

Chapter 11

TRIGONOMETRIC FUNCTIONS & THEIR GRAPHS

DOMAIN AND RANGE OF TRIGONOMETRIC FUNCTION

Function	Domain	Range
$y = \sin x$	$\mathbb{R}$	$-1 \leq y \leq 1$
$y = \cos x$	$\mathbb{R}$	$-1 \leq y \leq 1$
$y = \tan x$	$\forall x \in \mathbb{R}, \text{ but } x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$	$\mathbb{R}$
$y = \cot x$	$\forall x \in \mathbb{R}, \text{ but } x \neq n\pi, n \in \mathbb{Z}$	$\mathbb{R}$
$y = \sec x$	$\forall x \in \mathbb{R}, \text{ but } x \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$	$y \geq 1 \text{ or } y \leq -1$
$y = \operatorname{cosec} x$	$\forall x \in \mathbb{R}, \text{ but } x \neq n\pi, n \in \mathbb{Z}$	$y \geq 1 \text{ or } y \leq -1$

Period

Period of a trigonometric function is the smallest +ve number which, when added to the original circular measure of the angle, given the same value of the function.

Formula to Find Period

$$f(x) = f(x + P)$$

Standard Periods

$$\sin x = 2\pi$$

$$\tan x = \pi$$

$$\cos x = 2\pi$$

$$\cot x = \pi$$

$$\sec x = 2\pi$$

$$\operatorname{cosec} x = 2\pi$$

EXERCISE 11.1

Find the periods of the following functions:

Q.1 $\sin 3x$ (Lahore Board 2005)

Solution:

Given function is $\sin 3x$

Since period of $\sin$ is 2π

Therefore $f(x) = f(x + P)$

$$\sin 3x = \sin (3x + 2\pi)$$

$$\sin 3x = \sin 3 \left(x + \frac{2\pi}{3} \right)$$

Thus period of $\sin 3x$ is $\frac{2\pi}{3}$

Q.2 $\cos 2x$

Solution:

Given function is $\cos 2x$

Since period of $\cos$ is 2π

Therefore $f(x) = f(x + P)$

$$\cos 2x = \cos (2x + 2\pi)$$

$$\cos 2x = \cos 2(x + \pi)$$

Thus period of $\cos 2x$ is π .

Q.3 $\tan 4x$ (Gujranwala Board 2007)

Solution:

Given function is $\tan 4x$

Since period of $\tan x$ is π

Therefore $f(x) = f(x + P)$

$$\tan 4x = \tan (4x + \pi)$$

$$\tan 4x = \tan 4 \left(x + \frac{\pi}{4} \right)$$

Thus period of $\tan 4x$ is $\frac{\pi}{4}$.

Q.4 $\cot \frac{x}{2}$

Solution:

Given function is $\cot \frac{x}{2}$

Since period of $\cot$ is π

Therefore $f(x) = f(x + P)$

$$\cot \frac{x}{2} = \cot \left(\frac{x}{2} + \pi \right)$$

$$\cot \frac{x}{2} = \cot \frac{1}{2} (x + 2\pi)$$

Thus period of $\cot \frac{x}{2}$ is 2π

Q.5 $\sin \frac{x}{3}$ (Lahore Board 2006)

Solution:

Given function is $\sin \frac{x}{3}$

Since period of $\sin$ is 2π

Therefore $f(x) = f(x + P)$

$$\sin \frac{x}{3} = \sin \left(\frac{x}{3} + 2\pi \right)$$

$$\sin \frac{x}{3} = \sin \frac{1}{3} (x + 6\pi)$$

Thus period of $\sin \frac{x}{3}$ is 6π

Q.6 $\operatorname{cosec} \frac{x}{4}$

Solution:

Given function is $\operatorname{cosec} \frac{x}{4}$

Since period of cosec is 2π

Therefore $f(x) = f(x + P)$

$$\operatorname{cosec} \frac{x}{4} = \operatorname{cosec} \left(\frac{x}{4} + 2\pi \right)$$

$$\operatorname{cosec} \frac{x}{4} = \operatorname{cosec} \frac{1}{4} (x + 8\pi)$$

Thus period of $\operatorname{cosec} \frac{x}{4}$ is 8π

Q.7 $\sin \frac{x}{5}$ (Lahore Board 2010)

Solution:

Given function is $\sin \frac{x}{5}$

Since period of $\sin x$ is 2π

Therefore $f(x) = f(x + P)$

$$\sin \frac{x}{5} = \sin \left(\frac{x}{5} + 2\pi \right)$$

$$\sin \frac{x}{5} = \sin \frac{1}{5} (x + 10\pi)$$

Thus period of $\sin \frac{x}{5}$ is 10π

Q.8 $\cos \frac{x}{6}$

Solution:

Given function is $\cos \frac{x}{6}$

Since period of $\cos x$ is 2π

$f(x) = f(x + P)$

$$\text{Therefore } \cos \frac{x}{6} = \cos \left(\frac{x}{6} + 2\pi \right) = \cos \frac{1}{6} (x + 12\pi)$$

Thus period of $\cos \frac{x}{6}$ is 12π

Q.9 $\tan \frac{x}{7}$

Solution:

Given function is $\tan \frac{x}{7}$

Since period of $\tan x$ is π

$$f(x) = f(x + P)$$

$$\text{Therefore } \tan \frac{x}{7} = \tan \left(\frac{x}{7} + \pi \right)$$

$$\tan \frac{x}{7} = \tan \frac{1}{7} (x + 7\pi)$$

$$\text{Thus period of } \tan \frac{x}{7} \text{ is } 7\pi$$

Q.10 $\cot 8x$

Solution:

Given function is $\cot 8x$

Since period of $\cot x$ is π

$$f(x) = f(x + P)$$

$$\text{Therefore } \cot 8x = \cot (8x + \pi)$$

$$\cot 8x = \cot 8 \left(x + \frac{\pi}{8} \right)$$

$$\text{Thus period of } \cot 8x \text{ is } \frac{\pi}{8}$$

Q.11 $\sec 9x$

Solution:

Given function is $\sec 9x$

Since period of $\sec x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } \sec 9x = \sec (9x + 2\pi)$$

$$\sec 9x = \sec 9 \left(x + \frac{2\pi}{9} \right)$$

$$\text{Thus period of } \sec 9x \text{ is } \frac{2\pi}{9}$$

Q.12 $\operatorname{cosec} 10x$

Solution:

Given function is $\operatorname{cosec} 10x$

Since period of $\operatorname{cosec} x$ is 2π

$$f(x) = f(x + P)$$

Therefore $\operatorname{cosec} 10 x = \operatorname{cosec} (10x + 2\pi)$

$$\operatorname{cosec} 10 x = \operatorname{cosec} 10 \left(x + \frac{2 \pi}{10} \right)$$

Thus period of $\operatorname{cosec} 10 x$ is $\frac{\pi}{5}$

Q.13 $3 \sin x$

Solution:

Given function is $3 \sin x$

Since period of $\sin x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } 3 \sin x = 3 \sin (x + 2\pi)$$

Thus period of $3 \sin x$ is 2π .

Q.14 $2 \cos x$

Solution:

Given function is $2 \cos x$

Since period of $\cos x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } 2 \cos x = 2 \cos (x + 2\pi)$$

Thus period of $2 \cos x$ is 2π

Q.15 $3 \cos \frac{x}{5}$ (Lahore Board 2008)

Solution:

Given function is $3 \cos \frac{x}{5}$

Since period of $\cos x$ is 2π

$$f(x) = f(x + P)$$

$$\text{Therefore } 3 \cos \frac{x}{5} = 3 \cos \left(\frac{x}{5} + 2 \pi \right)$$

$$3 \cos \frac{x}{5} = 3 \cos \frac{1}{5} (x + 10 \pi)$$

Thus period of $3 \cos \frac{x}{5}$ is 10π .

EXERCISE 11.2

Q.1 Draw the graph of each of the following equations for the intervals mentioned against each:

(i) $y = -\sin x$; $x \in [-2\pi, 2\pi]$

Solution:

Given function is $y = -\sin x$

We want to draw the graph of $y = -\sin x$ from -360° to 360° .

Taking the sub-interval of 30° , we have the following table of values.

x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°	-90°	-60°	-30°
y	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1	0.9	0.5

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y	0	-0.5	0.9	-1	-0.9	-0.5	0	0.5	0.9	1	0.9	0.5	0

Pairs

$(-360^\circ, 0), (-330^\circ, -0.5), (-300^\circ, -0.9), (-270^\circ, -1), (-240^\circ, -0.9)$

$(-210^\circ, -0.5), (-180^\circ, 0), (-150^\circ, 0.5), (-120^\circ, 0.9), (-90^\circ, 1)$

$(-60^\circ, 0.9), (-30^\circ, 0.5), (0^\circ, 0), (30^\circ, -0.5), (60^\circ, -0.9)$

$(90^\circ, -1), (120^\circ, -0.9), (150^\circ, -0.5), (180^\circ, 0), (210^\circ, 0.5)$

$(240^\circ, 0.9), (270^\circ, 1), (300^\circ, 0.9), (330^\circ, 0.5), (360^\circ, 0)$

Scale:

0.1 = One small square

1 = One big square

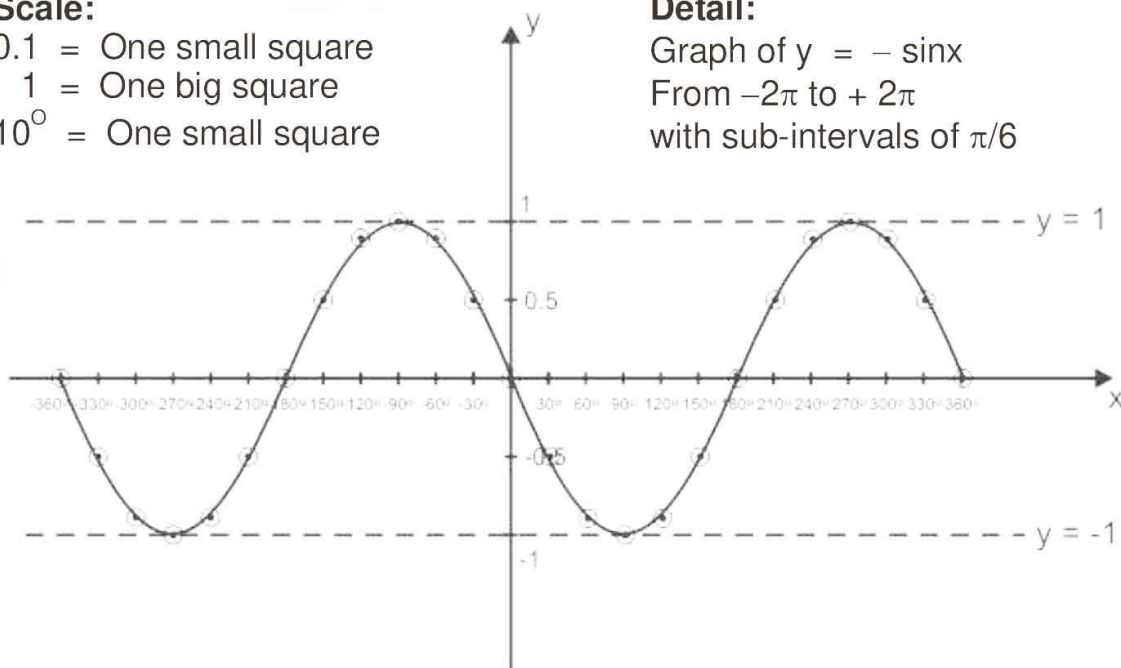
10° = One small square

Detail:

Graph of $y = -\sin x$

From -2π to $+2\pi$

with sub-intervals of $\pi/6$



(ii) $y = 2 \cos x$; $x \in [0, 2\pi]$

Solution:

Given function is $y = 2 \cos x$

We want to draw the graph of $y = 2 \cos x$ from 0 to 36.

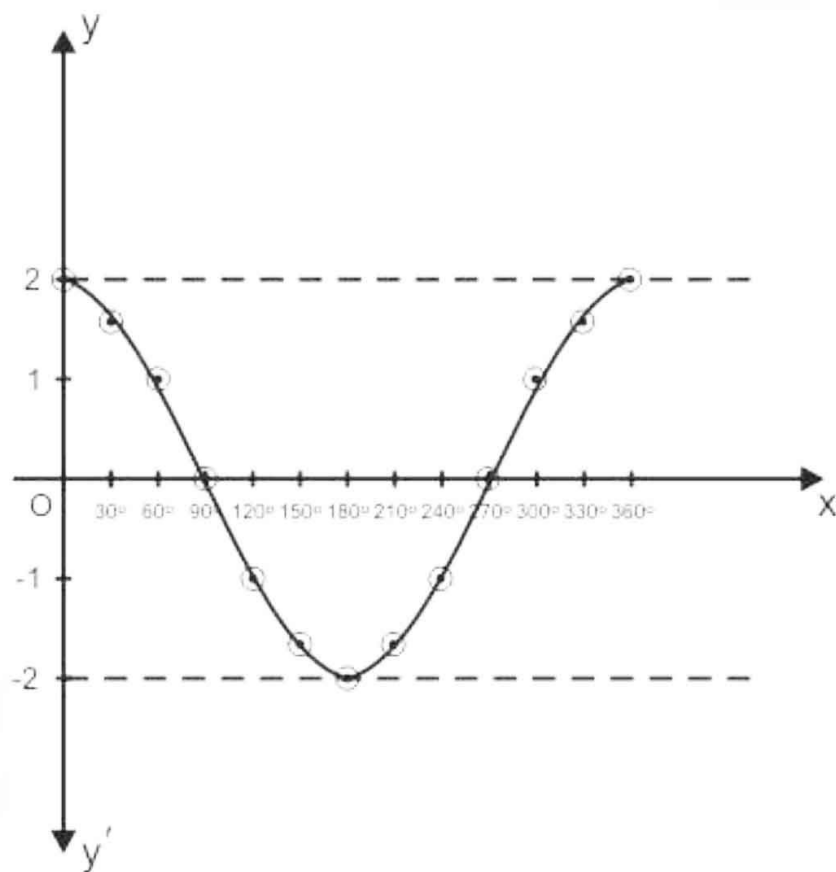
Taking the sub-interval of 30° , we have the following table of values.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \cos x$	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1
$y = 2 \cos x$	2	1.8	1	0	-1	-1.8	-2	-1.8	-1	0	1	1.8	2

Pairs

$(0^\circ, 2), (30^\circ, 1.8), (60^\circ, 1), (90^\circ, 0), (120^\circ, -1), (150^\circ, -1.8), (180^\circ, -2)$

$(210^\circ, -1.8), (240^\circ, -1), (270^\circ, 0), (300^\circ, 1), (330^\circ, 1.8), (360^\circ, 2)$



Scale:

0.1 = One small square

1 = One big square

10° = One small square

(iii) $y = \tan 2x$; $x \in [-\pi, \pi]$

Solution:

Given function is $y = \tan 2x$

We want to draw the graph of $y = \tan 2x$ from -180° to $+180^\circ$.

Taking the sub-interval of 30° , we have the following table of values.

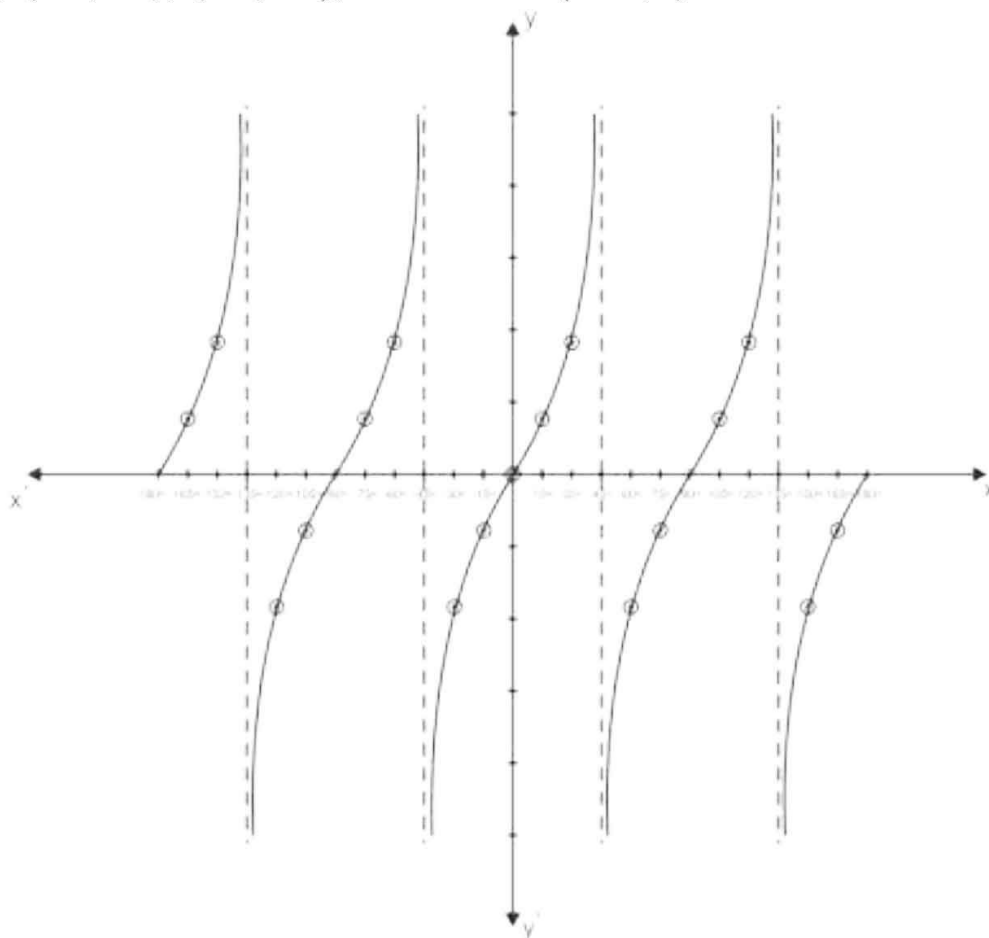
x	-180°	-165°	-150°	-135°	-120°	-105°	-90°	-75°	-60°	-45°	-30°	-15°	0°
2x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-125°	-90°	-60°	-30°	0
y = tan 2x	0	0.6	1.73	∞	-1.73	-0.6	0	0.6	1.7	∞	-1.7	-0.6	0

x	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
2x	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y = tan 2x	0.6	1.7	∞	-1.7	-0.6	0	0.6	1.7	∞	-1.7	-0.6	0

Pairs

$(-180^\circ, 0)$, $(-165^\circ, 0.6)$, $(-150^\circ, -300^\circ)$, ... $(-15^\circ, -0.6)$

$(0, 0)$, $(15^\circ, 0.6)$, $(30^\circ, 1.7)$, ... $(180^\circ, 0)$



Scale:

0.1 = One small square , 1 = One big square , 10° = One small square

(iv) $y = \tan x$; $x \in [-2\pi, 2\pi]$

Solution:

Given function is $y = \tan x$

We want to draw the graph of $y = \tan x$ from -360° to $+360^\circ$.

Taking the sub-interval of 30° , we have the following table of values.

x	-360°	-330°	-300°	-270°	-240°	-210°	-180°	-150°	-120°
y=tanx	0	+0.6	+1.7	∞	-1.7	-0.6	0	0.6	1.7

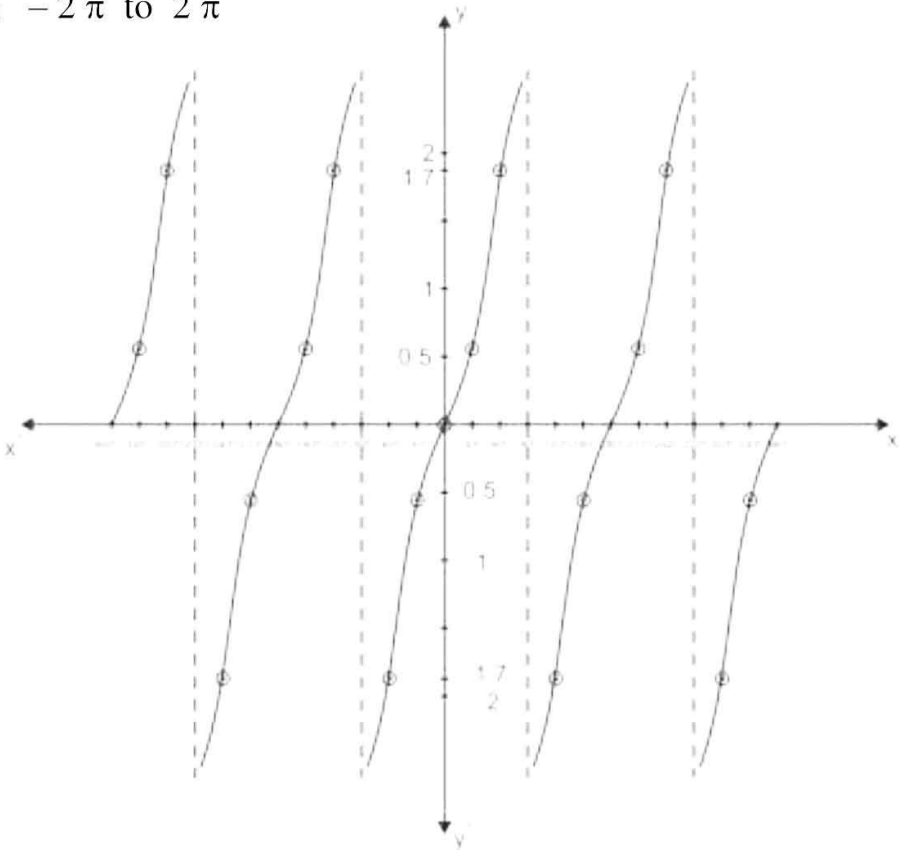
x	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°
y=tanx	$-\infty$	-1.7	-0.6	0	0.6	1.7	$+\infty$	-1.7	-0.6	0	0.6	1.7	$+\infty$	-1.7

x	330°	360°
y = tan x	-0.6	0

Pairs

$(-360^\circ, 0), (-330^\circ, 0.6), (-300^\circ, 1.7), \dots\dots\dots (360^\circ, 0)$

Graph of $y = \tan x$ from -2π to 2π



Scale:

$0.1 = \text{One small square}$, $1 = \text{One big square}$, $10^\circ = \text{One small square}$

(v) $y = \sin \frac{x}{2}; \quad x \in [0, 2\pi]$

Solution:

Given function is $y = \sin \frac{x}{2}$

We want to draw the graph of $y = \sin x$ from 0° to 360° .

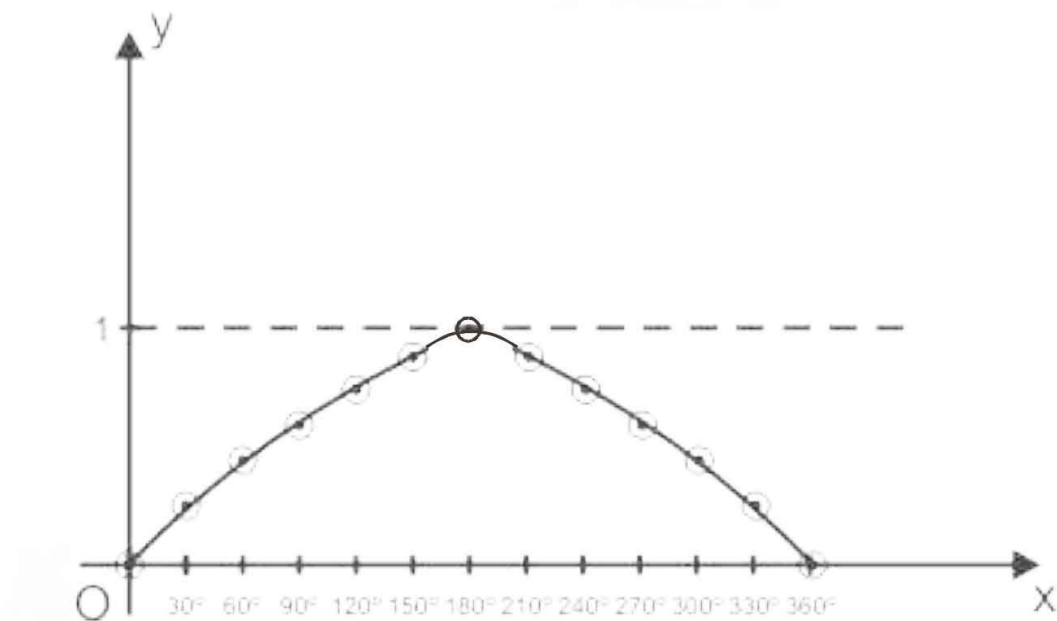
Taking the sub-interval of 30° , we have the following table of values.

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\frac{x}{2}$	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
$y = \sin \frac{x}{2}$	0	0.3	0.5	0.7	0.8	0.95	1	0.95	0.8	0.7	0.5	0.3	0

Pairs

$(0^\circ, 0), (30^\circ, 0.3), (60^\circ, 0.5), (90^\circ, 0.7), (120^\circ, 0.8), (150^\circ, 0.95)$

$(180^\circ, 1), (210^\circ, 0.95), (240^\circ, 0.8), (270^\circ, 0.7), (300^\circ, 0.5), (330^\circ, 0.3), (360^\circ, 0)$



Scale:

$0.1 = \text{One small square}$, $1 = \text{One big square}$, $10^\circ = \text{One small square}$

(vi) $y = \cos \frac{x}{2}; \quad x \in [-\pi, \pi]$

Solution:

Given function is $y = \cos \frac{x}{2}$

We want to draw the graph of $y = \cos \frac{x}{2}$ from -180° to 180° .

Taking the sub-interval of 30° , we have the following table of values.

$$y = \cos \frac{x}{2}$$

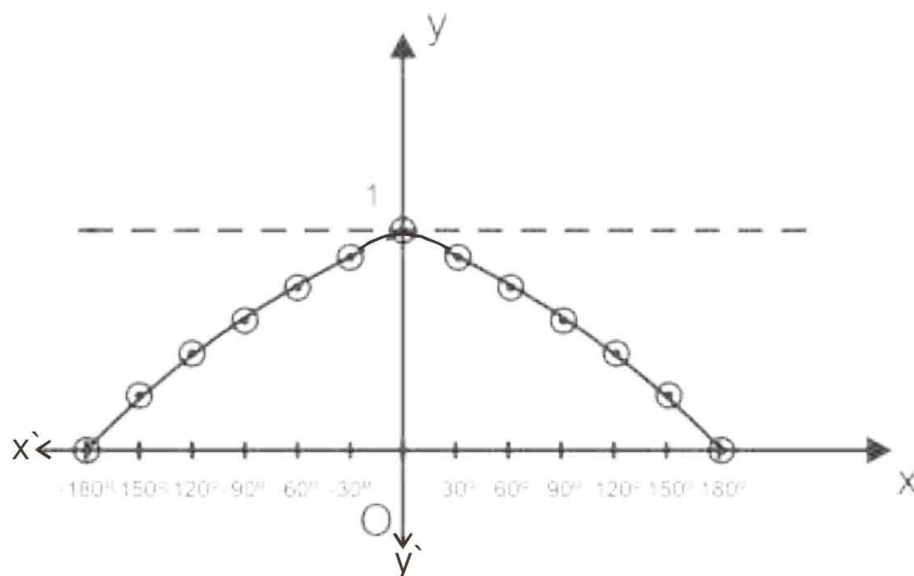
x	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°
$\frac{x}{2}$	-90°	-75°	-60°	-45°	-30°	-15°	0	15°	30°	45°	60°	75°	90°
$y = \cos \frac{x}{2}$	0	0.3	0.5	0.7	0.8	0.95	1	0.95	0.8	0.7	0.5	0.3	0

Pairs

$(-180^\circ, 0), (-150^\circ, 0.3), (-120^\circ, 0.5), (-90^\circ, 0.7), (-60^\circ, 0.8)$

$(-30^\circ, 0.95), (0^\circ, 1), (30^\circ, 0.95), (60^\circ, 0.8), (90^\circ, 0.7)$

$(120^\circ, 0.5), (150^\circ, 0.3), (180^\circ, 0)$



Scale:

$0.1 =$ One small square , $1 =$ One big square , $10^\circ =$ One small square

Q.2 On the same axes and to the same scale, draw the graphs of the following equations.

(i) $y = \sin x$ and $y = \sin 2x$

Solution:

Given functions are $y = \sin x$ and $y = \sin 2x$

As complete period of $\sin x$ is 2π

So we want to draw the graph of $y = \sin x$ from 0 to 360° .

As complete period of $\sin 2x$ is π

So we want to draw the graph of $y = \sin 2x$ from 0 to 180° .

Taking sub interval of 30° . We have the following tables of values.

$$y = \sin x$$

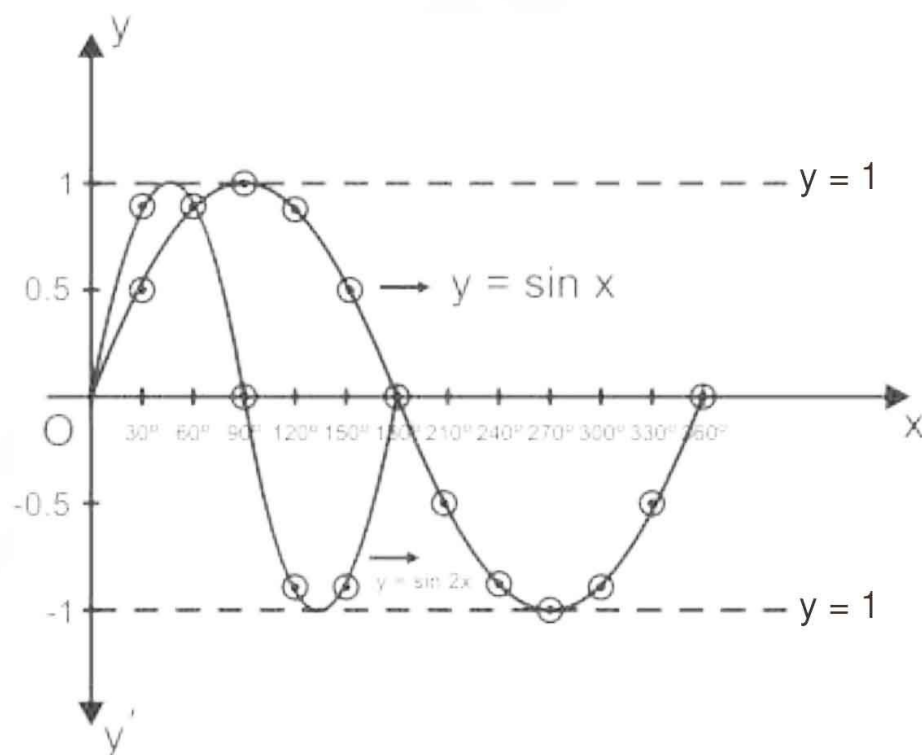
$$y = \cos \frac{x}{2}$$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \sin x$	0	0.5	0.9	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0

x	0°	30°	60°	90°	120°	150°	180°
2x	0	60°	120°	180°	240°	300°	360°
$y = \sin 2x$	0	0.8	0.9	0	-0.9	-0.8	0

Pairs

$(0^\circ, 0)$, $(30^\circ, 0.8)$, $(60^\circ, 0.9)$, $(90^\circ, 0)$, $(120^\circ, 0.9)$, $(150^\circ, 0.8)$, $(180^\circ, 0)$



Scale:

$0.1 =$ One small square , $1 =$ One big square , $10^\circ =$ One small square

(ii) On the same axes and to the same scale, draw the graphs of the following equations for their complete period.

$$y = \cos x \quad \text{and} \quad y = \cos 2x$$

Solution:

Given functions are $y = \cos x$ and $y = \cos 2x$

Firstly, $y = \cos x$, as complete period of $\cos x$ is 2π

So, we want to draw the graph of $y = \cos x$ from 0° to 360° .

Secondly, $y = \cos 2x$, as complete period of $\cos 2x$ is π

So we want to draw the graph of $y = \cos 2x$ from 0 to 180° .

Taking sub-interval of 30° , we have the following tables of values.

$$y = \cos x$$

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = \cos x$	1	0.9	0.5	0	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1

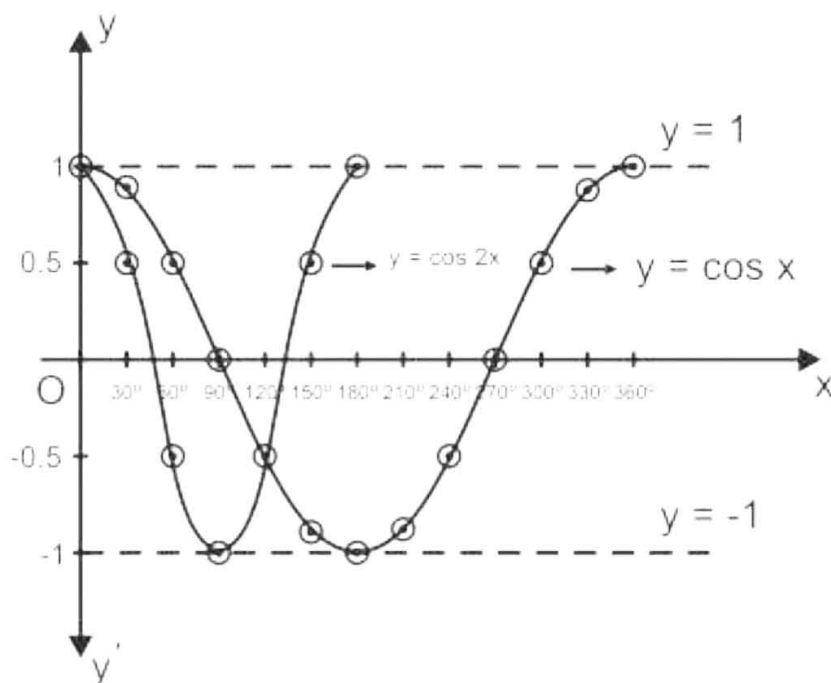
Pairs

$(0^\circ, 1), (30^\circ, 0.9), (60^\circ, 0.5), \dots, (360^\circ, 1)$

x	0°	30°	60°	90°	120°	150°	180°
2x	0	60°	120°	180°	240°	300°	360°
$y = \cos 2x$	1	0.8	-0.5	1	-0.5	0.5	1

Pairs

$(0^\circ, 1), (30^\circ, 0.5), (60^\circ, -0.5), (120^\circ, -0.5), (150^\circ, 300^\circ), (180^\circ, 1)$



Scale:

$0.1 =$ One small square , $1 =$ One big square , $10^\circ =$ One small square

Q.3 Solve graphically

(i) $\sin x = \cos x; \quad x \in [0, \pi]$

Solution:

Given equation is $\sin x = \cos x$

The solution of this equation will be the point of intersection of the graphs of $y = \sin x$ and $y = \cos x$

We want to draw the graph of $y = \sin x$ and $y = \cos x$ from 0 to 180° .

Taking the sub-interval of 30° , we have the following table of values:

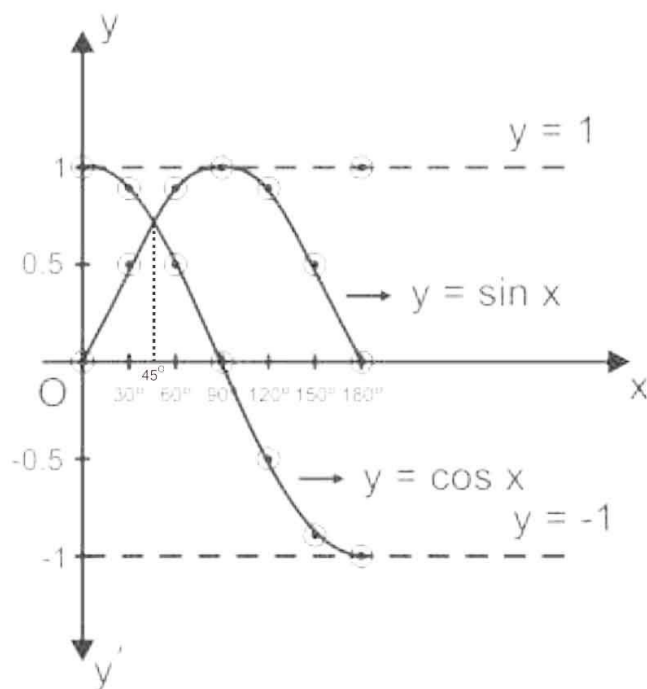
x	0°	30°	60°	90°	120°	150°	180°
$y = \sin x$	0	0.5	0.9	1	0.9	0.5	0
$y = \cos x$	1	0.9	0.5	0	-0.5	-0.9	-1

The pairs for $y = \sin x$ are

$(0^\circ, 0), (30^\circ, 0.5), (60^\circ, 0.9), (90^\circ, 1), (120^\circ, 0.9), (150^\circ, 0.5), (180^\circ, 0)$

The pairs for $y = \cos x$ are

$(0^\circ, 1), (30^\circ, 0.9), (60^\circ, 0.5), (90^\circ, 0), (120^\circ, -0.5), (150^\circ, -0.9), (180^\circ, -1)$



From graph, the point of intersection of $y = \sin x$ and $y = \cos x$ is $x = 45^\circ$
Thus solution set of equation $\sin x = \cos x$ is $\{\pi/4\}$. Ans.

Scale:

$0.1 =$ One small square , $1 =$ One big square , $10^\circ =$ One small square

(ii) Solve graphically

$$\sin x = x; \quad x \in [0, \pi]$$

Solution:

Given equation is $\sin x = x$

The solution of this equation will be the point of intersection of the graphs of $y = \sin x$ and $y = x$

Here we want to draw the graph of $y = \sin x$ and $y = x$ from 0° to 180° .

Taking the sub-interval of 30° , we have the following table of values:

x	0°	30°	60°	90°	120°	150°	180°
$y = \sin x$	0	0.5	0.9	1	0.9	0.5	0
x in radians	0	0.5	1.1	1.6	2.1	2.6	3.1
$y = x$	0	0.5	1.1	1.6	2.1	2.6	3.1

Pairs for $y = \sin x$ are

$(0^\circ, 0), (30^\circ, 0.5), (60^\circ, 0.9), (90^\circ, 1), (120^\circ, 0.9), (150^\circ, 0.5), (180^\circ, 0)$

The pairs for $y = x$ are

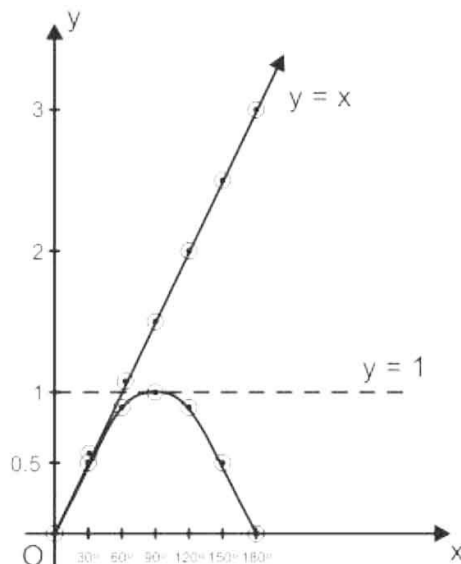
$(0^\circ, 0), (30^\circ, 0.5), (60^\circ, 1.1), (90^\circ, 1.6), (120^\circ, 2.1), (150^\circ, 2.6), (180^\circ, -1)$

From graph, the point of intersection of the equations.

$y = \sin x$ and $y = x$ is 0.

So solution set of the equation $\sin x = x$ is $\{0\}$.

Note: To convert degrees into radians, multiply then by $\frac{\pi}{180}$



Scale:

0.1 = One small square , 1 = One big square , 10° = One small square

TRIGONOMETRIC FUNCTIONS AND THEIR GRAPHS

MULTIPLE CHOICE QUESTIONS

(1) Period of $3 \sin \frac{x}{3}$ is:

- | | |
|------------|------------|
| (a) π | (b) 2π |
| (c) 3π | (d) 6π |

[Lahore Board 2005]

(2) Range of $7 \sin x$ is:

- | | |
|---------------|-------------------|
| (a) $[-7, 7]$ | (b) $(-7, 7)$ |
| (c) $[-1, 1]$ | (d) None of these |

[Lahore Board 2005]

(3) Cotangent is periodic function of period:

- | | |
|------------|---------------------|
| (a) π | (b) 2π |
| (c) 3π | (d) $\frac{\pi}{2}$ |

[Lahore Board 2005]

(4) Period of $\tan x$ is:

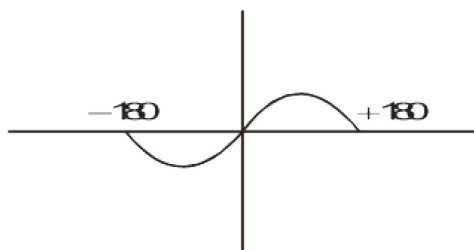
- | | |
|------------|-------------------------------|
| (a) 2π | (b) $2n\pi, n \in \mathbb{Z}$ |
| (c) π | (d) None of these |

[Gujranwala Board 2005]

- (5) Sine is periodic function whose period is:
- | | |
|------------|---------------------|
| (a) π | (b) 2π |
| (c) 3π | (d) $\frac{\pi}{2}$ |

[Gujranwala Board 2005]

- (6) The graph



represents the function from -180 to $+180$

- | | |
|--------------|--------------|
| (a) $\sin x$ | (b) $\cos x$ |
| (c) $\tan x$ | (d) $\cot x$ |

[Gujranwala Board 2005, Lahore Board 2015]

- (7) Range of $\sin x$ is:

- | | |
|------------------|-------------------|
| (a) π | (b) $[-1, 1]$ |
| (c) $\mathbb{R}$ | (d) None of these |

[Gujranwala Board 2005]

- (8) The range of $\cos x$ is:

- | | |
|---------------|------------------|
| (a) $[0, 1]$ | (b) $\mathbb{R}$ |
| (c) $[-1, 1]$ | (d) $[-1, 0]$ |

[Gujranwala Board 2006]

- (9) Domain of $\cot \theta$ is:

- | | |
|--|---|
| (a) $\theta \in \mathbb{R}$ but $\theta \neq n\pi, n \in \mathbb{Z}$ | (b) $\theta \in \mathbb{R}$ but $\theta \neq (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ |
| (c) $\theta \notin \mathbb{R}$ | (d) None of these |

[Gujranwala Board 2006]

- (10) The period of $\cos \frac{x}{5}$ is:

- | | |
|-------------|----------------------|
| (a) 10π | (b) $\frac{2\pi}{5}$ |
| (c) 2π | (d) None of these |

[Gujranwala Board 2006]

(11) The period of $\cot \pi x$ is:

- (a) 1
- (c) π

- (b) 2
- (d) None

[Gujranwala Board 2006]

(12) Domain of $\cos x$ is:

- (a) $\mathbb{Z}$
- (c) $\mathbb{R}$

- (b) $\mathbb{Q}$
- (d) None of these

[Lahore Board 2006]

(13) Period of $y = \sec x$ is:

- (a) π
- (c) 3π

- (b) 2π
- (d) $\frac{\pi}{2}$

[Lahore Board 2006]

(14) Period of $\tan\left(\frac{x}{7}\right)$ is:

- (a) π
- (c) 7π

- (b) $\frac{\pi}{7}$
- (d) $\pi + 7$

[Gujranwala Board 2007]

(15) The period of $\tan 4x$ is:

- (a) $\frac{\pi}{4}$
- (c) π

- (b) $\frac{\pi}{2}$
- (d) 2π

[Gujranwala Board 2007]

(16) Period of $\sin 3x$ is:

- (a) 3π
- (c) $\frac{\pi}{3}$

- (b) 2π
- (d) $\frac{2\pi}{3}$

[Lahore Board 2007]

(17) Domain of $y = \cos x$ is:

- (a) $-\infty < x < +\infty$
- (c) $0 < x < \infty$

- (b) $-1 \leq y \leq 1$
- (d) $-\infty < x < 0$

[Lahore Board 2008]

(18) Period of sine function is:

- (a) π
- (c) 2π

- (b) $-\pi$
- (d) 0

[Lahore Board 2008]

(19) The period of $\cos x$ is:

- | | |
|------------|---------------------|
| (a) π | (b) 2π |
| (c) 3π | (d) $\frac{\pi}{2}$ |

[Gujranwala Board 2008]

(20) Range of $y = \sin x$ equals:

- | | |
|----------------------------|-------------------------------|
| (a) $-1 < y < 1$ | (b) $y \geq 1$ or $y \leq -1$ |
| (c) $-\infty < y < \infty$ | (d) $-1 \leq y \leq 1$ |

[Gujranwala Board 2008]

(21) Domain of $\cot \theta =$

- | | |
|---|---|
| (a) $-\infty < \theta < \infty, \theta \neq n\pi, n \in \mathbb{Z}$ | (b) $-\infty < \theta < \infty, \theta \neq \left(\frac{2n+1}{2}\right)\pi$ |
| (c) $-1 < \theta < 1$ | (d) $\theta \geq 1$ or $\theta \leq -1$ |

[Lahore Board 2010]

(22) The range of $y = \cos x$ is:

- | | |
|------------------------|-----------------------------|
| (a) $-1 \leq x \leq 1$ | (b) $-\infty < x < +\infty$ |
| (c) $-1 \leq y \leq 1$ | (d) $-\infty < y < +\infty$ |

[Lahore Board 2010]

(23) The smallest positive integer P for which $f(P+x) = f(x)$ is called:

- | | |
|---------------|------------|
| (a) Domain | (b) Range |
| (c) Co-domain | (d) Period |

[Gujranwala Board 2010]

(24) The domain of $\sin x$ is:

- | | |
|------------------|--|
| (a) $[-1, 1]$ | (b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ |
| (c) $\mathbb{R}$ | (d) $\mathbb{Q}$ |

[Lahore Board 2011]

(25) The graphs of trigonometric functions, within their domains are:

- | | |
|-----------------|-------------------|
| (a) Continuous | (b) Discontinuous |
| (c) Co-terminal | (d) None of these |

(26) Domain of $y = \cos x$ is:

- | | |
|--|---|
| (a) $-\infty < x < \infty$ | (b) $-1 \leq x \leq 1$ |
| (c) $\mathbb{R} - \{x x = n\pi\} n \in \mathbb{Z}$ | (d) $\mathbb{R} - \{x x = 2n\pi\} n \in \mathbb{Z}$ |

[Lahore Board 2012]

- (27) The period of $\tan \frac{x}{7}$ is:
- (a) π (b) 7π
 (c) $\frac{\pi}{7}$ (d) $\pi + 7$ **[Lahore Board 2012]**
- (28) The period of $\tan \frac{x}{3}$ is:
- (a) π (b) 2π
 (c) 3π (d) None of these **[Lahore Board 2013]**
- (29) Period of $3 \sin 3x$ is:
- (a) 2π (b) $\frac{2\pi}{3}$
 (c) $\frac{3\pi}{2}$ (d) 3π **[Lahore Board 2013]**
- (30) In the graphs of $\tan x$; the dashes lines are:
- (a) Horizontal asymptotes (b) Lines
 (c) Vertical asymptotes (d) Periodic
- (31) Point $(30^\circ, 0.87)$ lies on:
- (a) $y = \cos x$ (b) $y = \sin x$
 (c) $y = \tan x$ (d) None of these
- (32) Trigonometric function $y = \operatorname{cosec} x$ meet x-axis at $x =$
- (a) 30° (b) 60°
 (c) 90° (d) None
- (33) All the trigonometric functions repeat their value. This behaviour is called:
- (a) Period (b) Periodicity
 (c) Continuity (d) None of these
- (34) $\sin(2\pi + x) =$
- (a) $\sin(x + 4\pi)$ (b) $\sin(x + 3\pi)$
 (c) $\cos(2\pi + x)$ (d) $\cos(3\pi + x)$
- (35) Period of $\cos \frac{x}{2} =$ _____
- (a) π (b) 2π
 (c) 4π (d) 6π **[Lahore Board 2014]**
- (36) Period of $\sin \frac{x}{3}$ is:
- (a) π (b) 3π
 (c) $\frac{2\pi}{3}$ (d) 6π **[Lahore Board 2014]**

Answers

(1)	d	(2)	a	(3)	a	(4)	c	(5)	b
(6)	a	(7)	c	(8)	b	(9)	a	(10)	a
(11)	a	(12)	c	(13)	b	(14)	c	(15)	a
(16)	d	(17)	a	(18)	c	(19)	b	(20)	d
(21)	a	(22)	c	(23)	d	(24)	c	(25)	a
(26)	a	(27)	b	(28)	c	(29)	b	(30)	c
(31)	a	(32)	d	(33)	b	(34)	a	(35)	c
(36)	d								

SOLUTION

Q.1 As period of $\sin x$ is 2π .

$$\begin{aligned}\Rightarrow 3\sin \frac{x}{3} &= 3\sin \left(\frac{x}{3} + 2\pi \right) \\ &= 3\sin \frac{1}{3}(x + 6\pi)\end{aligned}$$

$$\Rightarrow \text{Period of } 3\sin \frac{x}{3} \text{ is } 6\pi.$$

Q.2 As range of $\sin x$ is $[-1, 1]$

$$\Rightarrow \text{range of } 7\sin x \text{ is } [-7, 7]$$

Q.10 As period of $\cos x$ is 2π .

$$\begin{aligned}\Rightarrow \cos \frac{x}{5} &= \cos \left(\frac{x}{5} + 2\pi \right) \\ &= \cos \frac{1}{5}(x + 10\pi)\end{aligned}$$

$$\Rightarrow \text{Period of } \cos \frac{x}{5} \text{ is } 10\pi.$$

Q.11 The period of $\cot \pi x$ is π .

$$\begin{aligned}\Rightarrow \cot \pi x &= \cot(\pi x + \pi) \\ &= \cot \pi (x + 1)\end{aligned}$$

Period of $\cot \pi x$ is 1.

Q.29 Put, $(30^\circ, 0.87)$ in $y = \cos x$

$$\Rightarrow 0.87 = \cos 30^\circ$$

$$\Rightarrow 0.87 = 0.87 \text{ (Satisfy)}$$

$$\Rightarrow (30^\circ, 0.87) \text{ lies on } y = \cos x$$

Q.30 The trigonometric function $y = \operatorname{cosec} x$ does not meet x -axis at any point.

Q.32 As period of $\sin x$ is 2π .

$$\begin{aligned}\Rightarrow \sin(2\pi + x) \\ &= \sin(x + 2\pi) \\ &= \sin(x + 2\pi + 2\pi) \\ &= \sin(x + 4\pi)\end{aligned}$$

Q.35 As period of $\cos x$ is 2π .

$$\cos x = \cos(2\pi + x)$$

$$\begin{aligned}\cos \frac{x}{2} &= \cos \left(\frac{x}{2} + 2\pi \right) \\ &= \cos \frac{1}{2}(x + 4\pi)\end{aligned}$$

Period is 4π .

Q.36 As period of $\sin x$ is 2π .

$$\begin{aligned}\sin \frac{x}{3} &= \sin \left(\frac{x}{3} + 2\pi \right) \\ &= \sin \frac{1}{3}(x + 6\pi)\end{aligned}$$

Period is 6π .