

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

MULTIPLE CHOICE QUESTIONS

(1) $\sin x = \frac{1}{2}$ then $x =$

(a) $\frac{\pi}{6}, \frac{5\pi}{6}$

(b) $-\frac{\pi}{6}, \frac{-5\pi}{6}$

(c) $-\frac{\pi}{6}$

(d) $-\frac{5\pi}{6}$

[Lahore Board 2014, Lahore Board 2005]

[Gujranwala Board 2006-08]

(2) The solution set of the equation $\tan 2x = 1$ in $[0, \pi]$ is:

(a) $\frac{\pi}{4}, \frac{5\pi}{4}$

(b) $-\frac{\pi}{4}, \frac{5\pi}{4}$

(c) $\frac{\pi}{8}, \frac{5\pi}{8}$

(d) None of these

[Lahore Board 2005]

(3) The solution set of the equation $\sin x = 0$ is _____, $n \in \mathbb{Z}$

(a) $\{2n\pi\}$

(b) $\{\pi + 2n\pi\}$

(c) $\{n\pi\}$

(d) $\{3n\pi\}$

[Lahore Board 2005]

(4) If $2\cos^2\theta - \cos\theta = 0$ and $\theta \in \left] \frac{\pi}{2}, \frac{3\pi}{2} \right[$ then the solution set of this equation is:

- (a) $\left\{ \frac{\pi}{2}, \frac{\pi}{3} \right\}$ (b) ϕ
(c) $\left\{ \frac{3\pi}{2}, \frac{5\pi}{3} \right\}$ (d) None of these

[Gujranwala Board 2005]

(5) If $\tan x = \frac{1}{\sqrt{3}}$ then $x =$

- (a) $\left\{ \frac{\pi}{6} + n\pi \right\}, n \in \mathbb{Z}$ (b) $\left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{7\pi}{6} + n\pi \right\}, n \in \mathbb{Z}$
(c) $\left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{4\pi}{3} + n\pi \right\}, n \in \mathbb{Z}$ (d) None of these

[Gujranwala Board 2006]

(6) Solution of the equation $1 + \cos\theta = 0$ is in quadrant:

- (a) I (b) II
(c) III (d) None

[Lahore Board 2006, 2015]

(7) If $1 + \cos x = 0$, $x \in [0, 2\pi]$ then $x =$

- (a) π (b) 2π
(c) $\frac{\pi}{4}$ (d) 3π

[Lahore Board 2006]

(8) The solution of the equation $\sec x = -2$ lies in quadrant:

- (a) I, II (b) II, III
(c) III, IV (d) IV, I

[Lahore Board 2006]

(9) The solution set $\left\{ \frac{\pi}{4} + 2n\pi \right\} \cup \left\{ \frac{3\pi}{4} + 2n\pi \right\}$ is of trigonometric:

- (a) $\sin x = \frac{1}{\sqrt{2}}$ (b) $\cos x = \frac{1}{\sqrt{2}}$
(c) $\tan x = 1$ (d) $\cos x = \frac{-1}{\sqrt{2}}$

[Gujranwala Board 2007]

(10) An equation containing trigonometric function is called:

- | | |
|--------------------------|----------------------------|
| (a) Exponential equation | (b) Algebraic equation |
| (c) Radical equation | (d) Trigonometric equation |

[Lahore Board 2007]

[Gujranwala Board 2010]

(11) If $\cos x = \frac{-\sqrt{3}}{2}$ then reference angle of $\cos x$ is:

- | | |
|---------------------|----------------------|
| (a) $\frac{\pi}{3}$ | (b) $\frac{\pi}{4}$ |
| (c) $\frac{\pi}{6}$ | (d) $-\frac{\pi}{6}$ |

[Lahore Board 2008]

(12) Reference angle lies in quadrant:

- | | |
|---------|--------|
| (a) I | (b) II |
| (c) III | (d) IV |

[Lahore Board 2008]

(13) If one solution of $\cos x = 1$ in $x \in [0, 2\pi]$ is zero then other is:

- | | |
|---------------------|------------|
| (a) $\frac{\pi}{2}$ | (b) π |
| (c) $-\pi$ | (d) 2π |

[Gujranwala Board 2008]

(14) The reference angle of $\tan\theta = -1$ equals:

- | | |
|---------------------|----------------------|
| (a) $\frac{\pi}{4}$ | (b) $-\frac{\pi}{4}$ |
| (c) $-\pi$ | (d) π |

[Lahore Board 2009]

(15) Solution of the equation $\cos x - 1 = 0$ in $[0, 2\pi]$ is:

- | | |
|---------------------------------------|--|
| (a) $\{0, \pi\}$ | (b) $\{0, 2\pi\}$ |
| (c) $\left\{0, \frac{\pi}{2}\right\}$ | (d) $\left\{\frac{\pi}{3}, \frac{3\pi}{2}\right\}$ |

[Lahore Board 2010]

(16) The equation $\cos^2 x = \frac{3}{4}$ has solutions:

- | | |
|--------------|----------|
| (a) One | (b) Two |
| (c) Infinite | (d) Four |

[Lahore Board 2010]

(17) $\cos x = \frac{1}{2}$ has a solution:

- | | |
|---------------------|---------------------|
| (a) $\frac{\pi}{2}$ | (b) $\frac{\pi}{3}$ |
| (c) $\frac{\pi}{4}$ | (d) $\frac{\pi}{6}$ |

[Lahore Board 2011]

- (18) Which of the following is true solution of $\sec x = \sqrt{2}$, $x \in [0, 2\pi]$.
- (a) $\frac{-\pi}{4}, \frac{\pi}{4}$ (b) $\frac{\pi}{4}, \frac{7\pi}{4}$
 (c) $\frac{\pi}{4}, \frac{5\pi}{4}$ (d) $\frac{\pi}{3}, \frac{\pi}{4}$ [Lahore Board 2012]
- (19) Solution of equation $\tan x = \frac{1}{\sqrt{3}}$ lie in:
- (a) I and II quadrant (b) I and III quadrant
 (c) II and IV quadrant (d) I and IV quadrant
 [Lahore Board 2012, 2015]
- (20) Solution set of $1 + \cos x = 0$ is:
- (a) $\{2n\pi\}$ (b) $\{\pi + 2n\pi\}$
 (c) $\{\pi\}$ (d) $\{2\pi + n\pi\}$ [Lahore Board 2013]
- (21) The equation $\cos^2 x = \frac{3}{4}$ has solutions:
- (a) One (b) Two
 (c) Four (d) Infinite [Lahore Board 2013]
- (22) A trigonometric equation has infinite number of solutions due to the _____ of the trigonometric functions:
- (a) Periodicity (b) Exponent
 (c) One-to-one (d) Inverse
- (23) $\cot x + \operatorname{cosec} x = 0$ is a _____ equation:
- (a) Reciprocal (b) Exponential
 (c) Trigonometric (d) None of these
- (24) $\theta = \frac{(2n+1)\pi}{2}$, $n \in \mathbb{Z}$ is a solution of:
- (a) $\cos\theta = 0$ (b) $\sin\theta = 0$ (c) $\tan\theta = 0$ (d) $\cot\theta = 0$
- (25) If $\cos x = \frac{1}{\sqrt{2}}$, then reference angle is:
- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$ [Lahore Board 2014]

Answers

(1)	a	(2)	c	(3)	c	(4)	b	(5)	b
(6)	d	(7)	a	(8)	b	(9)	a	(10)	d
(11)	c	(12)	a	(13)	d	(14)	a	(15)	b
(16)	c	(17)	b	(18)	b	(19)	b	(20)	b
(21)	d	(22)	a	(23)	c	(24)	a	(25)	b

Q.1 $\sin x = \frac{1}{2}$

sine is +ve in I and II quadrant with reference angle $\frac{\pi}{6}$.

$$\Rightarrow x = \frac{\pi}{6} \quad \text{and} \quad x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Q.2 $\tan 2x = 1$

tan is +ve in I and III quadrant with reference angle $\frac{\pi}{4}$.

$$2x = \frac{\pi}{4} \quad \text{and} \quad 2x = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$x = \frac{\pi}{8} \quad \text{and} \quad x = \frac{5\pi}{8}$$

Q.4 $2\cos^2\theta - \cos\theta = 0$

$$\cos\theta(2\cos\theta - 1) = 0$$

$$\cos\theta = 0 \quad \text{or} \quad \cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

But,

$$\frac{\pi}{2}, \frac{3\pi}{2} \notin \left] \frac{\pi}{2}, \frac{3\pi}{2} \right[$$

Now, $\cos\theta = \frac{1}{2}$

cos is +ve in I and IV quadrant with reference angle $\frac{\pi}{3}$.

$$\theta = \frac{\pi}{3} \notin \left] \frac{\pi}{2}, \frac{3\pi}{2} \right[\quad \text{and} \quad \theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \notin \left] \frac{\pi}{2}, \frac{3\pi}{2} \right[$$

$\Rightarrow$ Solution is $\emptyset$.

$$\text{Q.5} \quad \tan x = \frac{1}{\sqrt{3}}$$

$\tan$ is +ve in I and III quadrant with reference angle $\frac{\pi}{6}$.

$$\Rightarrow \quad x = \frac{\pi}{6} \quad \text{and} \quad x = \pi + \frac{\pi}{6}$$

$$= \frac{7\pi}{6}$$

$$\Rightarrow \quad x \in \left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{7\pi}{6} + n\pi \right\}, n \in \mathbb{Z}$$

$$\text{Q.25} \quad \cos x = \frac{1}{\sqrt{2}}$$

$$x = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 45^\circ = \frac{\pi}{4}$$

Chapter 14

SOLUTIONS OF TRIGONOMETRIC EQUATIONS

EXERCISE 14.1

Trigonometric Equations:

The equations containing at least one trigonometric functions are called trigonometric equations.

e.g., $\sin x = \frac{2}{5}$, $\sec x = \tan x$

Q.1 Find the solutions of the following equation which lie in $[0, 2\pi]$

(i) $\sin x = \frac{-\sqrt{3}}{2}$

(ii) $\operatorname{cosec} \theta = 2$ (Gujranwala Board 2005, Lahore Board 2006)

(iii) $\sec x = -2$

(iv) $\cot \theta = \frac{1}{\sqrt{3}}$ (Lahore Board 2010)

Solution:

(i) $\sin x = \frac{-\sqrt{3}}{2}$

$$x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ = \frac{\pi}{3}$$

Since $\sin x$ is -ve in III & IV Quadrants with the reference angle $\pi/3$ thus we have

For III-Quadrant

$$x = \pi + \theta$$

$$x = \pi + \frac{\pi}{3},$$

$$x = \frac{4\pi}{3},$$

For IV-Quadrant

$$x = 2\pi - \theta$$

$$x = 2\pi - \frac{\pi}{3}$$

$$x = \frac{5\pi}{3}$$

So thus the required solution is $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

(ii) $\operatorname{cosec} \theta = 2$

$$\frac{1}{\sin \theta} = 2$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$$\theta = \sin^{-1} \left(\frac{1}{2} \right) = 30^\circ = \frac{\pi}{6}$$

Since $\sin \theta$ is +ve in I and II Quadrants with reference angle $\frac{\pi}{6}$ Thus

For I-Quadrant

$$x = \frac{\pi}{6},$$

For II-Quadrant

$$x = \pi - \theta$$

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6}$$

So thus the required solution is $x = \frac{\pi}{6}, \frac{5\pi}{6}$.

(iii) $\sec x = -2$

$$\frac{1}{\cos x} = -2 \Rightarrow \cos x = -\frac{1}{2}$$

$$x = \cos^{-1} \left(-\frac{1}{2} \right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos x$ is -ve in II & III Quadrants with reference angle $\frac{\pi}{3}$ thus we have

For II-Quadrant

$$x = \pi - \theta,$$

$$x = \pi - \frac{\pi}{3},$$

$$x = \frac{2\pi}{3},$$

For III-Quadrant

$$x = \pi + \theta$$

$$x = \pi + \frac{\pi}{3}$$

$$x = \frac{4\pi}{3}$$

Thus the required solution is $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

(iv) $\cot \theta = \frac{1}{\sqrt{3}}$

$$\tan \theta = \sqrt{3}$$

$$\theta = \tan^{-1} (\sqrt{3}) = 60^\circ = \frac{\pi}{3}$$

For I-Quadrant

$$x = \frac{\pi}{3}$$

Thus the required solution is $x = \frac{\pi}{3}, \frac{4\pi}{3}$

For III-Quadrant

$$x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

Ans.

Q.2 Solve the following trigonometric equations:

(i) $\tan^2 \theta = \frac{1}{3}$

(ii) $\operatorname{cosec}^2 \theta = \frac{4}{3}$

(iii) $\sec^2 \theta = \frac{4}{3}$

(iv) $\cot^2 \theta = \frac{1}{3}$ (Lahore Board 2007)

Solution:

(i) $\tan^2 \theta = \frac{1}{3}$

$$\tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = \frac{1}{\sqrt{3}} \quad \text{and} \quad \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ = \frac{\pi}{6}$$

Since $\tan \theta$ is +ve in I & III Quadrants
with reference angle $\frac{\pi}{6}$

Therefore

For I-Quad

For III-Quad

$$\theta = \frac{\pi}{6}, \quad \theta = \pi + \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6} + n\pi, \quad \theta = \frac{7\pi}{6}$$

$$\theta = \frac{7\pi}{6} + n\pi$$

$$\left\{ \frac{\pi}{6} + n\pi \right\} \cup \left\{ \frac{7\pi}{6} + n\pi \right\} \cup \left\{ \frac{5\pi}{6} + n\pi \right\} \cup \left\{ \frac{11\pi}{6} + n\pi \right\}, \forall n \in \mathbb{Z}$$

(ii) $\operatorname{cosec}^2 \theta = \frac{4}{3}$

$$\theta = \sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = 60^\circ = \frac{\pi}{3}$$

$$\operatorname{cosec} \theta = \pm \frac{2}{\sqrt{3}}$$

$$\operatorname{cosec} \theta = \frac{2}{\sqrt{3}}$$

$$\sin \theta = \frac{\sqrt{3}}{2}$$

Since $\sin \theta$ is +ve in I & II Quadrants
with reference angle $\frac{\pi}{3}$

Since $\tan \theta$ is -ve in II & IV Quadrants with
reference angle $\frac{\pi}{6}$

For II-Quad

For IV-Quad

$$\theta = \pi - \frac{\pi}{6}, \quad \theta = 2\pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6}, \quad \theta = \frac{11\pi}{6}$$

$$\theta = \frac{5\pi}{6} + n\pi, \quad \theta = \frac{11\pi}{6} + n\pi \quad \forall n \in \mathbb{Z}$$

$$\operatorname{cosec} \theta = \frac{-2}{\sqrt{3}}$$

$$\sin \theta = \frac{-\sqrt{3}}{2}$$

Since $\sin \theta$ is -ve in III & IV Quadrants
with reference angle $\frac{\pi}{3}$

For I-Quad For II-Quad

$$\theta = \frac{\pi}{3} \quad , \quad \theta = \pi - \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2n\pi \quad \theta = \frac{2\pi}{3} + 2n\pi$$

For III-Quad

$$\theta = \pi + \frac{\pi}{3} \quad , \quad \theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{4\pi}{3} + 2n\pi \quad , \quad \theta = \frac{5\pi}{3} + 2n\pi$$

$$\text{S.S} = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{2\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}, \forall n \in \mathbb{Z} \quad \text{Ans.}$$

(iii) $\sec^2 \theta = \frac{4}{3}$ $\theta = \cos^{-1} \left(\frac{\sqrt{3}}{2} \right) = 30^\circ = \frac{\pi}{6}$

$$\sec \theta = \pm \frac{2}{\sqrt{3}}$$

$$\sec \theta = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

Since $\cos \theta$ is +ve in I & IV Quadrants,

with reference angle $\frac{\pi}{6}$ therefore we have

For I-Quad For IV-Quad

$$\theta = \frac{\pi}{6} \quad , \quad \theta = 2\pi - \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6} + 2n\pi \quad , \quad \theta = \frac{11\pi}{6} + 2n\pi$$

For II-Quad

$$\theta = \pi - \frac{\pi}{6} \quad , \quad \theta = \pi + \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6} + 2n\pi \quad , \quad \theta = \frac{7\pi}{6} + 2n\pi$$

$$\text{S.S} = \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{4\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{7\pi}{6} + 2n\pi \right\}, \forall n \in \mathbb{Z} \quad \text{Ans.}$$

$$(iv) \quad \cot^2 \theta = \frac{1}{3}$$

(Lahore Board 2007)

$$\Rightarrow \cot \theta = \pm \frac{1}{\sqrt{3}}$$

$$\tan \theta = \pm \sqrt{3}$$

$$\tan \theta = \sqrt{3}$$

Since $\tan \theta$ is +ve in I & III

Quadrants with reference angle $\frac{\pi}{3}$

therefore we have

For I-Quad For III-Quad

$$\theta = \frac{\pi}{3}, \quad \theta = \pi + \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + n\pi, \quad \theta = \frac{4\pi}{3} + n\pi$$

$$\theta = \tan^{-1}(\sqrt{3}) = 60^\circ = \frac{\pi}{3}$$

$$\tan \theta = -\sqrt{3}$$

Since $\tan \theta$ is -ve in II & IV

Quadrants, with reference angle $\frac{\pi}{3}$

therefore we have

For II-Quad

For IV-Quad

$$\theta = \pi - \frac{\pi}{3}, \quad \theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{2\pi}{3} + n\pi, \quad \theta = \frac{5\pi}{3} + n\pi$$

$$S.S = \left\{ \frac{\pi}{3} + n\pi \right\} \cup \left\{ \frac{4\pi}{3} + n\pi \right\} \cup \left\{ \frac{2\pi}{3} + n\pi \right\} \cup \left\{ \frac{5\pi}{3} + n\pi \right\}, \forall n \in \mathbb{Z} \text{ Ans.}$$

Q.3 Find the values of θ satisfying the following equations:

$$3 \tan^2 \theta + 2\sqrt{3} \tan \theta + 1 = 0 \quad (\text{Gujranwala Board 2006})$$

Solution:

$$3 \tan^2 \theta + 2\sqrt{3} \tan \theta + 1 = 0$$

$$a = 3, \quad b = 2\sqrt{3}, \quad c = 1 \quad \text{by quadratic formula}$$

$$\tan \theta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{(2\sqrt{3})^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{-2\sqrt{3} \pm \sqrt{12 - 12}}{6} = \frac{-2\sqrt{3}}{6} = \frac{-1}{\sqrt{3}}$$

$$\Rightarrow \tan \theta = -\frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$$

Since $\tan \theta$ is -ve in II & IV Quadrants, with reference angle $\frac{\pi}{6}$ therefore we have

For II-Quad**For IV-Quad**

$$\theta = \pi - \frac{\pi}{6} \quad , \quad \theta = 2\pi - \frac{\pi}{6}$$

$$\theta = \frac{5\pi}{6} + n\pi \quad , \quad \theta = \frac{11\pi}{6} + n\pi$$

$$S.S = \left\{ \frac{5\pi}{6} + n\pi \right\} \cup \left\{ \frac{11\pi}{6} + n\pi \right\}, \forall n \in \mathbb{Z} \quad \text{Ans.}$$

$$\text{Q.4} \quad \tan^2 \theta - \sec \theta - 1 = 0$$

Solution:

$$\tan^2 \theta - \sec \theta - 1 = 0$$

$$\sec^2 \theta - 1 - \sec \theta - 1 = 0$$

$$\sec^2 \theta - \sec \theta - 2 = 0$$

$$\sec^2 \theta - 2 \sec \theta + \sec \theta - 2 = 0$$

$$\sec \theta (\sec \theta - 2) + 1 (\sec \theta - 2) = 0$$

$$(\sec \theta - 2) (\sec \theta + 1) = 0$$

$$\sec \theta = 2$$

$$\cos \theta = \frac{1}{2}$$

$$\sec \theta = -1$$

$$\cos \theta = -1$$

$$\Rightarrow \theta = \cos^{-1}(-1)$$

$$\theta = \cos^{-1}(-1)$$

$$\theta = \pi + 2n\pi$$

Since $\cos \theta$ is +ve in I & IV QuadrantsWith reference angle $\frac{\pi}{3}$ **For I-Quad. For IV-Quad.**

$$\theta = \frac{\pi}{3} \quad , \quad \theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2n\pi, \quad \theta = \frac{5\pi}{3} + 2n\pi$$

$$S.S = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\} \cup \{(2n+1)\pi\}, \forall n \in \mathbb{Z}$$

Ans.

$$\text{Q.5} \quad 2\sin\theta + \cos^2\theta - 1 = 0$$

Solution:

$$2\sin\theta + 1 - \sin^2\theta - 1 = 0$$

$$2\sin\theta - \sin^2\theta = 0$$

$$\sin\theta(2 - \sin\theta) = 0$$

$$\Rightarrow \sin\theta = 0, 2 - \sin\theta = 0$$

$$\Rightarrow \begin{array}{l|l} \sin\theta = 0 & \sin\theta = 2 \\ \theta = n\pi & 2 - \sin\theta = 0 \\ & \sin\theta = 0 \end{array}$$

Which is not possible because $-1 \leq \sin\theta \leq 1$

S.S = $\{n\pi, \forall n \in \mathbb{Z}\}$

Q.6 $2\sin^2\theta - \sin\theta = 0$

Solution:

$$\begin{array}{l} \sin\theta(2\sin\theta - 1) = 0 \\ \Rightarrow \sin\theta = 0 \text{ and } 2\sin\theta - 1 = 0 \\ \qquad \qquad \qquad 2\sin\theta = 1 \\ \qquad \qquad \qquad \sin\theta = 1/2 \end{array}$$

$$\Rightarrow \begin{array}{l|l} \sin\theta = 0 & \sin\theta = 1/2 \\ \theta = n\pi & \text{Since } \sin\theta \text{ is positive in I and II quadrants with the reference angle } \pi/6. \end{array}$$

I quad.	II quad.
$\theta = \pi/6 + 2n\pi, \forall n \in \mathbb{Z}$	$\theta = \pi - \pi/6$
	$\theta = 5\pi/6$
	$\theta = 5\pi/6 + 2n\pi, \forall n \in \mathbb{Z}$

S.S = $\{n\pi\} \cup \{\pi/6 + 2n\pi\} \cup \{5\pi/6 + 2n\pi\}, \forall n \in \mathbb{Z}$

Q.7 $3\cos^2\theta - 2\sqrt{3} \sin\theta \cos\theta - 3\sin^2\theta = 0$

Solution:

$$\begin{aligned} 3\cos^2\theta - 2\sqrt{3} \sin\theta \cos\theta - \sqrt{3} \sin\theta \cos\theta - 3\sin^2\theta &= 0 \\ 3\cos\theta(\cos\theta - \sqrt{3} \sin\theta) + \sqrt{3} \sin\theta (\cos\theta - \sqrt{3} \sin\theta) &= 0 \\ (\cos\theta - \sqrt{3} \sin\theta) (3\cos\theta + \sqrt{3} \sin\theta) &= 0 \end{aligned}$$

$\cos\theta - \sqrt{3} \sin\theta = 0$ $\cos\theta = \sqrt{3} \sin\theta$ $\tan\theta = \frac{1}{\sqrt{3}}$	$3\cos\theta + \sqrt{3} \sin\theta = 0$ $\sqrt{3} \sin\theta = -3\cos\theta$ $\tan\theta = -\sqrt{3}$
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Since $\tan\theta$ is positive in I to III quadrants with the reference angle $\pi/6$

Since $\tan\theta$ is negative in III and IV quadrants with the reference angle $\pi/3$.

I quad.	III quad	I quad.	III quad
$\theta = \pi/6 + n\pi$	$\theta = \pi + \pi/6$	$\theta = \pi - \pi/3$	$\theta = 2\pi - \pi/3$
	$\theta = 7\pi/6$	$\theta = 2\pi/3$	$\theta = 5\pi/3$
	$\theta = 7\pi/6 + n\pi$	$\theta = 2\pi/3 + n\pi$	$\theta = 5\pi/3 + n\pi$

S.S = $\{\pi/6 + n\pi\} \cup \{7\pi/6 + n\pi\} \cup \{2\pi/3 + n\pi\} \cup \{5\pi/3 + n\pi\}, \forall n \in \mathbb{Z}$

Q.8 Find the values of θ $4 \sin^2 \theta - 8 \cos \theta + 1 = 0$

Solution:

$$4 \sin^2 \theta - 8 \cos \theta + 1 = 0$$

$$4 (1 - \cos^2 \theta) - 8 \cos \theta + 1 = 0$$

$$4 - 4 \cos^2 \theta - 8 \cos \theta + 1 = 0$$

$$4 \cos^2 \theta + 8 \cos \theta - 5 = 0$$

$$4 \cos^2 \theta + 10 \cos \theta - 2 \cos \theta - 5 = 0$$

$$2 \cos \theta (2 \cos \theta + 5) - 1 (2 \cos \theta + 5) = 0$$

$$(2 \cos \theta + 5) (2 \cos \theta - 1) = 0$$

$$2 \cos \theta + 5 = 0$$

$$\cos \theta = \frac{-5}{2}$$

i.e. solution is impossible.

$$2 \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos \theta$ is +ve in I & IV Quadrants with reference angle $\frac{\pi}{3}$

For I-Quad.

$$\theta = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2n\pi ,$$

For IV-Quad.

$$\theta = 2\pi - \frac{\pi}{3}$$

$$= \frac{5\pi}{3} + 2n\pi$$

$$\text{S.S} = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\} , \forall n \in \mathbb{Z} \text{ Ans.}$$

Q.9 Find the solution set of the following equations.

$$\sqrt{3} \tan x - \sec x - 1 = 0$$

(Gujranwala Board 2004)

Solution:

$$\sqrt{3} \tan x = 1 + \sec x$$

$$(\sqrt{3} \tan x)^2 = (1 + \sec x)^2$$

$$3 \tan^2 x = 1 + \sec^2 x + 2 \sec x$$

$$3 (\sec^2 x - 1) = 1 + \sec^2 x + 2 \sec x$$

$$3 \sec^2 x - 3 - 1 - \sec^2 x - 2 \sec x = 0$$

$$2 \sec^2 x - 2 \sec x - 4 = 0$$

$$\sec^2 x - \sec x - 2 = 0$$

Q.8 Find the values of θ $4 \sin^2 \theta - 8 \cos \theta + 1 = 0$

Solution:

$$4 \sin^2 \theta - 8 \cos \theta + 1 = 0$$

$$4 (1 - \cos^2 \theta) - 8 \cos \theta + 1 = 0$$

$$4 - 4 \cos^2 \theta - 8 \cos \theta + 1 = 0$$

$$4 \cos^2 \theta + 8 \cos \theta - 5 = 0$$

$$4 \cos^2 \theta + 10 \cos \theta - 2 \cos \theta - 5 = 0$$

$$2 \cos \theta (2 \cos \theta + 5) - 1 (2 \cos \theta + 5) = 0$$

$$(2 \cos \theta + 5) (2 \cos \theta - 1) = 0$$

$$2 \cos \theta + 5 = 0$$

$$\cos \theta = \frac{-5}{2}$$

i.e. solution is impossible.

$$2 \cos \theta - 1 = 0$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos \theta$ is +ve in I & IV Quadrants with reference angle $\frac{\pi}{3}$

For I-Quad.

$$\theta = \frac{\pi}{3}$$

$$\theta = \frac{\pi}{3} + 2n\pi ,$$

For IV-Quad.

$$\theta = 2\pi - \frac{\pi}{3}$$

$$= \frac{5\pi}{3} + 2n\pi$$

$$\text{S.S} = \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\} , \forall n \in \mathbb{Z} \text{ Ans.}$$

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$$\sqrt{3} \tan x - \sec x - 1 = 0$$

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Solution:

$$\sqrt{3} \tan x = 1 + \sec x$$

$$(\sqrt{3} \tan x)^2 = (1 + \sec x)^2$$

$$3 \tan^2 x = 1 + \sec^2 x + 2 \sec x$$

$$3 (\sec^2 x - 1) = 1 + \sec^2 x + 2 \sec x$$

$$3 \sec^2 x - 3 - 1 - \sec^2 x - 2 \sec x = 0$$

$$2 \sec^2 x - 2 \sec x - 4 = 0$$

$$\sec^2 x - \sec x - 2 = 0$$

By quadratic formula

$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin x = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)} = \frac{-2 \pm \sqrt{4 + 16}}{8}$$

$$\sin x = \frac{-2 \pm \sqrt{20}}{8}$$

$$x = \sin^{-1}\left(\frac{\sqrt{20} - 2}{8}\right)$$

$$x = 18^\circ = \frac{\pi}{10}$$

Since $\sin x$ is +ve in the I and II Quadrants
with reference angle $\frac{\pi}{10}$

For I-Quad. For II-Quad.

$$x = \frac{\pi}{10}, \quad x = \pi - \frac{\pi}{10}$$

$$x = \frac{\pi}{10} + 2n\pi, \quad x = \frac{9\pi}{10} + 2n\pi$$

Also $\sin x = 1$

$$x = \frac{\pi}{2} + 2n\pi, \quad \forall n \in \mathbb{Z}$$

Hence solution set is

$$\left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\} \quad n \in \mathbb{Z} \quad \text{Ans}$$

Q.11 $\sec 3\theta = \sec \theta$

Solution:

$$\sec 3\theta = \sec \theta$$

$$\frac{1}{\cos 3\theta} = \frac{1}{\cos \theta}$$

$$\cos 3\theta = \cos \theta$$

$$\cos 3\theta - \cos \theta = 0$$

$$\sin x = \frac{-\sqrt{20} - 2}{8} = -\left(\frac{\sqrt{20} + 2}{8}\right)$$

$$x = \sin^{-1}\left(\frac{\sqrt{20} + 2}{8}\right)$$

$$x = 54^\circ = \frac{3\pi}{10}$$

Since $\sin x$ is -ve in III & IV Quadrants
with reference angle $\frac{3\pi}{10}$

For III-Quad.

For IV-Quad.

$$x = \pi + \frac{3\pi}{10},$$

$$x = 2\pi - \frac{3\pi}{10}$$

$$x = \frac{13\pi}{10} + 2n\pi,$$

$$x = \frac{17\pi}{10} + 2n\pi$$

$$-2 \sin\left(\frac{3\theta + \theta}{2}\right) \sin\left(\frac{3\theta - \theta}{2}\right) = 0$$

$$-2 \sin 2\theta \sin \theta = 0$$

$$\Rightarrow \sin 2\theta \sin \theta = 0$$

$$\Rightarrow \sin 2\theta = 0$$

$$\Rightarrow 2\theta = n\pi$$

$$\Rightarrow \theta = \frac{n\pi}{2}$$

$$\sin \theta = 0$$

$$\Rightarrow \theta = n\pi$$

Solution set is $\left\{\frac{n\pi}{2}\right\} \cup \{n\pi\}$, $n \in \mathbb{Z}$ Ans.

Q.12 $\tan 2\theta + \cot \theta = 0$

Solution:

$$\tan 2\theta = -\cot \theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = -\frac{\cos \theta}{\sin \theta}$$

$$\sin 2\theta \sin \theta = -\cos 2\theta \cos \theta$$

$$\sin 2\theta \sin \theta + \cos 2\theta \cos \theta = 0$$

$$\Rightarrow \cos (2\theta - \theta) = 0$$

$$\Rightarrow \cos \theta = 0$$

$$\Rightarrow \theta = (2n + 1) \frac{\pi}{2}$$

Solution set is $\left\{(2n + 1) \frac{\pi}{2}\right\}$, $n \in \mathbb{Z}$

Q.13 $\sin 2x + \sin x = 0$

Solution:

$$\sin 2x + \sin x = 0$$

$$2 \sin x \cos x + \sin x = 0$$

$$\sin x (2 \cos x + 1) = 0$$

$$\sin x = 0$$

$$x = n\pi$$

$$2 \cos x + 1 = 0$$

$$\cos x = \frac{-1}{2}$$

As $\cos x$ is +ve in I & IV Quadrants
with reference angle $\frac{\pi}{3}$

For I-Quad.

$$x = \frac{\pi}{3},$$

$$x = \frac{\pi}{3} + 2n\pi,$$

For II-Quad.

$$x = 2\pi - \frac{\pi}{3}$$

$$x = \frac{5\pi}{3} + 2n\pi$$

Therefore solution set is

$$\{n\pi\} \cup \left\{ \frac{\pi}{3} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{3} + 2n\pi \right\}, \quad n \in \mathbb{Z}$$

Q.14 $\sin 4x - \sin 2x = \cos 3x$

Solution:

$$\sin 4x - \sin 2x = \cos 3x$$

$$2 \cos \left(\frac{4x + 2x}{2} \right) \sin \left(\frac{4x - 2x}{2} \right) = \cos 3x$$

$$2 \cos 3x \sin x - \cos 3x = 0$$

$$\cos 3x [2 \sin x - 1] = 0$$

$$\cos 3x = 0$$

$$\Rightarrow 3x = (2n + 1) \frac{\pi}{2}$$

$$\Rightarrow x = (2n + 1) \frac{\pi}{6}$$

$$2 \sin x - 1 = 0$$

$$\Rightarrow 2 \sin x = 1$$

$$\Rightarrow \sin x = \frac{1}{2} \quad \boxed{x = \sin^{-1} \left(\frac{1}{2} \right) = 30^\circ = \frac{\pi}{6}}$$

Since $\sin x$ is +ve in I and II Quadrants

with reference angle $\frac{\pi}{6}$ so

For I-Quad.

$$x = \frac{\pi}{6}$$

$$x = \frac{\pi}{6} + 2n\pi,$$

For II-Quad.

$$x = \pi - \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} + 2n\pi, \quad \forall n \in \mathbb{Z}$$

Hence solution set is

$$\left\{ (2n + 1) \frac{\pi}{6} \right\} \cup \left\{ \frac{\pi}{6} + 2n\pi \right\} \cup \left\{ \frac{5\pi}{6} + 2n\pi \right\}, \quad n \in \mathbb{Z}$$

Ans.

Q.15 $\sin x + \cos 3x = \cos 5x$

Solution:

$$\sin x + \cos 3x = \cos 5x$$

$$\sin x = \cos 5x - \cos 3x$$

$$\sin x = -2 \sin \left(\frac{5x + 3x}{2} \right) \sin \left(\frac{5x - 3x}{2} \right)$$

$$= -2 \sin 4x \sin x$$

$$\sin x + 2 \sin 4x \sin x = 0$$

$$\sin x (1 + 2 \sin 4x) = 0$$

$$\sin x = 0$$

$$x = n\pi$$

$$\sin 4x = -\frac{1}{2} \Rightarrow \boxed{4x = \sin^{-1}\left(-\frac{1}{2}\right) = 30^\circ = \frac{\pi}{6}}$$

since $\sin x$ is -ve in III & IV Quadrants

with reference angle $\frac{\pi}{6}$

For III-Quad.

For II-Quad.

$$4x = \pi + \frac{\pi}{6}, \quad 4x = 2\pi - \frac{\pi}{6}$$

$$4x = \frac{7\pi}{6} + 2n\pi, \quad 4x = \frac{11\pi}{6} + 2n\pi$$

$$x = \frac{7\pi}{24} + \frac{n\pi}{2}, \quad x = \frac{11\pi}{24} + \frac{n\pi}{2}$$

Hence solution set $\{n\pi\} \cup \left\{\frac{7\pi}{24} + \frac{n\pi}{2}\right\} \cup \left\{\frac{11\pi}{24} + \frac{n\pi}{2}\right\}, \quad n \in \mathbb{Z}$ Ans.

Q.16 $\sin 3x + \sin 2x + \sin x = 0$

Solution:

$$\sin 3x + \sin 2x + \sin x = 0$$

$$\sin 3x + \sin x + \sin 2x = 0$$

$$2 \cos\left(\frac{3x-x}{2}\right) \sin\left(\frac{3x+x}{2}\right) + \sin 2x = 0$$

$$2 \sin 2x \cos x + \sin 2x = 0$$

$$\sin 2x (2 \cos x + 1) = 0$$

$$\sin 2x = 0$$

$$2x = n\pi$$

$$x = \frac{n\pi}{2}$$

$$2 \cos x + 1 = 0$$

$$\boxed{x = \cos^{-1}\left(-\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}}$$

$$\cos x = -\frac{1}{2}$$

Since $\cos$ is -ve in II & III Quadrants

with reference angle $\frac{\pi}{3}$

For II-Quad.

For III-Quad.

$$x = \pi - \frac{\pi}{3}, \quad x = \pi + \frac{\pi}{3}$$

$$x = \frac{2\pi}{3} + 2n\pi, \quad x = \frac{4\pi}{3} + 2n\pi, \quad \forall n \in \mathbb{Z}$$

Solution set is $\left\{\frac{n\pi}{2}\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}, \quad n \in \mathbb{Z}$ Ans.

Q.17 $\sin 7x - \sin x = \sin 3x$

Solution:

$$\sin 7x - \sin x = \sin 3x$$

$$2 \cos \left(\frac{7x+x}{2} \right) \sin \left(\frac{7x-x}{2} \right) = \sin 3x$$

$$2 \cos 4x \sin 3x = \sin 3x$$

$$2 \cos 4x \sin 3x - \sin 3x = 0$$

$$\sin 3x (2 \cos 4x - 1) = 0$$

$$\Rightarrow \sin 3x = 0$$

$$3x = n\pi$$

$$x = \frac{n\pi}{3}$$

$$2 \cos 4x - 1 = 0$$

$$2 \cos 4x = 1$$

$$\cos 4x = \frac{1}{2}$$

$$4x = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos x$ is +ve in I & IV Quadrants.

with reference angle $\frac{\pi}{3}$

For I-Quad.

For IV-Quad.

$$4x = \frac{\pi}{3}$$

$$, \quad 4x = 2\pi - \frac{\pi}{3}$$

$$, \quad 4x = \frac{5\pi}{3}$$

$$4x = \frac{\pi}{3} + 2n\pi$$

$$, \quad 4x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{12} + \frac{n\pi}{2}$$

$$, \quad x = \frac{5\pi}{12} + \frac{n\pi}{2}$$

Therefore solution set is $\left\{ \frac{n\pi}{3} \right\} \cup \left\{ \frac{\pi}{12} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{5\pi}{12} + \frac{n\pi}{2} \right\} \quad n \in \mathbb{Z} \quad \text{Ans.}$

Q.18 $\sin x + \sin 3x + \sin 5x = 0$

Solution:

$$\sin 5x + \sin x + \sin 3x = 0$$

$$2 \sin \left(\frac{5x+x}{2} \right) \cos \left(\frac{5x-x}{2} \right) + \sin 3x = 0$$

$$2 \sin 3x \cos 2x + \sin 3x = 0$$

$$\sin 3x (2 \cos 2x + 1) = 0$$

$$\sin 3x = 0$$

$$3x = n\pi$$

$$x = \frac{n\pi}{3}$$

$$2 \cos 2x + 1 = 0$$

$$\cos 2x = \frac{-1}{2}$$

$$2x = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ = \frac{\pi}{3}$$

Since $\cos x$ is -ve in II & III Quadrants,
with reference angle $\frac{\pi}{3}$ so

For I-Quad.

For III-Quad.

$$2x = \pi - \frac{\pi}{3}, \quad 2x = \pi + \frac{\pi}{3}$$

$$2x = \frac{2\pi}{3}, \quad 2x = \frac{4\pi}{3}$$

$$2x = \frac{2\pi}{3} + 2n\pi, \quad 2x = \frac{4\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{3} + \frac{2n\pi}{2}, \quad x = \frac{2\pi}{3} + \frac{2n\pi}{2}$$

$$x = \frac{\pi}{3} + n\pi, \quad x = \frac{2\pi}{3} + n\pi$$

Hence solution set is

$$\left\{\frac{n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{2\pi}{3} + \frac{2n\pi}{3}\right\}, \quad n \in \mathbb{Z} \quad \text{Ans.}$$

Q.19 $\sin \theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 0$

Solution:

$$[\sin 7\theta + \sin \theta] + [\sin 5\theta + \sin 3\theta] = 0$$

$$\left[2 \sin\left(\frac{7\theta + \theta}{2}\right) \cos\left(\frac{7\theta - \theta}{2}\right)\right] + \left[2 \sin\left(\frac{5\theta + 3\theta}{2}\right) \cos\left(\frac{5\theta - 3\theta}{2}\right)\right] = 0$$

$$2 \sin 4\theta \cos 3\theta + 2 \sin 4\theta \cos \theta = 0$$

$$2 \sin 4\theta (\cos 3\theta + \cos \theta) = 0$$

$$2 \sin 4\theta \left[2 \cos\left(\frac{3\theta + \theta}{2}\right) \cos\left(\frac{3\theta - \theta}{2}\right)\right] = 0$$

$$2 \times 2 \sin 4\theta (\cos 2\theta \cos \theta) = 0$$

$$\sin 4\theta \cos 2\theta \cos \theta = 0$$

$$\Rightarrow \sin 4\theta = 0 \quad \left| \quad \cos 2\theta = 0 \quad \right| \quad \cos \theta = 0$$

$$4\theta = n\pi \quad \left| \quad 2\theta = (2n+1)\frac{\pi}{2} \quad \right| \quad \theta = (2n+1)\frac{\pi}{2}$$

$$\theta = \frac{n\pi}{4} \quad \left| \quad \theta = (2n+1)\frac{\pi}{4} \quad \right|$$

Hence solutions set is

$$\left\{ \frac{n\pi}{4} \right\} \cup \left\{ (2n+1)\frac{\pi}{4} \right\} \cup \left\{ (2n+1)\frac{\pi}{2} \right\}, n \in \mathbb{Z} \quad \text{Ans.}$$

OR

$$\left\{ \frac{n\pi}{4} \right\} \cup \left\{ \frac{\pi}{4} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{2} + n\pi \right\}, n \in \mathbb{Z}$$

Q.20 $\cos \theta + \cos 3\theta + \cos 5\theta + \cos 7\theta = 0$

Solution:

$$[\cos 7\theta + \cos \theta] + [\cos 5\theta + \cos 3\theta] = 0$$

$$2 \cos \left(\frac{7\theta + \theta}{2} \right) \cos \left(\frac{7\theta - \theta}{2} \right) + 2 \cos \left(\frac{5\theta + 3\theta}{2} \right) \cos \left(\frac{5\theta - 3\theta}{2} \right) = 0$$

$$2 \cos 4\theta \cos 3\theta + 2 \cos 4\theta \cos \theta = 0$$

$$2 \cos 4\theta (\cos 3\theta + \cos \theta) = 0$$

$$2 \cos 4\theta \left[2 \cos \left(\frac{3\theta + \theta}{2} \right) \cos \left(\frac{3\theta - \theta}{2} \right) \right] = 0$$

$$2 \times 2 \cos 4\theta (\cos 2\theta \cos \theta) = 0$$

$$\Rightarrow \cos 4\theta = 0 \quad \left| \quad \cos 2\theta = 0 \quad \right| \quad \cos \theta = 0$$

$$4\theta = (2n+1)\frac{\pi}{2} \quad \left| \quad 2\theta = (2n+1)\frac{\pi}{2} \quad \right| \quad \theta = (2n+1)\frac{\pi}{2}$$

$$\theta = (2n+1)\frac{\pi}{8} \quad \left| \quad \theta = (2n+1)\frac{\pi}{4} \quad \right|$$

Hence solution set is $\left\{ (2n+1)\frac{\pi}{8} \right\} \cup \left\{ (2n+1)\frac{\pi}{4} \right\} \cup \left\{ (2n+1)\frac{\pi}{2} \right\}, n \in \mathbb{Z}$

OR

$$\text{S.S} = \left\{ \frac{\pi}{8} + \frac{n\pi}{4} \right\} \cup \left\{ \frac{\pi}{4} + \frac{n\pi}{2} \right\} \cup \left\{ \frac{\pi}{2} + n\pi \right\}, n \in \mathbb{Z}$$