), we get

$$\frac{n!}{n!} = \frac{210\,(n-r)!}{35r!\,(n-r)!}$$

$$1 = \frac{6}{r!}$$

$\Rightarrow$ $r! = 6 = 3!$

$\Rightarrow$ $r! = 3!$

$\Rightarrow$ $\boxed{r = 3}$

Put $r = 3$ in (2), we get

$n! = 210\,(n-3)!$

$\Rightarrow$ $n\,(n-1)\,(n-2)\,(n-3)! = 210\,(n-3)!$

$\Rightarrow$ $n\,(n-1)\,(n-2) = 210 = 7.6.5$

$\Rightarrow$ $\boxed{n = 7}$

(ii) $^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 3 : 6 : 11$

$^{n-1}C_{r-1} : {}^nC_r = 3 : 6$

$\Rightarrow \quad \dfrac{^{n-1}C_{r-1}}{{}^nC_r} = \dfrac{3}{6}$

$\Rightarrow \quad \dfrac{\dfrac{(n-1)!}{(r-1)!\,(n-1-r+1)!}}{\dfrac{n!}{r!\,(n-r)!}} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{(n-1)!}{(r-1)!\,(n-r)!} \dfrac{r\,(r-1)!\,(n-r)!}{n\,(n-1)!} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{r}{n} = \dfrac{1}{2}$

$\Rightarrow \quad n = 2r \qquad \ldots\ldots\ldots (1)$

Now $^nC_r : {}^{n+1}C_{r+1} = 6 : 11$

$\dfrac{^nC_r}{^{n+1}C_{r+1}} = \dfrac{6}{11}$

$\Rightarrow \quad \dfrac{\dfrac{n!}{r!\,(n-r)!}}{\dfrac{(n+1)!}{(r+1)!\,(n+1-r-1)!}} = \dfrac{6}{11}$

$\Rightarrow \quad \dfrac{n!}{r!\,(n-r)!} \times \dfrac{(r+1)\,r!\,(n-r)!}{(n+1)\,n!} = \dfrac{6}{11}$

$\Rightarrow \quad \dfrac{r+1}{n+1} = \dfrac{6}{11}$

$\Rightarrow \quad 6\,(n+1) = 11\,(r+1)$

$\Rightarrow \quad 6n + 6 - 11r - 11 = 0$

$\Rightarrow \quad 6n - 11r - 5 = 0$

$\Rightarrow \quad 6n - 11r = 5 \qquad \ldots\ldots\ldots (2)$

Put $n = 2r$ from (1) in (2), we get

$\Rightarrow \quad 6\,(2r) - 11r = 5$

$\Rightarrow \quad 12r - 11r = 5$

$\Rightarrow \quad \boxed{r = 5}$

Put $r = 5$ in (1), we get

$\Rightarrow \quad n = 2\,(5)$

$\boxed{n = 10}$

Q.4 How many (a) diagonals and (b) triangles can be formed by joining the vertices of polygon having

(i) 5 sides (ii) 8 sides (iii) 12 sides

Solution:

(a) As we know that the diagonal is a line segment joining two points. So

(i) We will find combinations of 5 taken 2 at a time

$\Rightarrow$ Total number of line segments $= {}^5C_2 = 10$

But Number of sides $= 5$

$\Rightarrow$ Number of diagonals $= 10 - 5 = 5$

(ii) Here we will find combinations of 8 taken two at a time

$\Rightarrow$ Total number of line segments $= {}^8C_2 = 28$

But Number of sides $= 8$

$\Rightarrow$ Number of diagonals $= 28 - 8 = 20$

(iii) Here we will find combinations of 12 taken two at a time

$\Rightarrow$ Total number of line segments $= {}^{12}C_2 = 66$

But Number of sides $= 12$

$\Rightarrow$ Number of diagonals $= 66 - 12 = 54$

(b) As we know that triangle has three vertices

(i) 5–sided

Number of triangles $= {}^5C_3 = 10$

(ii) 8–sided

Number of triangles $= {}^8C_3 = 56$

(iii) 12–sided

Number of triangles $= {}^{12}C_3 = 220$

Q.5 The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

Solution:

Number of committees having 3 boys out of 12 and 2 girls out of 8 $= {}^{12}C_3 \cdot {}^8C_2 = 6160$

Q.6 How many committees of 5 members can be chosen from a group of 8 persons when each committee must include 2 particular persons?

Solution:

If 2 particular persons are in each committee. Then

Number of ways to select 3 persons out of 6 $= {}^6C_3 = 20$

Q.7 In how many ways can a hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?

Solution:

Number of ways to select 11 players out of 15 $= {}^{15}C_{11} = 1365$

If one particular player is to be included in every team, then

Number of ways to select 10 players out of 14 $= {}^{14}C_{10} = 1001$

Q.8 Show that ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$.

Solution:

$${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$$

$$\text{L.H.S.} = {}^{16}C_{11} + {}^{16}C_{10}$$

$$= \frac{16!}{11!\,(16-11)!} + \frac{16!}{10!\,(16-10)!}$$

$$= \frac{16!}{11!\,5!} + \frac{16!}{10!\,6!}$$

$$= \frac{16!}{11.10!\,5!} + \frac{16!}{10!\,6.5!}$$

$$= \frac{16!}{10!\,5!}\left[\frac{1}{11} + \frac{1}{6}\right]$$

$$= \frac{16!}{10!\,5!}\left[\frac{6+11}{(11)\,(6)}\right]$$

$$= \frac{16!}{10!\,5!}\,\frac{17}{(11)\,(6)}$$

$$= \frac{17.16!}{11.10!\,6.5!} = \frac{17!}{11!\,6!}$$

$$= \frac{17!}{11!\,(17-11)!} = {}^{17}C_{11} = \text{R.H.S.}$$

Hence proved.

Q.9 There are 8 men and 10 women members of a club. How many committees of 7 can be formed, having;

(i) 4 women (ii) at the most 4 women (iii) at least 4 women?

Solution:

(i) 4 women

Number of committees having 4 women out of 10 and 3 men out of 8 $= {}^{10}C_4 \,.\, {}^{8}C_3 = 11760$

(ii) at the most 4 women

At the most 4 women means

(4W, 3M), (3W, 4M), (2W, 5M), (1W, 6M) (0W, 7M)

So Number of committees $= {}^{10}C_4\ {}^{8}C_3 + {}^{10}C_3\ {}^{8}C_4 + {}^{10}C_2\ {}^{8}C_5 + {}^{10}C_1\ .\ {}^{8}C_6 + {}^{10}C_0\ {}^{8}C_7$

$= 11760 + 8400 + 2520 + 280 + 8$

$= 22968$

(iii) at least 4 women

At least 4 women means

(4W, 3M), (5W, 2M), (6W, 1M), (7W, 0M)

So Number of committees $= {}^{10}C_4\ {}^{8}C_3 + {}^{10}C_5\ {}^{8}C_2 + {}^{10}C_6\ {}^{8}C_1 + {}^{10}C_7\ {}^{8}C_0$

$= 11760 + 7056 + 1680 + 120$

$= 20616$

Q.10 Prove that $\mathbf{{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r}$.

Solution:

$$\text{L.H.S.} = {}^{n}C_r + {}^{n}C_{r-1}$$

$$= \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$= \frac{n!}{r\,(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)\,(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n-r+1+r}{r\,(n-r+1)}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n+1}{r\,(n-r+1)}\right]$$

$$= \frac{(n+1)\,n!}{r\,(r-1)!\,(n-r+1)\,(n-r)!}$$

$$= \frac{(n+1)!}{r!\,(n-r+1)!} = {}^{n+1}C_r = \text{R.H.S.}$$

Hence proved.

PROBABILITY

Probability is the numerical evaluation of a chance that a particular event would occur.

SAMPLE SPACE AND EVENTS

The set S consisting of all possible outcomes of a given experiment is called the sample space.

A particular outcome is called an event and usually denoted by E.

An event is the subset of the sample space S.

PROBABILITY OF AN EVENT

To find the probability of an event we use the formula

$$P(E) = \frac{n(E)}{n(S)} = \frac{\text{no. of ways in which event occurs}}{\text{no. of the elements of the sample space}}$$

Since the number of outcomes in an event is less than or equal to the number of outcomes in the sample space, the probability of an event must be a number between 0 and 1.

i.e. $0 \leq P(E) \leq 1$

PROBABILITY THAT AN EVENT DOES NOT OCCUR

The formula to find the probability that an event does not occur is given by

$$P(\bar{E}) = 1 - P(E)$$

EXERCISE 7.5

For the following experiments, find the probability in each case:

Q.1 Experiment:

From a box containing orange flavoured sweets, Bilal takes out one sweet without looking

Events Happening:

(i) The sweet is orange flavoured **(ii) The sweet is lemon flavoured**

Solution:

As the box contain just orange flavoured sweets

$\Rightarrow$ $n(S) = 1$

(i) Let A = Event: The sweet is orange – flavoured

$\Rightarrow$ $n(A) = 1$

So $P(A) = \frac{n(A)}{n(S)} = \frac{1}{1} = 1$

(ii) Let B = Event: The sweet is lemon – flavoured

$\Rightarrow$ $n(B) = 0$

So $P(B) = \frac{n(B)}{n(S)} = \frac{0}{1} = 0$

Q.2 Experiment:

Pakistan and India play a cricket match. The result is:

Events Happening:

(i) Pakistan wins (ii) India does not lose.

Solution:

Here total possible outcomes of match are three

$\Rightarrow$ $n(S) = 3$

(i) Let

A = Event: Pakistan wins

$\Rightarrow$ $n(A) = 1$

So $P(A) = \frac{n(A)}{n(S)} = \frac{1}{3}$

(ii) Let

B = Event: India does not lose

$\Rightarrow$ $n(B) = 2$

So $P(B) = \frac{n(B)}{n(S)} = \frac{2}{3}$

Q.3 Experiment

There are 5 green and 3 red balls in a box, one ball is taken out.

Events Happening:

(i) The ball is green (ii) The ball is red.

Solution:

As there are 5 green and 3 red balls in box

$\Rightarrow$ $n(S) = 8$

(i) Let

A = Event: The ball is green.

$\Rightarrow$ $n(A) = 5$

So $P(A) = \frac{n(A)}{n(S)} = \frac{5}{8}$

(ii) Let

B = Event: The ball is red

$\Rightarrow$ $n(B) = 3$

So $P(B) = \frac{n(B)}{n(S)} = \frac{3}{8}$

Q.4 Experiment

A fair coin is tossed three times. It shows

Events Happening:

(i) one tail (ii) at least one head.

Solution:

If a fair coin is tossed three times, we get 8 outcomes such that

$S = \{HHH, HHT, HTH, THH, THH, TTH, THT, HTT, TTT\}$

(i) Let

A = Event: One tail $= \{HHT, HTH, THH\}$

$\Rightarrow \quad n(A) = 3$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{8}$

(ii) Let

B = Event: at least one head $= \{HHH, HHT, HTH, THH, TTH, THT, HTT\}$

$\Rightarrow \quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{7}{8}$

Q.5 A die is rolled. The top shows

Events Happening:

(i) 3 or 4 dots (ii) dots less than 5

Solution:

As we know that a die has six faces

$\Rightarrow \quad S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

(i) Let

A = Event: 3 or 4 dots $= \{3, 4\}$

$\Rightarrow \quad n(A) = 2$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{2}{6} = \dfrac{1}{3}$

(ii) Let

B = Event: dots less than 5 $= \{1, 2, 3, 4\}$

$\Rightarrow \quad n(B) = 4$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{6} = \dfrac{2}{3}$

Q.6 Experiment:

From a box containing slips numbered 1, 2, 3, 4, 5 one slip is picked up.

Event Happening:

(i) The number on the slip is a prime number

(ii) The number on the slip is a multiple of 3.

Solution:

Here $S = \{1, 2, 3, 4, 5\}$

$\Rightarrow \quad n(S) = 5$

(i) Let

A = Event: The number on the slip is a prime number $= \{2, 3, 5\}$

$\Rightarrow \quad n(A) = 3$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{3}{5}$

(ii) Let

B = Event: The number on the slip is a multiple of 3 $= \{3\}$

$\Rightarrow \quad n(B) = 1$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{1}{5}$

Q.7 Experiment:

Two dice, one red and the other blue, are rolled simultaneously. The number of dots on the tops are added. The total of two score is:

Events Happening:

(i) 5 (ii) 7 (iii) 11

Solution:

When two dice are rolled, the total possible outcomes are

$$S = \left\{ \begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix} \right\}$$

$\Rightarrow \quad n(S) = 36$

(i) Let

A = Event: The total of two scores is 5

$= \{(1, 4), (2, 3), (3, 2), (4, 1)\}$

$\Rightarrow \quad n(A) = 4$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{36} = \dfrac{1}{9}$

(ii) Let

B = Event: The total of two scores is 7

= {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

$\Rightarrow$ n (B) = 6

So $P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(iii) Let

C = Event: The total of two scores is 11

= {(5, 6), (6, 5)}

$\Rightarrow$ n (C) = 2

So $P(C) = \frac{n(C)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

Q.8 Experiment:

A bag contains 40 balls out of which 5 are green, 15 are black and the remaining are yellow. A ball is taken out of the bag.

Events Happening:

(i) The ball is black (ii) The ball is green (iii) The ball is not green

Solution:

Total balls = 40

$\Rightarrow$ n (S) = 40

(i) Let

A = Event: The ball is black

$\Rightarrow$ n (A) = 15

So $P(A) = \frac{n(A)}{n(S)} = \frac{15}{40} = \frac{3}{8}$

(ii) Let

B = Event: The ball is green

$\Rightarrow$ n (B) = 5

So $P(B) = \frac{n(B)}{n(S)} = \frac{5}{40} = \frac{1}{8}$

(iii) Let

C = Event: The ball is not green

$\Rightarrow$ n (C) = 35

So $P(C) = \frac{n(C)}{n(S)} = \frac{35}{40} = \frac{7}{8}$

Q.9 Experiment:

One chit out of 30 containing the names of 30 students of a class of 18 boys and 12 girls is taken out at random, for nomination as the nominator of the class.

Events Happening:

(i) The monitor is a boy (ii) The monitor is a girl

Solution:

Total students = 30

$\Rightarrow$ $n(S) = 30$

(i) Let

A = Event: The monitor is a boy

$\Rightarrow$ $n(A) = 18$

So $P(A) = \frac{n(A)}{n(S)} = \frac{18}{30} = \frac{3}{5}$

(ii) Let

B = Event: The monitor is a girl

$\Rightarrow$ $n(B) = 12$

So $P(B) = \frac{n(B)}{n(S)} = \frac{12}{30} = \frac{2}{5}$

Q.10 A coin is tossed four times. The tops show

Events Happening:

(i) All heads (ii) Two heads and two tails

Solution:

If a coin is tossed four times the total number of outcomes are 16.

$\Rightarrow$ $n(S) = 16$

(i) Let

A = Event: The top shows all heads

= {HHHH}

$\Rightarrow$ $n(A) = 1$

So $P(A) = \frac{n(A)}{n(S)} = \frac{1}{16}$

(ii) Let

B = Event: The top shows 2 heads and 2 tails

= {HHTT, HTHT, THTH, THHT, HTTH, TTHH}

$\Rightarrow$ $n(B) = 6$

So $P(B) = \frac{n(B)}{n(S)} = \frac{6}{16} = \frac{3}{8}$

EXERCISE 7.6

Q.1 A fair coin is tossed 30 times, the result of which is tabulated below. Study the table and answer the questions given below the table.

Events	Tally Marks	Frequency
Head	𝍸 𝍸 ||||	14
Tail	𝍸 𝍸 𝍸 |	16

(i) How many time does "head" appear?

(ii) How many time does tail appears?

(iii) Estimate the probability of the appearance of head?

(iv) Estimate the probability of the appearance of tail?

Solution:

Total outcomes = 30

$\Rightarrow$ n (S) = 30

(i) Head appears = 14 times

(ii) Tail appears = 16 times

(iii) Probability of appearance of head = $P(H) = \frac{14}{30} = \frac{7}{15}$

(iv) Probability of appearance of tail = $P(T) = \frac{16}{30} = \frac{8}{15}$

Q.2 A die is tossed 100 times. The result is tabulated below. Study the table and answer the question given below:

Solution:

Events	Tally Marks	Frequency
1	𝍸 𝍸 ||||	14
2	𝍸 𝍸 𝍸 ||	17
3	𝍸 𝍸 𝍸 𝍸	20
4	𝍸 𝍸 𝍸 |||	18
5	𝍸 𝍸 𝍸	15
6	𝍸 𝍸 𝍸 |	16

(i) How many times do 3 dots appear?

(ii) How many times 5 dots appear?

(iii) How many times does an even number of dots appear?

(iv) How many times does a prime number of dots appear?

(v) Find the probability of each one of the above cases.

Solution:

Total outcomes = 30

$\Rightarrow$ n (S) = 30

From table, we see that

(i) 3 dots appear = 20 times

(ii) 5 dots appear = 15 times

(iii) Even number of dots (2, 4, 6) appear = 17 + 18 + 16 = 51 times

(iv) Prime number of dots (2, 3, 5) appear = 17 + 20 + 15 = 52 times

(v) $P(3 \text{ dots appear}) = \frac{20}{100} = \frac{1}{5}$

$P(5 \text{ dots appear}) = \frac{15}{100} = \frac{3}{20}$

$P(2, 4, 6) = \frac{51}{100}$

$P(2, 3, 5) = \frac{52}{100} = \frac{18}{25}$

Q.3 The eggs supplied by a poultry farm during a week broke during transit as follows:

$$\mathbf{1\%, 2\%, 1\frac{1}{2}\%, \frac{1}{2}\%, 1\%, 2\%, 1\%}$$

Find the probability of the eggs that broke in a day. Calculate the number of eggs that will be broken in transiting the following number of eggs:

(i) 7,000 (ii) 8,400 (iii) 10,500

Solution:

Eggs that broke in a week $= 1\% + 2\% + 1\frac{1}{2}\% + \frac{1}{2}\% + 1\% + 2\% + 1\% = 9\%$

Eggs that broke in a day $= \frac{9}{7}\%$

(i) Number of eggs that will be broken out of 7000 $= 7000 \times \frac{9}{7}\%$

$= 7000 \times \frac{9}{7} \times \frac{1}{100} = 90$

(ii) Number of eggs that will be broken out of 8400 $= 8400 \times \frac{9}{7}\%$

$= 8400 \times \frac{9}{7} \times \frac{1}{100} = 108$

(iii) Number of eggs that will be broken out of 10500 $= 10500 \times \frac{9}{7}\%$

$= 10500 \times \frac{9}{7} \times \frac{1}{100} = 135$

MUTUALLY EXCLUSIVE EVENTS

If a sample space $S = \{1, 3, 5, 7, 9\}$ and an event $A = \{1, 3, 5\}$ and another event $B = \{9\}$, then A and B are disjoint sets and they are said to be mutually exclusive events.

EQUALLY LIKELY EVENTS

If two events A and B occur in an experiment then A and B are said to be equally likely events if each one of them has equal number of chances of occurrence.

ADDITION OF PROBABILITIES

If A and B are two events, then the formulas for the addition of probabilities are:

(i) $P(A \cup B) = P(A) + P(B)$, when A and B are disjoint.

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ when A and B are overlapping or $B \subseteq A$.

EXERCISE 7.7

Q.1 If sample spaces = {1, 2, 3, ... 9}, Event A = {2, 4, 6, 8} and Event B = {1, 3, 5} find $P(A \cup B)$

Solution:

Here $S = \{1, 2, 3, \ldots\ldots 9\} \Rightarrow n(S) = 9$

$A = \{2, 4, 6, 8\} \Rightarrow n(A) = 4$

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{9}$

Also $B = \{1, 3, 5\} \Rightarrow n(B) = 3$

$\Rightarrow P(B) = \frac{n(B)}{n(S)} = \frac{3}{9} = \frac{1}{3}$

As A and B are disjoint or mutually exclusive events. So

$P(A \cup B) = P(A) + P(B)$

$= \frac{4}{9} + \frac{1}{3} = \frac{4+3}{9} = \frac{7}{9}$

Q.2 A box contains 10 red, 30 white, and 20 black marbles. A marble is drawn at random. Find the probability that it is either red or white.

Solution:

Total marbles $= 10 + 30 + 20 = 60$

$\Rightarrow$ $n(S) = 60$

Let A = Event: the drawing marble is red

$\Rightarrow$ $n(A) = 10$

So $P(A) = \frac{n(A)}{n(S)} = \frac{10}{60} = \frac{1}{6}$

Let B = Event: The drawing marble is white

$\Rightarrow$ $n(B) = 30$

So $P(B) = \frac{n(B)}{n(S)} = \frac{30}{60} = \frac{1}{2}$

As A and B are disjoint or mutually exclusive events.

So $P(A \cup B) = P(A) + P(B)$

$= \frac{1}{6} + \frac{1}{2} = \frac{1+3}{6} = \frac{4}{6} = \frac{2}{3}$

Q.3 A natural number is chosen out of the first fifty natural numbers. What is the probability that the chosen numbers is a multiple of 3 or of 5?

Solution:

$S = \{1, 2, 3, \ldots\ldots, 50\} \quad \Rightarrow \quad n(S) = 50$

Let A = Event: the chosen number is a multiple of 3

$= \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48\}$

$\Rightarrow$ $n(A) = 16$

So $P(A) = \frac{n(A)}{n(S)} = \frac{16}{50} = \frac{8}{25}$

Let B = Event: The chosen number is a multiple of 5

$= \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$

$\Rightarrow$ $n(B) = 10$

So $P(B) = \frac{n(B)}{n(S)} = \frac{10}{50} = \frac{1}{5}$

Now $A \cap B = \{15, 30, 45\} \quad \Rightarrow \quad n(A \cap B) = 3$

So $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{50}$

As A and B are not mutually exclusive events. So

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{8}{25} + \frac{1}{5} - \frac{3}{50} = \frac{16 + 10 - 3}{50} = \frac{23}{50}$

Q.4 A card is drawn from a deck of 52 playing cards. What is the probability that it is a diamond card or an ace?

Solution:

Total cards = 52 $\Rightarrow$ n (S) = 52

Let A = Event: the drawing card is a diamond card

$\Rightarrow$ n (A) = B

so $P(A) = \frac{n(A)}{n(S)} = \frac{13}{52}$

Let B = Event: the drawing card is an ace card

$\Rightarrow$ n (B) = 4

so $P(B) = \frac{n(B)}{n(S)} = \frac{4}{52}$

Now $A \cap B$ = The ace of diamond

$\Rightarrow$ $n(A \cap B) = 1$

$\Rightarrow$ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{52}$

As A and B are not mutually exclusive events.

$\Rightarrow$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Q.5 A die is thrown twice. What is the probability that the sum of the number of dots shown is 3 or 11?

Solution:

When two die is thrown twice, the possible outcomes are

$$S = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$\Rightarrow$ n (S) = 36

Let A = Event: sum of the number of dots is 3

= {(1, 2), (2, 1)}

$\Rightarrow$ n (A) = 2

So $P(A) = \frac{n(A)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

Let B = Event: sum of the number of dots is 11

$= \{(5, 6), (6, 5)\}$

$\Rightarrow \quad n(B) = 2$

so $P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

As A and B are mutually exclusive events so

$P(A \cup B) = P(A) + P(B)$

$= \frac{1}{18} + \frac{1}{18} = \frac{2}{18} = \frac{1}{9}$

Q.6 Two dice are thrown. What is the probability that the sum of the number of dots appearing on them is 4 or 6?

Solution:

When two dice are thrown, the possible outcomes are

$$S = \left\{ \begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix} \right\}$$

$\Rightarrow \quad n(S) = 36$

Let A = Event: sum of the number of dots is 4

$= \{(1, 3), (2, 2), (3, 1)\}$

$\Rightarrow \quad n(A) = 3$

So $P(A) = \frac{n(A)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

Let B = Event: sum of the number of dots is 6

$= \{(1, 5), (2, 4), (3,3), (4, 2), (5, 1)\}$

$\Rightarrow \quad n(B) = 5$

so $P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

As A and B are mutually exclusive events so

$P(A \cup B) = P(A) + P(B)$

$= \frac{1}{12} + \frac{5}{36} = \frac{3+5}{36} = \frac{8}{36} = \frac{2}{9}$

Q.7 Two dice are thrown simultaneously. If the event A is that the sum of the number of dots shown is an odd number and the event B is that the number of dots shown on at least one die is 3. Find P (A∪B).

Solution:

When two dice are thrown, then possible outcomes are

$$S = \left\{ \begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix} \right\}$$

$\Rightarrow \quad n(S) = 36$

Let A = Event: sum of the number of dots is odd

$= \{(1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), (4,5)$

$(5,2), (5,4), (5,6), (6,1), (6,3), (6,5)\}$

$\Rightarrow \quad n(A) = 18$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{18}{36} = \dfrac{1}{2}$

Let B = Event: number of dots shown on at least one die is 3

$= \{(1,3), (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,3), (5,3), (6,3)\}$

$\Rightarrow \quad n(B) = 11$

so $P(B) = \dfrac{n(B)}{n(S)} = \dfrac{11}{36}$

Now $A \cap B = \{(2,3), (3,2), (3,4), (3,6), (4,3), (6,3)\}$

$n(A \cap B) = 6$

So $P(A \cap B) = \dfrac{n(A \cap B)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

As A and B are not mutually exclusive events so

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{11}{36} - \frac{1}{6} = \frac{18 + 11 - 6}{36} = \frac{23}{36}$$

Q.8 There are 10 girls and 20 boys in a class. Half of the boys and half of the girls have blue eyes. Find the probability that that one student chosen as monitor is either a girl or has blue eyes.

Solution:

Total number of students $= 10 + 20 = 30$

$\Rightarrow$ $n(S) = 30$

Let A = Event: the chosen student is a girl

$n(A) = 10$

So $P(A) = \frac{n(A)}{n(S)} = \frac{10}{30} = \frac{1}{3}$

Let B = Event: the chosen student has blue eyes

$\Rightarrow$ $n(B) = 15$

so $P(B) = \frac{n(B)}{n(S)} = \frac{15}{30} = \frac{1}{2}$

Let $A \cap B$ = The chosen student is a girl and has blue eyes

$n(A \cap B) = 5$

So $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{5}{30} = \frac{1}{6}$

As A and B are not mutually exclusive events

So $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{3} + \frac{1}{2} - \frac{1}{6}$$

$$= \frac{2 + 3 - 1}{6} = \frac{4}{6} = \frac{2}{3}$$

MULTIPLICATION OF PROBABILITIES

Two events A and B are said to be independent, if the occurrence of any one of them does not influence the occurrence of the other event.

THEOREM

If A and B are two independent events, the probability that both of them occur is equal to the probability of the occurrence of A multiplied by the probability of the occurrence of B. Symbolically, it is denoted as:

$P(A \cap B) = P(A) \cdot P(B)$

EXERCISE 7.8

Q.1 The probability that a person A will be alive 15 years hence is $\frac{5}{7}$ and the probability that another person B will be alive 15 years hence $\frac{7}{9}$. Find the probability that both will be alive 15 years hence.

Solution:

Let A = Event: person A will be alive for 15 years given that $P(A) = \frac{5}{7}$

Let B = Event: person B will be alive for 15 years given that $P(B) = \frac{7}{9}$

So $P(A \cap B) = P(A) \cdot P(B) = \frac{5}{7}\,\frac{7}{9} = \frac{5}{9}$

Q.2 A die is rolled twice: Event E_1 is the appearance of even number of dots and event E_2 is the appearance of more than 4 dots prove that

$P(E_1 \cap E_2) = P(E_1)\,P(E_2)$

Solution:

When a die is rolled twice, the possible outcomes are

$$S = \left\{ \begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix} \right\}$$

$\Rightarrow$ $n(S) = 36$

Let E_1 = Event of appearance of even number of dots

= {(2, 2), (2, 4), (2, 6), (4, 2), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}

$\Rightarrow$ $n(E_1) = 9$

So $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

Let E_2 = Event of appearance of more than 4 dots

= {(5, 5), (5, 6), (6, 5), (6, 6)}

$\Rightarrow$ $n(E_2) = 4$

so $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Now $E_1 \cap E_2 = \{(6, 6)\}$

$\Rightarrow$ $n(E_1 \cap E_2) = 1$

So $P(E_1 \cap E_2) = \dfrac{n(E_1 \cap E_2)}{n(S)} = \dfrac{1}{36}$ ..... (1)

and $P(E_1) \cdot P(E_2) = \dfrac{1}{4} \cdot \dfrac{1}{9} = \dfrac{1}{36}$ ..... (2)

From (1) and (2).

It is clear that

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

Hence proved.

Q.3 Determine the probability of getting 2 heads in two successive tosses of a balanced coin.

Solution:

When two coins are tossed the possible outcomes are

$S = \{HH, HT, TH, TT\}$

$\Rightarrow$ $n(S) = 4$

Let A = Event: getting two heads

$\Rightarrow$ $n(A) = 1$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{1}{4}$

Q.4 Two coins are tossed twice each. Find the probability that the head appears on the first toss and the same faces appear in the two tosses.

Solution:

Possible outcomes in first and second toss are same

i.e. $S = \{HH, HT, TH, TT\}$

$\Rightarrow$ $n(S) = 4$

Let A = Event: head appears in the first toss

$= \{TH, HT\}$

$\Rightarrow$ $n(A) = 2$

So $P(A) = \dfrac{n(A)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$

Let B = Event: the same faces appear in the second toss

$= \{HH, TT\}$

$\Rightarrow \quad n(B) = 2$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{4} = \dfrac{1}{2}$

So $\quad P(A \cap B) = P(A) \cdot P(B)$

$= \dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{1}{4}$

Q.5 Two cards are drawn from a deck of 52 playing cards. If one card is drawn and replaced before drawing the second card, find the probability that both the cards are aces.

Solution:

Total cards $= 52 \quad \Rightarrow \quad n(S) = 52$

Let $\quad A =$ Event: drawing card is an ace

$\Rightarrow \quad n(A) = 4$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

Let $\quad B =$ Event: drawing card is an ace

$\Rightarrow \quad n(B) = 4$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

and $\quad P(A \cap B) = P(A) \cdot P(B) = \dfrac{1}{13} \cdot \dfrac{1}{13} = \dfrac{1}{169}$

Q.6 Two cards from a deck of 52 playing cards are drawn in such a way that card is replaced after the first draw. Find the probability in the following cases:

(i) first card is king and second is queen.

(ii) both the cards are faced cards i.e. king, queen, jack.

Solution:

Total cards $= 52 \quad \Rightarrow \quad n(S) = 52$

(i) Let $\quad A =$ Event: the first card is king

$\Rightarrow \quad n(A) = 4$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

Let $\quad B =$ Event: second card is a queen

$\Rightarrow \quad n(B) = 4$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{4}{52} = \dfrac{1}{13}$

$\therefore \quad P(A \cap B) = P(A) \cdot P(B)$

$= \dfrac{1}{13} \cdot \dfrac{1}{13} = \dfrac{1}{169}$

(ii) Let

A = Event: the first card is faced card

$\Rightarrow \quad n(A) = 12$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{12}{52} = \dfrac{3}{13}$

Let $\quad B$ = Even: the second card is also faced card

$n(B) = 12$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{12}{52} = \dfrac{3}{13}$

$\therefore \quad P(A \cap B) = P(A) \cdot P(B)$

$= \dfrac{3}{13} \cdot \dfrac{3}{13} = \dfrac{9}{169}$

Q.7 Two dice are thrown twice. What is the probability that sum of the dots shown in the first throw is 7 and that of the second throw is 11?

Solution:

When two dice are thrown twice, the possible outcomes are

$$S = \left\{ \begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix} \right\}$$

$\Rightarrow \quad n(S) = 36$

Let $\quad A$ = Event: sum of dots is 7

$= \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$\Rightarrow \quad n(A) = 6$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

Let $\quad B$ = Event: sum of dots is 11

$= \{(5,6), (6,5)\}$

$\Rightarrow \quad n(B) = 2$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{2}{36} = \dfrac{1}{18}$

$\therefore \quad P(A \cap B) = P(A) \cdot P(B)$

$= \dfrac{1}{6} \cdot \dfrac{1}{18} = \dfrac{1}{108}$

Q.8 Find the probability that the sum of dots appearing in two successive throws of two dice is every time 7.

Solution:

The possible outcomes are

$$S = \left\{ \begin{matrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{matrix} \right\}$$

$\Rightarrow \quad n(S) = 36$

Let $\quad A =$ Event: sum of dots is 7

$= \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$\Rightarrow \quad n(A) = 6$

So $\quad P(A) = \dfrac{n(A)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

Let $\quad B =$ Event: sum of dots in second throw is 7

$= \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$

$\Rightarrow \quad n(B) = 6$

So $\quad P(B) = \dfrac{n(B)}{n(S)} = \dfrac{6}{36} = \dfrac{1}{6}$

$\therefore \quad P(A \cap B) = P(A) \cdot P(B)$

$= \dfrac{1}{6} \cdot \dfrac{1}{6} = \dfrac{1}{36}$

Q.9 A fair die is thrown twice. Find the probability that a prime number of dots appear in the first throw and the number of dots in the second throw is less than 5.

Solution:

We have same outcomes in the first and second throw.

When a die is throw the possible number of dots are

$S = \{1, 2, 3, 4, 5, 6\} \quad \Rightarrow \quad n(S) = 6$

Let $\quad A =$ Event: prime dots appear

$= \{2, 3, 5\}$

$\Rightarrow \quad n(A) = 3$

So $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

Let B = Event: number of dots are less than 5

$= \{1, 2, 3, 4\}$

$\Rightarrow n(B) = 4$

So $P(B) = \frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$= \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

Q.10 A bag containing 8 red, 5 white and 7 black balls. 3 balls are drawn from the bag. What is the probability that the first ball is red, the second ball is white and the third ball is black, when every time the ball drawn is replaced?

Solution:

Total balls $= 8 + 5 + 7 = 20$

$\Rightarrow n(S) = 20$

Let A = Event: drawing ball is red

$\Rightarrow n(A) = 8$

So $P(A) = \frac{n(A)}{n(S)} = \frac{8}{20} = \frac{2}{5}$

Let B = Event: drawing ball is white

$\Rightarrow n(B) = 5$

So $P(B) = \frac{n(B)}{n(S)} = \frac{5}{20} = \frac{1}{4}$

Let C = Event: drawing ball is black

$\Rightarrow n(C) = 7$

So $P(C) = \frac{n(C)}{n(S)} = \frac{7}{20}$

$\therefore P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$= \frac{2}{5} \cdot \frac{1}{4} \cdot \frac{7}{20} = \frac{7}{200}$

Chapter 7

PERMUTATION, COMBINATION AND PROBABILITY

MULTIPLE CHOICE QUESTIONS

(1) ${}^6P_4$ = ————

(a) 36 (b) 360

(c) 6 (d) 4

[Lahore Board 2005]

(2) The value of ${}^{10}P_2$ is:

(a) 0 (b) 20

(c) 90 (d) 80

[Lahore Board 2005]

(3) ${}^nC_{r-1} + {}^nC_{r-2}$ is equal to:

(a) ${}^nC_{r-1}$ (b) ${}^{n+1}C_{r-1}$

(c) ${}^{n+1}C_{r-2}$ (d) ${}^nC_{r-2}$

[Gujranwala Board 2005]

(4) If ${}^nP_2 = 30$ then n = ————

(a) 2 (b) 4

(c) 6 (d) 8

[Gujranwala Board 2005]

(5) ${}^6P_3$ is equal to:

(a) 36 (b) 360

(c) 6 (d) 120

[Gujranwala Board 2005]

(6) n different objects can be arranged taken all at a time, in:

(a) $(n-1)!$ ways (b) $(n+1)!$ ways

(c) $n!$ ways (d) $(2n)!$ ways

[Gujranwala Board 2006]

(7) ${}^{n+1}C_r =$

(a) ${}^{n}C_r + {}^{n}C_{r+1}$ (b) ${}^{n}C_r + {}^{n}C_{r-1}$

(c) ${}^{n}C_r + {}^{n}C_r$ (d) None of these

[Gujranwala Board 2006]

(8) ${}^{n}C_r \times r! =$

(a) ${}^{n}P_r$ (b) ${}^{n+1}C_r$

(c) ${}^{n}C_{r+1}$ (d) None of these

[Gujranwala Board 2006]

(9) ${}^{n}P_n =$

(a) $n!$ (b) $0!$

(c) $1!$ (d) None of these

[Gujranwala Board 2006]

(10) ${}^{n}C_{n-r} =$

(a) ${}^{n+1}C_r$ (b) ${}^{n}C_r$

(c) ${}^{n-1}C_{n-r}$ (d) None of these

[Lahore Board 2006]

(11) Number of diagonals of a 3-sided figure are _____

(a) 0 (b) 1

(c) 2 (d) 3

[Lahore Board 2006]

(12) $\frac{(n+1)!}{(n-1)!} =$ _____

(a) $n^2(n+1)$ (b) $n^2 + n$

(c) $n + 1$ (d) None of these

[Lahore Board 2006]

(13) ${}^{n}C_0 =$ _____

(a) 0 (b) + 1

(c) n (d) 2

[Lahore Board 2006]

(14) ${}^{n}P_r$ = _____

(a) $r \cdot {}^{n}C_r$ (b) $\frac{1}{r} {}^{n}C_r$

(c) $\lfloor r \, {}^{n}C_r$ (d) $\frac{{}^{n}C_r}{r}$

(15) If A and B are disjoint events, then $P(A \cup B)$ = _____

(a) $P(A) + P(B)$ (b) $P(A) - P(B)$

(c) $P(A) \backslash P(B)$ (d) $P(A) + P(B) = P(A \cap B)$

[Gujranwala Board 2007, Lahore Board 2007]

(16) There are 5 green and 3 red balls in a box. One ball is taken out, the probability that the ball drawn is yellow is:

(a) $\frac{5}{8}$ (b) $\frac{3}{8}$

(c) 1 (d) Zero

[Gujranwala Board 2007]

(17) Factorial form of $n(n-1)(n-2) \ldots\ldots\ldots (n-r+1)$ is:

(a) $\lfloor n$ (b) $\lfloor n-1$

(c) $\frac{\lfloor n}{\lfloor n-r}$ (d) $\frac{\lfloor n}{\lfloor n-r+1}$

[Gujranwala Board 2007]

(18) If $S = \{1, 2, 3, 4\}$, $A = \{1, 2\}$, $P(A)$ is:

(a) 2 (b) 4

(c) 1 (d) 1/2

[Lahore Board 2007]

(19) In how many ways can 6 keys be arranged on a key ring:

(a) 60 (b) 840

(c) 120 (d) 5

[Lahore Board 2007]

(20) 0! is equal to:

(a) 0 (b) 1

(c) 10 (d) 100

[Lahore Board 2007]

(21) With usual notation ${}^{6}P_{4}$ equals:

(a) 160 (b) 260

(c) 360 (d) 340

[Lahore Board 2008]

(22) A die is rolled then n(S) equals:

(a) 36 (b) 6

(c) 1 (d) 9

[Lahore Board 2008]

(23) With usual notation, ${}^{n}C_{r}$ equals:

(a) ${}^{n}C_{r-n}$ (b) ${}^{r}C_{n}$

(c) ${}^{n}C_{n-r}$ (d) ${}^{n-r}C_{n}$

[Lahore Board 2008]

(24) If n(S) = 20, n(B) = 2 then P(B) equals:

(a) 10 (b) $\frac{1}{10}$

(c) $-\frac{1}{10}$ (d) 1

[Lahore Board 2008]

(25) What is the probability that a slip of numbers divisible by 4 are picked from the slips bearing numbers 1, 2, 3, ... 10?

(a) $\frac{1}{5}$ (b) $\frac{5}{2}$

(c) $\frac{2}{5}$ (d) $-\frac{1}{5}$

[Gujranwala Board 2008]

(26) With usual notation ${}^{6}C_{2}$ = _____

(a) 30 (b) 20

(c) 12 (d) 15

[Gujranwala Board 2008]

(27) The number of 3-digit numbers formed by using each of the digits 2, 3, 5, 7, 9 only once is:

(a) 120 (b) 360

(c) 60 (d) 10

[Gujranwala Board 2009]

(28) From a box containing 5 green and 3 red balls, one ball is taken out. The probability that the ball drawn is black is:

(a) 1 (b) 1/2

(c) 1/8 (d) Zero

[Gujranwala Board 2009]

(29) ${}^{n}C_{r} + {}^{n}C_{r-1}$ equals:

(a) ${}^{n}C_{r}$ (b) ${}^{n+1}C_{r}$

(c) ${}^{n+1}C_{r+1}$ (d) ${}^{n-1}C_{r-1}$

[Lahore Board 2009]

(30) For an event E which one is true:

(a) $0 \leq P(E) \leq 1$ (b) $1 \leq P(E) \leq 2$

(c) $0 \geq P(E) \leq 1$ (d) $-1 \leq P(E) \leq 1$

[Lahore Board 2009]

(31) ${}^{4}P_{3}$ equals:

(a) ${}^{4}P_{4}$ (b) ${}^{4}P_{2}$

(c) ${}^{4}P_{1}$ (d) ${}^{5}P_{4}$

[Lahore Board 2009]

(32) $P(\bar{E})$ equals:

(a) $1 + P(E)$ (b) $P(E) - 1$

(c) $2 - P(E)$ (d) $1 - P(E)$

[Lahore Board 2009-10]

(33) 5 keys can be arranged in a circular ring in number of ways:

(a) 24 (b) 12

(c) 6 (d) 5

[Lahore Board 2010, Lahore Board 2015]

(34) A die is rolled once. The probability that the dots on the top are greater than four is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) 3! (d) $\frac{6!}{2!}$

[Gujranwala Board 2010]

(35) The factorial form of 6.5.4 is:

(a) $\frac{6!}{3!}$ (b) $6!$

(c) $3!$ (d) $\frac{6!}{2!}$

[Gujranwala Board 2010]

(36) If a coin is tossed the probability of head is:

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{2}$

[Lahore Board 2010-11]

(37) Factorial form of $n(n-1)(n-2)$ = _____

(a) $\frac{n!}{(n-1)!}$ (b) $\frac{n!}{(n-2)!}$

(c) $\frac{n!}{(n-3)!}$ (d) $\frac{n!}{(n+3)!}$

[Lahore Board 2011]

(38) The probability to get an odd number in a dice thrown once is:

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) 2 (d) $\frac{1}{3}$

[Lahore Board 2012]

(39) ${}^{n}p_{n}$ =

(a) $n!$ (b) $\frac{(n-1)!}{(n+1)!}$

(c) $\frac{n!}{(n-1)!}$ (d) $(n+1)!$

[Lahore Board 2012]

(40) The probability that the dots on the top are greater than 4 when a die is rolled is:

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

[Lahore Board 2012]

(41) ${}^{n}C_{n}$ is equal to:

(a) 0 (b) 1

(c) n (d) n!

[Lahore Board 2013]

(42) Factorial form of $(n + 2)(n + 1)n$, is:

(a) $\frac{(n+2)!}{(n-1)!}$ (b) $\frac{(n+2)!}{n!}$

(c) $\frac{n!}{(n+2)!}$ (d) $(n + 2)!$

[Lahore Board 2013]

(43) $5! =$

(a) 140 (b) −120

(c) 120 (d) 0

[Lahore Board 2013]

(44) How many signals can be given by 5 flags of different colours, using 3 flags at a time?

(a) 40 (b) 50

(c) 60 (d) 70

(45) ${}^{n}P_{r} =$ ————

(a) $n\ {}^{n-1}P_{r-1}$ (b) $n\ {}^{n}P_{r}$

(c) ${}^{n-1}P_{r-1}$ (d) None of these

(46) $\binom{n}{n_1, n_2} =$ ————

(a) $\frac{n}{n_1 \cdot n_2}$ (b) $\frac{n!}{n_1!\, n_2!}$

(c) ${}^{n}C_{n1} \cdot {}^{n}C_{n2}$ (d) None of these

(47) In how many ways can a necklace of 8 beads of different colours be made?

(a) 1250 (b) 12

(c) 2620 (d) 2520

(48) $C(n, r) =$ ————

(a) $\frac{n!}{(n-r)!}$ (b) $\frac{1}{n!\, r!}$

(c) $\frac{n!}{r!\,(n-r)!}$ (d) $\frac{r!}{(n-r)!}$

(49) If $A \subseteq B$ then $P(A \cup B)$ = ————

(a) $P(A) + P(B)$ (b) $P(A) + P(B) - P(A \cap B)$

(c) $P(A) \cdot P(B)$ (d) None of these

(50) If $A_1, A_2, A_3, \ldots\ldots A_n$ are independent events then $P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n)$ = ————

(a) $P(A_1) + P(A_2) + \ldots\ldots P(A_n)$ (b) $P(A_1) \cdot P(A_2) \ldots\ldots P(A_n)$

(c) $P(A_1 \cdot A_2 \cdot A_3 \ldots\ldots A_n)$ (d) None of these

(51) If ${}^nC_8 = {}^nC_{12}$ then n = ———— **[Lahore Board 2014]**

(a) 10 (b) 15

(c) 20 (d) 25

(52) In tossing a fair coin, n(S) = ————

(a) 0 (b) 2

(c) 4 (d) 6

(53) Number of the diagonals of a 6-sided figure is:

(a) 15 (b) 30

(c) 16 (d) 9

(54) If $\bar{E}$ is the non occurrence of the event E then $P(\bar{E})$ = ————

(a) $1 - P(E)$ (b) $1 + P(E)$

(c) $2 - P(E)$ (d) $2 + P(E)$

(55) The greater the number of trials, the more accurate is the estimate of the ————

(a) Permutation (b) Combination

(c) Probability (d) None of these

(56) If the occurrence of 1st event does not influence the occurrence of the other event then events are called:

(a) Independent (b) Dependent

(c) Equal (d) None of these

(57) In tossing a coin if event A = {H} and event B = {T} then A and B are:

(a) Mutual exclusive events (b) Dependent events

(c) Both 'a' and 'b' (d) None of these

(58) A coin is tossed three times. The probability of getting at least one tail is:

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) 1

(59) The probability of getting a jack, from 52 playing cards is:

(a) $\frac{1}{52}$ (b) $\frac{1}{13}$

(c) $\frac{4}{13}$ (d) $\frac{1}{4}$

(60) The probability of drawing a card of diamond from a deck of 52 playing cards is:

(a) $\frac{1}{13}$ (b) $\frac{1}{4}$

(c) $\frac{1}{52}$ (d) $\frac{3}{4}$

(61) If $\frac{n!}{r!\,(n-r)!} = 20$ and $n = 6$ then $r =$

(a) 1 (b) 2

(c) 3 (d) None of these

(62) If $\frac{(n+1)!}{3!\,(n-2)!}$ and $\frac{(n+1)!}{5!\,(n-r)!}$ are in ratio 10 : 3 , then $n =$

(a) 2 (b) 3

(c) 4 (d) 5

(63) The number of permutations of 7 different things taken 3 at a time are:

(a) ${}^{7}C_{3}$ (b) ${}^{7}P_{3}$

(c) 210 (d) Both (a) and (c)

(64) How many 6-digit numbers can be formed, without repeating any digit from the digits 0, 1, 2, 3, 4, 5:

(a) 400 (b) 600

(c) 800 (d) 1000

(65) In rolling two dice the number of possible outcomes are:

(a) 6 (b) 36

(c) 1 (d) 256

(66) If E is an impossible event then P(E) =

(a) 0 (b) 1

(c) $0 < P(E) < 1$ (d) None of these

(67) If 'E' is sure to occur then P(E) =

(a) 0 (b) 1

(c) $0 < P(E) < 1$ (d) None of these

(68) The greater the number of trials the more accurate is the estimate of the ————

(a) Permutation (b) Combination

(c) Probability (d) None of these

(69) If ${}^{n}P_2 = 6$ then n =

(a) 3 (b) 2

(c) –3 (d) –2

(70) If ${}^{12}P_n = 12.11.10.$ then n = ————

(a) 1 (b) 2

(c) 3 (d) 4

(71) ${}^{n}C_{n-r}$ = ————

(a) ${}^{n}C_r$ (b) ${}^{n}P_r$

(c) ${}^{n}P_{n-r}$ (d) $r!\ {}^{n}P_r$

(72) ${}^{16}C_{11} + {}^{16}C_{10}$ =

(a) ${}^{16}C_{11}$ (b) ${}^{17}C_{10}$

(c) ${}^{17}C_{11}$ (d) ${}^{16}C_{10}$

(73) Number of triangles can be formed by joining the vertices of the polygon having 5 sides are:

(a) 5 (b) 10

(c) 15 (d) 20

(74) A particular outcome is called a/an ————

(a) Permutation (b) Probability

(c) Event (d) Combination

(75) The factorial form of $\frac{10.9}{2.1}$ is:

(a) $\frac{\lfloor 10}{\lfloor 2}$ (b) $\frac{\lfloor 10}{\lfloor 8 \lfloor 2}$

(c) $\frac{\lfloor 10 \lfloor 8}{\lfloor 2}$ (d) $\frac{\lfloor 8}{\lfloor 4}$

(76) The value of ${}^{5}C_{2}$ is:

(a) 1 (b) 10

(c) 20 (d) 30

(77) If n is negative integer then $\lfloor n$ is: **[Lahore Board 2015]**

(a) 1 (b) Not defined

(c) Zero (d) n

(78) Pakistan and India play a hockey match, probability that Pakistan will win:

[Lahore Board 2015]

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{3}$ (d) 1

(79) A fair coin is tossed then probability of head is: **[Lahore Board 2015]**

(a) 1 (b) $\frac{1}{2}$

(c) 2 (d) $\frac{1}{3}$

Answers

(1)	b	*(2)*	c	*(3)*	b	*(4)*	c	*(5)*	d
(6)	c	*(7)*	b	*(8)*	a	*(9)*	a	*(10)*	b
(11)	a	*(12)*	b	*(13)*	b	*(14)*	c	*(15)*	a
(16)	d	*(17)*	c	*(18)*	d	*(19)*	a	*(20)*	b
(21)	c	*(22)*	b	*(23)*	c	*(24)*	b	*(25)*	a
(26)	d	*(27)*	c	*(28)*	d	*(29)*	b	*(30)*	a
(31)	a	*(32)*	d	*(33)*	b	*(34)*	b	*(35)*	a
(36)	d	*(37)*	c	*(38)*	a	*(39)*	a	*(40)*	b
(41)	b	*(42)*	a	*(43)*	c	*(44)*	c	*(45)*	a
(46)	b	*(47)*	d	*(48)*	c	*(49)*	b	*(50)*	b
(51)	c	*(52)*	b	*(53)*	d	*(54)*	a	*(55)*	c
(56)	a	*(57)*	a	*(58)*	c	*(59)*	b	*(60)*	b
(61)	c	*(62)*	d	*(63)*	d	*(64)*	b	*(65)*	b
(66)	a	*(67)*	b	*(68)*	c	*(69)*	a	*(70)*	d
(71)	a	*(72)*	c	*(73)*	b	*(74)*	c	*(75)*	b
(76)	b	*(77)*	b	*(78)*	c	*(79)*	b		

SOLUTION

Q.1 $^{6}P_{4} = \dfrac{6!}{(6-4)!} = \dfrac{6!}{2!} = \dfrac{6\times5\times4\times3\times2!}{2!}$

$= 6\times5\times4\times3 = 360$

Q.2 $^{10}P_{2} = \dfrac{10!}{(10-2)!}$

$= \dfrac{10!}{8!} = \dfrac{10\times9\times8!}{8!} = 90$

Q.3 $^{n}C_{r-1} + {}^{n}C_{r-2}$

$= \dfrac{n!}{(r-1)!\,(n-r+1)!} + \dfrac{n!}{(r-2)!\,(n-r+2)!}$

$= \dfrac{n!}{(r-1)(r-2)!\,(n-r+1)!} + \dfrac{n!}{(r-2)!\,(n-r+2)(n-r+1)!}$

$= \dfrac{n!}{(r-2)!\,(n-r+1)!}\left[\dfrac{1}{r-1} + \dfrac{1}{n-r+2}\right]$

$= \dfrac{n!}{(r-2)!\,(n-r+1)!}\left[\dfrac{n-r+2+r-1}{(r-1)(n-r+2)}\right]$

$= \dfrac{n!}{(r-2)!\,(n-r+1)!}\,\dfrac{n+1}{(r-1)(n-r+2)}$

$= \dfrac{(n+1)\,n!}{(r-1)(r-2)!\,(n-r+2)(n-r+1)!}$

$= \dfrac{(n+1)!}{(r-1)!\,(n-r+2)!}$

$= {}^{n+1}C_{r-1}$

Q.4 $^{n}P_{2} = 30$

$\dfrac{n!}{(n-2)!} = 30$

$\dfrac{n(n-1)(n-2)!}{(n-2)!} = 30$

$n(n-1) = 30$

$n^2 - n - 30 = 0$

$n^2 - 6n + 5n - 30 = 0$

$n(n-6) + 5(n-6) = 0$

$(n+5)(n-6) = 0$

$n = -5$ (Not possible) $\boxed{n = 6}$

Q.5 ${}^{6}P_3$

$$= \frac{6!}{(6-3)!} = \frac{6 \times 5 \times 4 \times 3!}{3!} = 120$$

Q.7 Proof is similar as Q3.

Q.8 ${}^{n}C_r \times r! = \frac{n!}{r!\,(n-r)!} \times r! = \frac{n!}{(n-r)!} = {}^{n}P_r$

Q.9 ${}^{n}P_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! \quad \because 0! = 1$

Q.10 As

$${}^{n}C_{n-r} = \frac{n!}{(n-r)!\,(n-n+r)!}$$

$$= \frac{n!}{(n-r)!\,r!} = \frac{n!}{r!\,(n-r)!} = {}^{n}C_r$$

Q.12 $\frac{(n+1)!}{(n-1)!}$

$$= \frac{(n+1)(n)(n-1)!}{(n-1)!} = n(n+1) = n^2 + n$$

Q.13 ${}^{n}C_0 = \frac{n!}{0!\,(n-0)!} = \frac{n!}{n!} = 1$

Q.14 ${}^{n}P_r = \frac{n!}{(n-r)!}$

and

$$\underline{|r}\ {}^{n}C_r = r! \frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!}$$

$$\Rightarrow \quad {}^{n}P_r = \underline{|r}\ {}^{n}C_r$$

Q.16 Given $5G + 3R = 8$

$$\Rightarrow \quad n(S) = 8$$

Let E be the event that the ball is yellow.

$$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{0}{8} = 0$$

Q.17 $n(n-1)(n-2) \ldots\ldots\ldots (n-r+1)$

$$= \frac{n(n-1)(n-2) \ldots\ldots\ldots (n-r+1)(n-r)!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!} = \frac{\underline{|n}}{\underline{|n-r}}$$

Q.18 $n(S) = 4 \;,\; n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

Q.19 No. of ways $= \frac{1}{2}(6-1)! = \frac{1}{2}\,5!$

$$= \frac{1}{2} \times 5 \times 4 \times 3 \times 2! = 60$$

Q.21 $${}^6P_4 = \frac{6!}{(6-r)!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360$$

Q.23 Same as Q.10.

Q.24 As, $$P(B) = \frac{n(B)}{n(S)} = \frac{2}{20} = \frac{1}{10}$$

Q.25 $S = \{1, 2, 3, \ldots\ldots, 10\} \Rightarrow n(S) = 10$

Let E be the event that the chosen number is divisible by $4 \Rightarrow E = \{4, 8\} \Rightarrow n(E) = 2$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{10} = \frac{1}{5}$$

Q.26 ${}^6C_2$

$$= \frac{6!}{2!\,(6-2)!} = \frac{6 \times 5 \times 4!}{2!\,4!} = \frac{6 \times 5}{2} = 3 \times 5 = 15$$

Q.27 $n = 5 \;,\; r = 3$

$${}^nP_r = {}^5P_3 = \frac{5!}{(5-3)!} = \frac{5 \times 4 \times 3 \times 2!}{2!} = 60$$

Q.28 Same as Q.16.

Q.29 ${}^nC_r + {}^nC_{r-1}$

$$= \frac{n!}{r!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$$

$$= \frac{n!}{r(r-1)!\,(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$$

$$= \frac{n!}{(r-1)!\,(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$= \frac{(n+1)\,n!}{r(r-1)!\,(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{r!\,(n+1-r)!} = {}^{n+1}C_r$$

Q.31 $${}^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 4!$$

And

$${}^4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = 4!$$

$$\Rightarrow \quad {}^4P_3 = {}^4P_4$$

Q.41 $$n = 5 \ , \ r = 3$$

$${}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2!}{2!} = 60$$

Q.42 $${}^nP_r = \frac{n!}{(n-r)!}$$

And

$$n \ {}^{n-1}P_{r-1} = \frac{n(n-1)!}{(n-1-r+1)!} = \frac{n(n-1)!}{(n-r)!} = \frac{n!}{(n-r)!}$$

$$\Rightarrow \quad {}^nP_r = n \ {}^{n-1}P_{r-1}$$

Q.44 Required number of ways $= \frac{1}{2} \ 7! = 2520$

Q.48 $${}^nC_8 = {}^nC_{12}$$

As,

$${}^nC_r = {}^nC_{n-r}$$

$$\Rightarrow \quad {}^nC_8 = {}^nC_{n-8}$$

But $${}^nC_8 = {}^nC_{12}$$

$$\Rightarrow \quad {}^nC_{12} = {}^nC_{n-8} \Rightarrow 12 = n - 8$$

$$\Rightarrow \quad \boxed{n = 20}$$

Q.49 In tossing a fair coin. Possible outcomes are 2 (Head and Tail).

$$\Rightarrow \quad n(S) = 2$$

Q.50 6-sided figure has 6-vertices. Joining any two vertices, we get a line segment:

47 num of the line segments $= {}^6C_2 = 15$ but these line segments include 6 sides also.

$$\because \quad \text{Number of diagonals} = 15 - 6 = 9$$

Q.55 When a coin is tossed three times possible outcomes are:

$$S = \{HHH, HHT, HTH, THH, THT, TTH, HTT, TTT\}$$

$$n(S) = 8$$

Let E be the event of getting at least one tail then.

$$E = \{HHT, HTH, THH, THT, TTH, HTT, TTT\}$$

$$n(E) = 7$$

$$\Rightarrow \quad P(E) = \frac{n(E)}{n(S)} = \frac{7}{8}$$

Q.56 Let E be the event of getting jack.

As, $n(E) = 4$ and $n(S) = 52$

$\Rightarrow \quad p(E) = \frac{4}{52} = \frac{1}{13}$

Q.57 As there are 13 diamond cards in 52 playing cards.

$\Rightarrow$ If E be the event of getting diamond card then $n(E) = 13$.

and $n(S) = 52$

$p(E) = \frac{13}{52} = \frac{1}{4}$

Q.58 $\frac{n!}{r!\,(n-r)!} = 20$

As, $n = 6$

If we take $r = 3$ then.

$\frac{6!}{3!(6-3)!} = \frac{6 \times 5 \times 4 \times 3!}{3!\,3!} = 20$ (True)

$\Rightarrow \quad r = 3$

Q.59 $\frac{(n+1)!}{3!\,(n-2)!} : \frac{(n+1)!}{5!\,(n-4)!} = 10 : 3$

$\Rightarrow \quad \frac{\frac{(n+1)!}{3!\,(n-2)!}}{\frac{(n+1)!}{5!(n-4)!}} = \frac{10}{3}$

$\Rightarrow \quad \frac{5!(n-4)!}{3!\,(n-2)!} = \frac{10}{3}$

$\Rightarrow \quad \frac{5 \times 4 \times 3!\,(n-4)!}{3!(n-2)(n-3)(n-4)!} = \frac{10}{3}$

$\Rightarrow \quad \frac{20}{(n-2)(n-3)} = \frac{10}{3}$

$\Rightarrow \quad \frac{2}{n^2-5n+6} = \frac{1}{3}$

$\Rightarrow \quad n^2 - 5n + 6 = 6$

$\Rightarrow \quad n^2 - 5n = 0 \Rightarrow n(n-5) = 0$

$\Rightarrow \quad n = 0$ or $n = 5$

Q.61 Number of 6-digit numbers $= 5 \times 5 \times 4 \times 3 \times 2 \times 1 = 600$

Q.62 Possible outcomes will be:

$$S = \left\{ \begin{matrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix} \right\}$$

$\Rightarrow \quad n(S) = 36$

Q.66 $\quad {}^{n}P_2 = 6$

$\Rightarrow \quad \dfrac{n!}{(n-2)!} = 6$

$\Rightarrow \quad \dfrac{n(n-1)(n-2)!}{(n-2)!} = 6$

$\Rightarrow \quad n(n-1) = 6$

$\Rightarrow \quad n^2 - n - 6 = 0$

$\Rightarrow \quad n^2 - 3n + 2n - 6 = 0$

$\Rightarrow \quad n(n-3) + 2(n-3) = 0$

$\Rightarrow \quad (n+2)(n-3) = 0 \Rightarrow \boxed{n = 3}$

$n = -2$ (Not possible)

Q.67 $\quad {}^{12}P_n = 12 \,.\, 11 \,.\, 10 \,.\, 9$

$${}^{12}P_n = \frac{12 \,.\, 11 \,.\, 10 \,.\, 9 \,.\, 8!}{8!}$$

$${}^{12}P_n = \frac{12!}{8!}$$

$${}^{12}P_n = {}^{12}P_4$$

$\Rightarrow \quad \boxed{n = 4}$

Q.69 ${}^{16}C_{11} + {}^{16}C_{10}$

$$= \frac{16!}{11!(16-11)!} + \frac{16!}{10!(16-10)!}$$

$$= \frac{16!}{11!\,5!} + \frac{16!}{10!\,6!}$$

$$= \frac{16!}{11 \times 10! \times 5!} \times \frac{16!}{10! \times 6 \times 5!}$$

$$= \frac{16!}{10! \times 5!}\left[\frac{1}{11} + \frac{1}{6}\right]$$

$$= \frac{16!}{10! \times 5!}\,\frac{6+11}{11 \times 6}$$

$$= \frac{17 \times 16!}{11 \times 10! \times 6 \times 5!}$$

$$= \frac{17!}{11! \times 6!} = {}^{17}C_{11}$$

Q.70 ${}^{5}C_{3} = \dfrac{5!}{3!(5-3)!} = \dfrac{5 \times 4 \times 3!}{3! \times 2!} = 10$

Q.71 ${}^{n}C_{n-r} = {}^{n}C_{r}$

Q.72 ${}^{16}C_{11} + {}^{16}C_{10}$

$$= {}^{17}C_{11}$$

Q.73 ${}^{5}C_{3} = \dfrac{5!}{3!\,2!}$

$$\frac{5.4.3!}{3!\,2.1}$$

$$= 10$$

Q.75 $\dfrac{10.9}{2.1}$

$$\frac{10.9.8.7 \ldots\ldots 1}{2.1\ 8.7 \ldots\ldots 1}$$

$$= \frac{10!}{2!\ 8!}$$

Q.76 ${}^5C_2$

$$= \frac{5!}{2!\,(5-2)!}$$

$$= \frac{5.4.3}{2.3!}$$

$$= 10$$

Q.78

$$S.S = \{W, L, T\}$$

$$n(S) = 3$$

$$P(A) = \text{Pax will win} = \frac{n(A)}{n(S)} = \frac{1}{3}$$

Q.79

$$S.S = \{H, T\}$$

$$n(S) = 2$$

$$P(A) = \text{Probability of head}$$

$$= \frac{n(A)}{n(S)} = \frac{1}{2}$$